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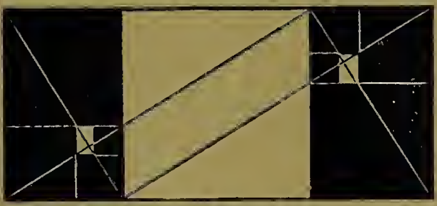
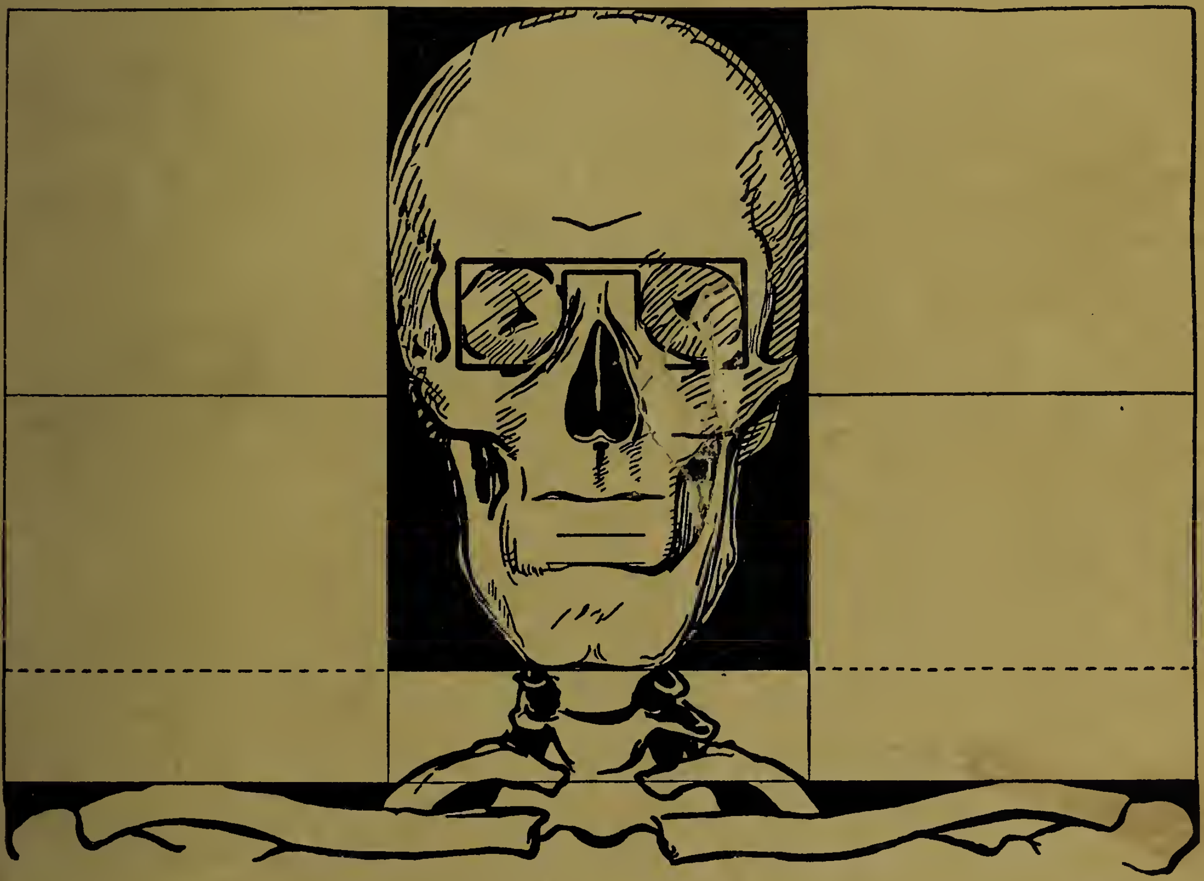
THE DIAGONAL

AN ILLUSTRATED MONTHLY MAGAZINE DEVOTED TO THE EXPLANATION OF THE REDISCOVERED PRINCIPLES OF GREEK DESIGN, THEIR APPEARANCE IN NATURE AND THEIR APPLICATION TO THE NEEDS OF MODERN ART

EDITED BY JAY HAMBIDGE





FRONT ELEVATION OF A SKULL FROM THE HARVARD MEDICAL SCHOOL
See article on the Human Figure in this number of THE DIAGONAL

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THE DIAGONAL

FOREWORD

THE basic principles underlying the greatest art so far produced in the world, may be found in the proportions of the human figure and in the growing plant. These principles have been reduced to working use and are being employed by a large number of the leading American artists, designers, and teachers of design and manual art. After two years lecturing at Yale and Harvard Universities and to special classes of artists, sculptors, designers and teachers in New York City and Boston, the discoverer of these principles, Mr. Jay Hambidge, is now lecturing in Europe and gathering from the museums there, fresh material for the artists and students of America. This material includes the designs to be found in the surviving masterpieces of classic art, such as bronzes, pottery, decoration, architecture and statuary. The principal results of this gleaning of the Museums of the old world will appear monthly in THE DIAGONAL accompanied by especially prepared lessons.

The principles of design to be found in the architecture of man and plant have been given the name "Dynamic Symmetry." This symmetry is identical with that used by Greek masters in almost all the art produced during the great classical period.

The synthetic use of these design principles is simple. The Greeks probably used a string held in the two hands. The Harpedonaptae or rope-stretchers of Egypt, had no other instrument for orientating and surveying or laying out temple plans. The recovery of these design principles by analysis is difficult, requiring special talent and training, considerable mathematical ability, much patience and sound aesthetic judgment. The analysis of the plan of a large building, such for example as the Parthenon, often is not so difficult as the recovery of the plans of many minor design forms. Sometimes a simple vase is most baffling, requiring days of intensive inspection before the design theme becomes manifest. To recover these themes of classic design it is necessary to use arithmetical analysis. Geometrical analysis is misleading and inexact. Necessity compelled the old artists to use simple and understandable shapes to correlate the elements of their design fabrics. It is evident that even the simplest pattern arrangement can become very complicated as a design develops. A few lines, simple as a synthetic evolution, may tax the utmost ingenuity to analyze.

The determination of the form principles in a specific example of design means, in a sense, the elimination of the personal element. With this element removed the residue represents merely the planning knowledge possessed by the artist. This residue is sometimes meager and more or less meaningless; often it is rich in suggestion and positive design knowledge. Invariably the higher or more perfect the art, the richer is the remainder when the personal element is removed. Also the degree of planning knowledge is positive evidence of the conscious or unconscious use of a scheme in a work of art. Saracenic, Mahomedan, Chinese, Japanese, Persian, Hindu, Assyrian, Coptic, Byzantine, or Gothic art analysis shows unmistakably the conscious use of plan schemes and all belong to the same type. Greek and Egyptian art analyses show an unmistakable conscious use of plan schemes of another type. There is no question as to the relative merit of the two types. The latter is immeasurably superior to the former. This is made manifest as soon as the two types are tested by nature. These plan schemes, which we find so abundantly in art, are nothing more than symmetry, using the word in the Greek sense of analogy; literally it signifies the relationship which the composing elements of form in design, or in an organism in nature, bear to the whole. In design it is the thing which governs the just balance of variety in unity. The investigation of this impersonal aspect of

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art in relation to the symmetry of natural form was begun some twenty years ago. The results of the labor showed clearly that there were but two types of symmetry in nature which could be utilized in design. One of these types, because of its character, was termed "static," the other "dynamic." Possibly the former is but a special case of the latter, as a circle for example, is a special case of an ellipse. At any rate there is no question of the superiority of the dynamic over the static. The static is the type which is used consciously or unconsciously in art. In fact, no design is possible without symmetry. The savage decorating his canoe or paddle, his pottery or his blanket, uses static symmetry unconsciously. The crude drawings of the cave-men disclose no design, consequently, no symmetry. As civilization advances the artist becomes more or less conscious of the necessity for symmetry. When we reach a period which is recognizable as an art epoch, when a people's character is shot through and through a great design fabric and the result stands as a national aesthetic expression, we find, invariably, a highly sophisticated use of symmetry. But still it is almost always static. It is impossible to use dynamic symmetry unconsciously. Curiously, there were but two peoples who did use dynamic symmetry, the Egyptians and the Greeks. It was developed by the former very early as a empiric or rule-of-thumb method of surveying. Possibly the date is as early as the first or second dynasty. Later it was taken over as a means of plan making in architecture and design in general. The Egyptians seemed to attach some sort of ritualistic significance to the idea as it is found used in this sense in temple and tomb, particularly in the bas-reliefs which were used so plentifully to adorn these. It is also curious that the Hindus, about the fifth or eighth century B. C., possessed a slight knowledge of dynamic symmetry. A few of the dynamic shapes were actually worked out and appear in the Sulvasutra, literally "the rules of the cord," and were part of a sacrificial altar ritual. But to what extent it may have been used in Hindu art is not known, because examples containing its presence have disappeared. The Greeks obtained knowledge of dynamic symmetry from the Egyptians some time during the sixth century B. C. It supplanted, probably rapidly, a sophisticated type of static symmetry then in general use. In Greece, as in India and in Egypt, the scheme was connected with altar ritual. Witness the Delian or the Duplication of the Cube problem. The Greeks, however, soon far outstripped their Egyptian masters and, within a few years after acquiring the knowledge, apparently made the astounding discovery that this symmetry was the symmetry of growth in man. According to Vitruvius the Greeks learned symmetry from the human figure and were most particular in applying it to their works of art, especially to their temples. This, however, is not more reliable than other Vitruvian statements. The Roman architect had no knowledge of symmetry beyond a crude form of the static. He declared that the Greeks used a modulus to determine the symmetry of their temples and gives most elaborate instructions as to how the plans were developed. No Greek design has been found which agrees with the Vitruvian statements. In fact, the modulus would produce a grade of static symmetry which would have afforded much amusement to a Greek.

THE DYNAMIC SYMMETRY OF THE PLANT

It has long been known that a peculiar series of numbers is connected with the phenomenon of the orderly distribution of the leaves of plants. The series is: 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, etc. This is called a summation series from the fact that each term is composed of the sum of the two preceding terms. For those who may be interested in the subject from the botanical standpoint reference is made to the elaborate work of Professor A. H. Church of Oxford on "Phyllotaxis in Relation to Mechanical Law." This series of numbers in reality represents the symmetry of the plant in the sense that the word is used as "analogy," which is its Greek meaning. From this series we obtain the entire machinery of dynamic symmetry, which applies not only to the architecture of the plant but to the architecture of man.

This summation series of numbers, because of its character, represents a ratio, *i. e.*, it is a

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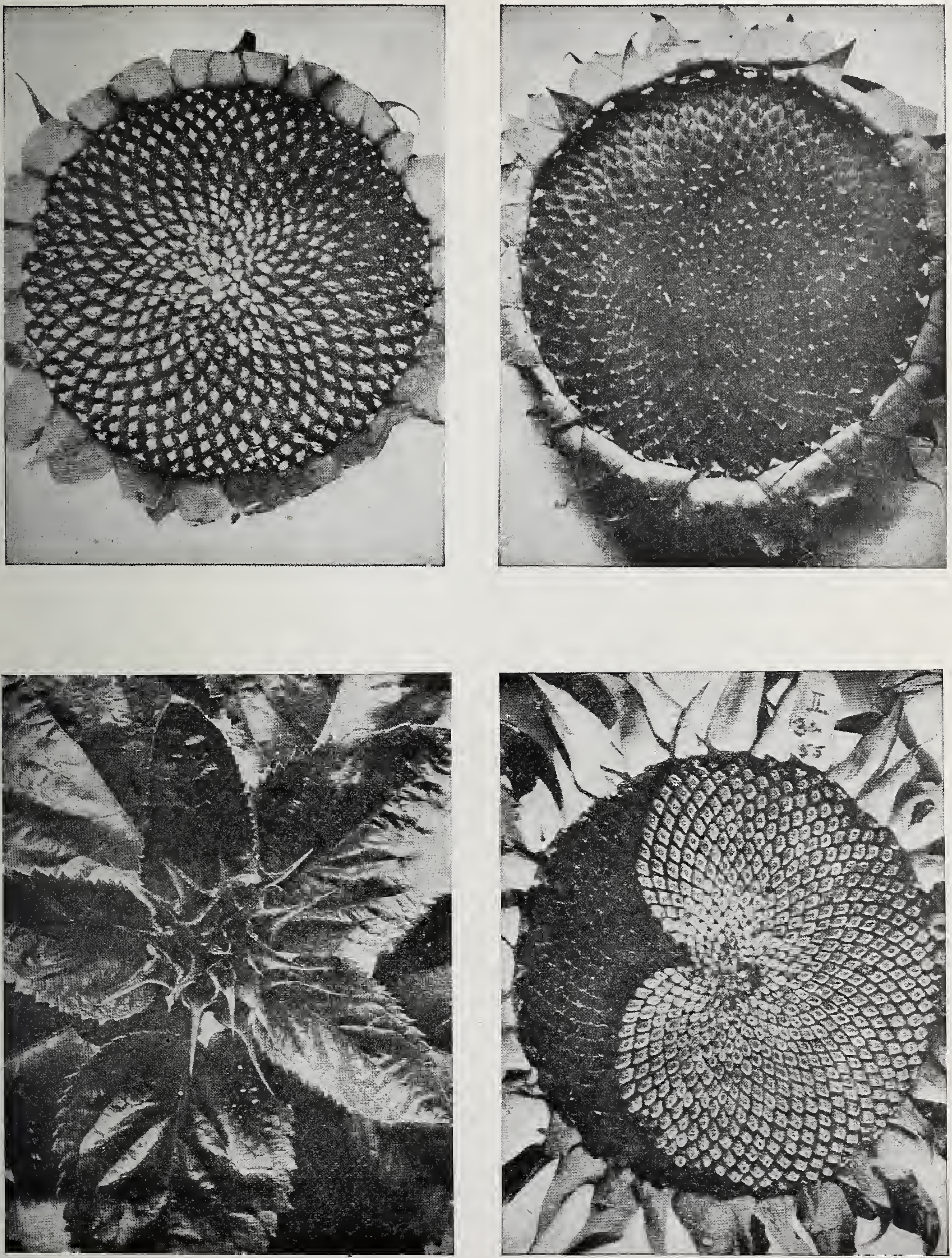
v.1



Sunflower heads used by Professor Church to exhibit the phyllotaxis phenomena geometrical progression. This ratio may be obtained by dividing one term into another, such as 34 into 55. The series does not represent the phenomenon exactly but only so far as it is representable by whole numbers. A much closer representation would be obtained by a substitute series such as: 118, 191, 309, 500, 809, 1309, 2118, 3427, 5545, 8972, 14517, etc. One term of this series divided into the other equals 1.6180, which is the ratio necessary to explain the symmetry of the plant design system.

The operation of the law of leaf distribution and its connection with the summation series of numbers is explained by Professor Church, who uses as illustration the disk of the sunflower. See page 3.

"The most perfect examples of phyllotaxis easily obtainable are afforded by the common sunflower, so frequently selected as a typical Angiosperm, both in anatomical and physiological observations, owing to the fact that it exhibits, *par excellence*, what is regarded as a normal structure little modified by specialization for any peculiar environment. Not only is the sunflower a leading type of the Compositae which holds the highest position among Angiosperm families, but amongst this family it flourishes in the best stations, in which sunlight, air, and water supply are perhaps at an optimum for modern vegetation. The very fact that it is as near an approximation to the typical Angiosperm as can perhaps be obtained, suggests that the phenomena of growth exhibited by it will also be normal, and from the time of Braun to that of Schwendener it has afforded a classical example of spiral phyllotaxis."

The sunflower heads "admit of ready observation. By taking a head in which the last flowers are withering, and clearing away the corolla tubes, the developing ovaries are seen to mark out rhomboidal facets, and when the fruits are ripened, and have been shed, the subtending bracts still form rhomboidal sockets. These sockets, with or without fruits, form a series of intersecting curves identical with those of the pine cone, only reduced to a horizontal plane.

"A fairly large head, 5-6 inches in diameter in the fruiting condition, will show exactly 55 long curves crossing 89 shorter ones. A head slightly smaller, 3-5 inches across the disk, exactly 34 long and 55 short; very large 11-inch heads give 89 long and 144 short; the smallest tertiary heads reduce to 21-34, and ultimately 13-21 may be found; but these, being developed late in the season, are frequently distorted and do not set fruit well.

"A record head grown at Oxford in 1899 measured 22 inches in diameter, and, though it was not counted, there is every reason to believe that its curves belonged to a still higher series, 144-233. The sunflower is thus limited in its inflorescence to certain set patterns, according to the strength of the inflorescence axis, *e.g.*, 13/21, 21/34, 34/55, 89/144. These were first observed by Braun (1835), and translated into terms of the Schimper-Braun series they would correspond to divergences of 13/34, 21/55, 34/89, 55/144, and 89/233, respectively. Under normal circumstances of growth, the ratio of the curves is practically constant. (Cf. Weisse. Out of 140 plants 6 only were anomalous, the error being thus only 4 per cent.)"

The ratio 1.6180, when reduced to a rectangular area makes a rectangle which has been given the name by the writer, of the "rectangle of the whirling squares." For a description of a root-five rectangle and the rectangle of the whirling squares, see lesson on the elements of symmetry in this number of The Diagonal.

[To be continued.]

DYNAMIC SYMMETRY OF THE HUMAN FIGURE FOR ADVANCED STUDENTS

Τὸ γὰε εὖ παεὰ μικεὸν διὰ πολλῶν ἀειδμῶν ἔφη γίγνεσδαι.—Philon *πεοὶ βελοποιϊκῶν* iv. 2.

Beauty, he said, was produced from a small unit through a long chain of numbers.
H. Stuart Jones, "Ancient Writers on Greek Sculpture," p. 129.

"*The earlier stages in sculpture are not so important. The real work of the true artist begins when it comes to the finest last point of finish, the finger nail point, when the clay adheres to the finger nail. There are the ordinary workers in any sphere and those who really advance their art in the world, those with high ideals, who must go further than what is "just good" or "just good enough," who use their ideal as if it were a measure, and give the most difficult last touch of perfection which makes the great work.*" Plutarch.

"*Chrysippos holds beauty to consist in the proportions not of the elements but of the parts, that is to say, of finger to finger, and of all the fingers to the palm and wrist, and of these to the forearm, and of the forearm to the upper arm, and of all the parts to each other, as they are set forth in the Canon of Polykleitos. For Polykleitos when he had taught us all the proportions of the human figure by means of that treatise, confirmed his theory by a practical illustration and made a statue according to the dictates of the theory, and called the statue, like the treatise, his 'Canon.'*" H. Stuart Jones, "Ancient Writers on Greek Sculpture," pp. 128-9.

THIS series of articles will be devoted to the explanation of the architecture of the human figure as it is disclosed by dynamic symmetry. The plan embraces an examination of the human structure in all its details, both in nature and in art. For this purpose the human skeleton will be first considered, then the live figure, both in repose and in action. Finally, Greek sculpture, from the artist's standpoint.

A rough outline of the history of man's attempts to reduce the human figure to some scheme which would help the artist to overcome the tremendous mechanical difficulty involved in securing adequate or just relationship between the members of the body and the structure as a whole, is as follows:

A number of Greek architects and sculptors are supposed to have written books upon the symmetry of the human figure, but these have all been lost.

Two supposed sayings by Polykleitos have survived. They are quoted above.

According to Plutarch, Polykleitos thought that excellence in sculpture consisted in the degree of perfection to which the statue could be carried after its plan was laid out or its proportions determined. This point of view seems strange to us, because we know from bitter experience that the proper adjustment of the proportions of a human figure, or indeed any design, is more difficult than the finish. If the Plutarch statement is true, then, what to us is an almost unsurmountable difficulty, to the Greek was a comparatively simple task. Dynamic symmetry shows us, however, that the Plutarch statement has in it an element of truth, because it describes the condition which would follow a use of the dynamic scheme for figure construction. The only work that has survived which treats of Greek symmetry is that of Vitruvius, a Roman architect and author of the first century B. C. He describes an elaborate scheme for determining the symmetry of temples, theatres and other buildings in the Greek style. Architectural symmetry, he says, was obtained by the Greeks, from the human figure; the determination of the principles involved followed the discovery that the members of the human structure were commensurate with the body as a whole.

The Roman reduces the symmetry concept to a modulus and this modulus or unit of length is used by him to determine the proportions of the composing units of an architectural design. He also reduces the human figure to a modulus. The Vitruvian rules, however, have been entirely discredited, because no Greek structure has been found to agree with them. That the modulus idea cannot be applied to the human figure is evident from the fact that no individual has been found whose overall plan can be determined by even multiples, or even fractional divisions of a multiple, of any member.* The recovery of the symmetry themes of Greek design shows us that the Roman writer was either misled or he wrongly interpreted the Greek tradition. The members of the human structure are, indeed, commensurate with the whole, but *not in line*. The Romans knew nothing of the Greek concept of the "application of areas" and the reduction of two-dimensional form to understandable commensurate units of area. Herein is the secret of dynamic design. Nature, in the architecture of man and plant, never uses commensurate lengths, but always employs commensurate areas. The Roman idea of the modulus, however, has persisted to our time in spite of the fact that it means nothing to design. The use of the modulus idea automatically produces static symmetry, and, as far as the present investigation has proceeded, the static type has not been found in man.

Later in this series of articles, when Greek statuary is considered in detail, the canons of proportion used in the French studios during the past hundred years will be examined as well as certain alleged recoveries of Egyptian schemes of figure construction. Present discussion of these phases of the subject would lead to confusion. It may be said in passing, however, that, without exception, all of the schemes which have been proposed since the first century B. C. would lead to static results.

With the idea of the correct drawing of the human figure clearly defined as essentially a matter of a type of symmetry, a mode of procedure suggests itself. It will be necessary, at first, to become familiar with the units of areas which dynamic symmetry discloses. To do this as rapidly as possible the student must familiarize himself with the metric scale. This scale is a natural scale, carefully worked out by scientists, and is related to the circumference of the earth itself. The area shapes of dynamic symmetry consist of a number of rectangles, and the relation of the ends and sides of these produce a set of ratios. For the present these ratios are used to identify the rectangles. As an illustration; if a rectangle is found which measures 2.36 by 4.27 we consider the smaller measurement as unity and divide it into the larger number to find the ratio. This is 1.809. This ratio is immediately recognizable as a familiar shape of the dynamic symmetry series. Again, if a rectangle measures 472 by 528, we find the ratio by dividing the lesser into the greater. In this case it is 1.118. Again, this ratio is recognized as a very important shape belonging to the dynamic series. If we measured a model and found the full stature, reading from the metric scale, was 472 and the full stretch of the arms, from middle finger tip to middle finger tip was 528, we would know that the rectangle of the great span, the Grande Envergere of the French, *i. e.*, the full wing spread, was a 1.118 or a double root-five rectangle. This rectangle is encountered in the architecture of the first skeleton to be analyzed in these lessons. When the student becomes familiar with the dynamic shapes he will also recognize the actual measurements of length 472 and 528, as also representing well-known ratios which will be encountered time and again in the human figure. He will also discover that the identical shapes which he finds so plentifully in the human structure were repeatedly used, daringly and masterfully, by the Greeks in the creation of their wonderful designs, not only in sculpture, but in architecture and the crafts. So far over five hundred Greek design themes have been recovered from the vases alone. These themes have been recovered in quantity, not only by the writer but by skilled, competent and unprejudiced investigators.

Having grasped the basic idea that the architecture of man and plant can be understood from a projection, *i. e.*, from a two-dimension drawing, that the projection always represents some arrangement of two-dimension form within a rectangle and that these rectangles, individually and as a group, possess marvelous properties of proportional subdivisions, the student will be prepared to investigate these area shapes for himself and to apply the knowledge thus obtained to his design needs.

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*"The canon of Vitruvius adopted by Gerdy and Cousin exists only in the imagination of the authors," and again, "the span of the arms used by artists is absolutely false, for it is equal to the height only once in every ten cases." DR. PAUL TOPINARD, "Anthropology."

Before beginning the area analysis of a specific skeleton, it may be said that the average student, by the time he has completed an ordinary school course of figure drawing, wherein dynamic symmetry is used instead of the methods at present employed, should be able to draw the human form accurately and well, in any position or action, and, if necessary, without the use of a model. Also, it may be said that until an artist can so draw the figure, design accomplishment of a high order with the human motif, is impossible. As far as great design is concerned, especially mural design, the usual art school methods of instruction have failed. The fact that no great figure design is being produced or has been produced for several hundred years, shows this. We moderns have lost the art of creating compositions of great pictorial force. The mere copying of the superficial aspects of a model will not produce the pictorial power that the well-understood construction of the human figure produces.

It may also be said that human anatomy, as it is now taught in art schools and has been taught for many generations, is of little if any use to the artist. The only anatomy the artist needs is that of a type which brings out the larger pattern forms of the structure of the human frame. Surgical anatomy leads to an overstress of detail and a violent squirm and twist of the figure incompatible with noble design. The Greeks used the correct type of anatomy. But we must not study Greek anatomy by itself; we must study nature and use Greek knowledge to explain something of the subtlety of construction and the beauty of man's structure.

In the Museum of the Harvard Medical School, at Boston, there is the complete skeleton of a man. It is called a typical male skeleton. However, it is not in any sense typical, indeed, very much the contrary. The subject was prepared and mounted in Paris by a well-known expert. In life the man must have been nearly six feet high and strongly individualized. His arms are unusually long and the hands are large, so that his great span, middle finger tip to middle finger tip with arms outstretched, with his full height, makes a double root-five rectangle. The side elevation of the skull is handsome. Every bone and almost all the teeth are perfect. The great span ratio or index for the average man, which is determined by dividing the height into the span, is, according to actual measurements of thousands of individuals, 1. to 1.045. The span ratio of this subject is 1. to 1.118. This ratio is the square root of five divided by two; 2.236 divided by 2 equals 1.118. This is the ratio of a rectangle which was used repeatedly by Greek artists, as is shown by some of the finest examples of design which have survived. The rectangle which encloses the side elevation of the head is also a double root-five rectangle. The rectangle which encloses the torso and head is a root-five rectangle. (For explanation of a root-five rectangle, see lesson in this number.) Moreover, the length of every bone in the example is expressible, arithmetically, in figures of the dynamic symmetry series if the measurements are made with a metric scale. For this reason the reader will be careful not to confuse the length measurements with the rectangles of the system. For example, the distance from the top of the head, the vertex, to the shoulder, the clavicle, reading from a metric scale, is .2618±. The symbol means plus or minus. The reading might be .2617 or .2619, but it might as well be .2618, because it would be impossible to show the difference by a pencil line. This number does not mean the rectangle 2.618, unless it is used with unity to form such a shape. The width of the shoulders reads, metrically, .4045. For the same reason this is not the rectangle .4045. To obtain the rectangle defined by the vertex-clavicle length and the width of the shoulder .2618 must be divided into .4045. This establishes the rectangle 1.545, an important shape of the dynamic series. That we are justified in making the reading .2618 is shown when the width of the front elevation of the skull is considered. This reads .1427. If this number is subtracted from .4045, the width of the shoulders, the remainder is .2618. It is now plain that the rectangle .2618 by .4045 is composed of a square plus a shape 1427 by 2618. This rectangle exactly contains the head and neck. (Fig. 1.) But the head and neck rectangle must be placed in the middle of the 1.545 rectangle. .1427 divided into .2618 furnishes a 1.8345 rectangle. It is now clear that the arrangement is as Fig. 1, *i. e.*, a 1.8345 rectangle with two squares on either side. The height of the skull reads from the metric scale, .2236. This figure is

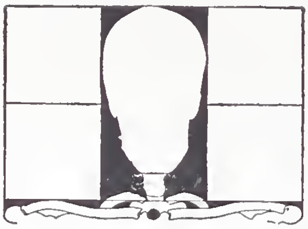


Fig. 1

the figure of the square root of 5, but in this case it is simply a length measurement. If we make a new rectangle, using the height of the skull and the width of the shoulders, .2236 by .4045, the result is 1.809. (.4045 divided by 2.236 equals 1.809.) Fig. 2*a*.

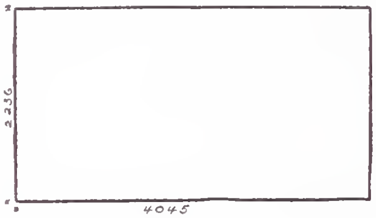


Fig. 2a

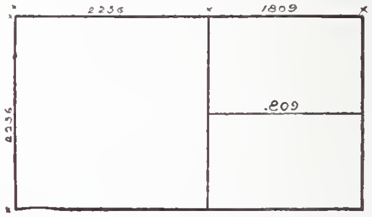


Fig. 2b

The width of the frontal head elevation is .1427, the height .2236, the lesser measurement divided into the greater, .1427 into .2236 equals 1.5669 the ratio of the front elevation of the skull.

Summing up, we have for the proportions of a specific human skeleton, as far as we have proceeded, a series of rectangles which evidently belong to a system of some sort. These are:

	mm.		mm.		ratio	
a shape	2618	by	4045	producing by division	1.545	plus
a shape	1427	by	2618	producing by division	1.8345	plus
a shape	2236	by	4045	producing by division	1.809	plus
a shape	1427	by	2236	producing by division	1.5669	plus

These ratios cannot be expressed exactly in arithmetical terms, because they are never ending fractions. It will not be necessary in using the scale, however, to go beyond the third or fourth decimal, because we cannot read closer than this. As areas, though, it is apparent that the troublesome never-ending fraction disappears, and the shapes may be constructed easily with the simplest instruments. It is doubtful whether the Greeks used a more complex instrument than a string and pins or a compass. It is also doubtful whether any one Greek designer ever knew the entire system of dynamic shapes. It would seem that the average classical artist's knowledge of symmetry was confined to a few shapes which were used over and over again and combined and varied in many ways, the craftsman not understanding anything whatever of the arithmetic connected with the subject. It is recommended, however, that the modern artist take advantage of simple arithmetic and use it as a corrective for geometrical construction.

The total height of the skeleton now being discussed, measured in meters, is 1.7662. The total width of the shoulders .4045. The lesser measurement divided into the greater produces the ratio 4.3664. The height of the torse, with the skull and neck, is .9045. The shoulder width .4045. The lesser divided into the greater supplies the ratio 2.236, or the square root of five. The torse and head of the skeleton, therefore, are exactly enclosed by a root-five rectangle.

The height of the pelvis is .2382; the distance from the clavicle or top of the shoulders to the top of the head is .2618. These two measurements added equal .5. If from .9045 the measurement .5 is subtracted the remainder .4045 equals the shoulder width. Therefore, in this root-five rectangle of the torse and head, if the pelvis height of .2382 and the head and neck height of .2618 be subtracted, there remains an area .4045 by .4045, or a square. Therefore, the head and neck height .2618 added to the pelvis height .2382 and divided by the shoulder width, produces two whirling square rectangles. The square root of five, 2.236 minus 1., in dynamic symmetry a square, equals 1.236.

The length of the upper arm bone is .3618; the lower arm bone .2618; the hand and wrist .2118; added these lengths make .8354; multiplied by 2 they equal 1.6708. When the arms are in position to produce the great span the distance through the chest, from the end of one upper arm bone to the other, is .3038; this measurement added to 1.6708 equals 1.9746. The total height of the figure, 1.7662 divided into the total arm spread 1.9746, furnishes the ratio 1.118. This ratio is an important one of the dynamic series and represents two root-five rectangles standing side by side. The height of the skull is .2236; the width of the side elevation .2; the lesser into the greater equals the ratio 1.118. The rectangle of the side elevation of the skull, therefore, is similar to the rectangle of the grand span, each being composed of two root-five rectangles standing side by side.

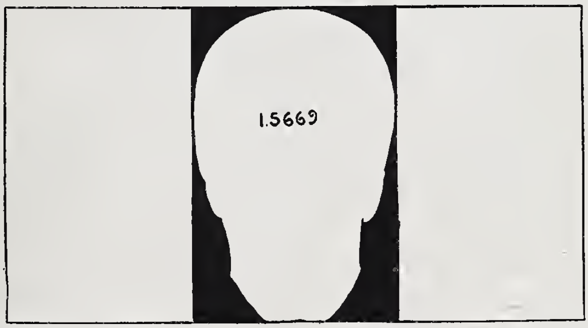


Fig. 3a—Front elevation of the skull and the ratio of its rectangle

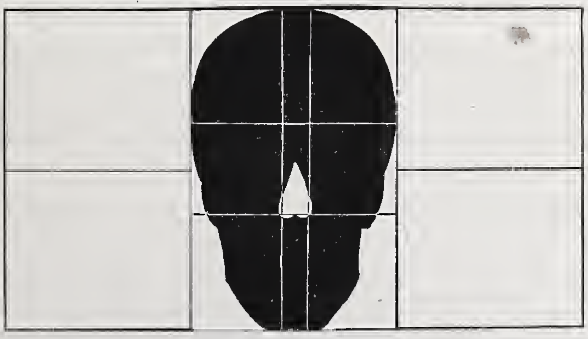


Fig. 3b—The lines drawn across the front elevation of this skull divide that area into a series of remarkable proportions. This diagram will be explained in the next article.

[To be continued.]

THE DIFFERENCE BETWEEN DYNAMIC AND STATIC SYMMETRY

SYMMETRY is that quality in a work of art or craft which we recognize as design. Static symmetry, as the name implies, is a symmetry which has a sort of fixed entity or state. It is the orderly arrangement of units of form about a center or plane as in the crystal. A snow crystal furnishes a perfect example. It is apparent in cross sections of certain fruits. Diatoms and radiolaria supply other examples. In art it is the symmetry used by all peoples except the Egyptians and the Greeks, although a small percentage is found in the design products of these two nations. The static is the spontaneous type; *i. e.*, an artist or craftsman may use it unconsciously. Static symmetry, as used by the Copts, Byzantines, Saracens, Mahomedans, and the Gothic and Renaissance designers, was based upon the pattern properties of the regular two-dimensional figures such as the square and equilateral triangle. The static symmetry used by the Greeks before they obtained knowledge of dynamic symmetry depended upon an area being divided into even multiple parts such as a square and a half, three-quarters, one-quarter, one-third, two-thirds, etc. The Tenea Apollo, a sixth century B. C. Greek statue, has a symmetry theme of three and two-thirds. This means that if a rectangle is made the area of which consists of three squares and two-thirds of a square it will be exactly the shape which will contain the projection of the statue. When we find that every member of the body is expressible in terms of this shape and that the theme produces a simple pattern form throughout we decide that this Apollo belongs to the static class. It is well understood that the Archaic Apollo statues of Greece closely followed an Egyptian prototype. A statue of Amenophis IV, of the 14th or 15th century B. C., which is such a prototype, discloses, quite unmistakably, a dynamic theme. Diodorus Siculus, the Sicilian Greek historian, says the early Greeks obtained their sculptural knowledge from Egypt and tells the story of a certain Rhoecus, a great sculptor who learned his art in Egypt, who had two sons, Telecus and Theodorus, who like their father, were sculptors. One of these sons worked at Samos, the other at Ephesus and between them they made a statue according to a prearranged plan. When the two parts of the figure were brought together they fitted exactly, so the statue appeared the work of one man. Stories of this character suggest that the poet was possibly near the truth when he said:

"From Egypt arts their progress made to Greece
Wrapt in the fable of the Golden Fleece."

Now that dynamic symmetry supplies a means by which we may closely inspect any design and classify it according to its symmetry theme, we see that the Tenean Apollo, though closely resembling the outward aspect of an Egyptian prototype, such as the statue of Amenophis IV, is in reality, as far as its symmetry is concerned, quite a different thing. The archaic Apollo sculptors of Greece apparently did not have knowledge of the Egyptian symmetry secret when the Tenea figure was made. When we find that the identical symmetry theme disclosed by the Tenea statue was used by Greek designers to a limited extent, less than five per cent, throughout the classical period, we conclude that this represents the work of a small number of designers who, apparently, did not know the dynamic scheme. Probably they did not belong to the craft guilds. After the decline of Athens and during the Hellenistic Age, we find this Tenea figure or static symmetry type, used more and more until by the first century B. C., it is the only type observable; the dynamic type in all design entirely disappears at this date.

Dynamic symmetry in nature is the type of orderly arrangement of members of an organism such as we find in a shell or the adjustment of leaves on a plant. There is a great difference between this and the static type. The dynamic is a symmetry suggestive of life and movement.

Its great value to design lies in its power of transition or movement from one form to another in the system. It produces the only perfect modulating process in any of the arts. This symmetry cannot be used unconsciously although many of its shapes are approximated by designers of great native ability whose sense of form is highly developed. It is the symmetry of man and the plant, and the phenomenon of our reaction to Greek classic art and to certain fine forms of other art is probably due to our subconscious feeling of the presence of the beautiful shapes of this symmetry.

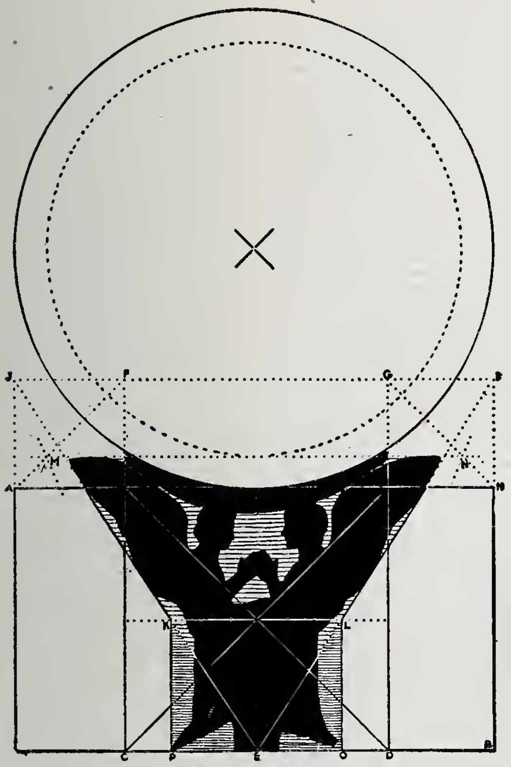


Fig. 4a—Greek Bronze Mirror

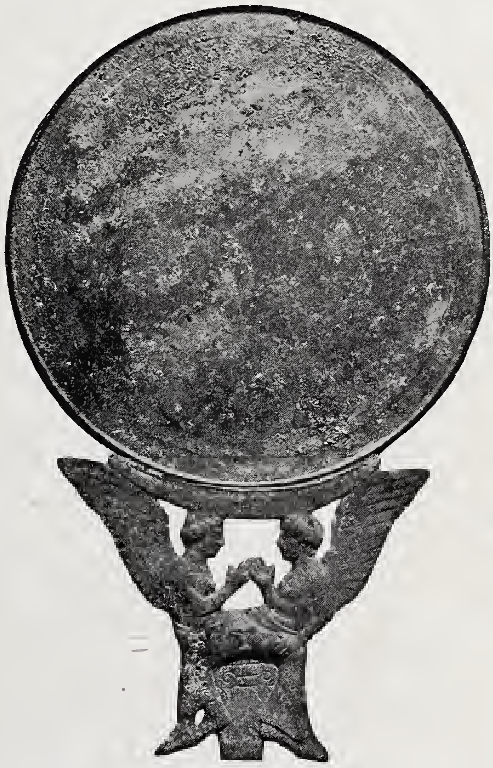


Fig. 4b—Greek Bronze Mirror

A GREEK BRONZE MIRROR. MUSEUM OF FINE ARTS, BOSTON

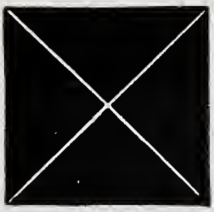


Fig. 1—Greek Bronze Mirror

DRAW a square and its two diagonals as Fig. 1.

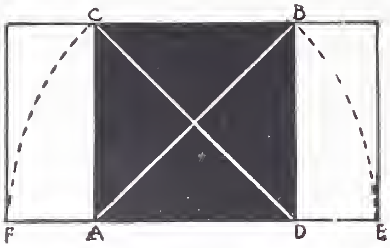


Fig. 2—Greek Bronze Mirror

Use these two diagonals of the square to construct the rectangle Fig. 2. DC is equal to DF and AB is equal to AE.

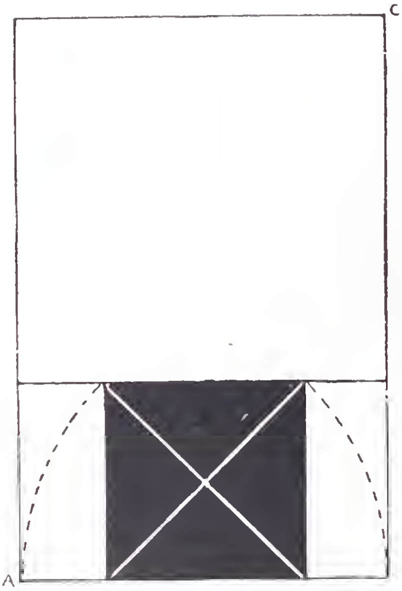


Fig. 3—Greek Bronze Mirror

Upon the side of the rectangle of Fig. 2 construct a square as in Fig. 3. This rectangle AC of Fig. 3 is the rectangle which exactly contains the bronze mirror with its two decorative figures of Cupid and Psyche playing a game of chance.

ANALYSIS OF THE DETAIL

The area AB, Fig. 4a, is composed of two root-two rectangles which overlap to the extent of a square. These figures follow from the constructive use of the diagonals of the primary square as shown in Fig. 2. Use one of the diagonals of this primary square to obtain the line DG or CF and draw the line JI. Complete the rectangle JB. This rectangle equals one-half the overall rectangle. AD, CH and DF are root-two rectangles and JD, CI are squares overlapping to the extent of the root-two rectangle DF. From E, the middle of the line BQ, draw the lines EJ and EI. These lines will cut a line marking the middle division of the primary square at K and L. Draw the lines KP and LO. Lines drawn from P to J and O to I cut the diagonals of the two squares AF and GH at M and N. Draw the line MN. The symmetry theme of this design is now developed. If necessary, the analysis might be carried further and the fact shown that every detail was expressible in terms of root two and square. This, relatively, minor design is characteristic of all Greek design. The symmetry theme is definite and consistently one thing throughout. The satisfying harmony found in all good Greek design is due more to this consistency of symmetry theme than to any other single factor.

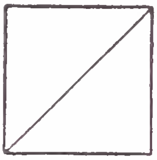
THE ARITHMETICAL ANALYSIS OF THE BRONZE MIRROR DESIGN

If we consider the side of the primary square as unity or 1. then a diagonal of this square is equal to the square root of 2. or 1.4142, *i. e.*, the line CD is equal to 1. and CQ, consequently, the line BQ is equal to 1.4142 plus .4142 or 1.8284. The line AQ is equal to unity or 1. The ratio of the area AB is found by dividing AQ into BQ, 1. divided into 1.8284 equals 1.8284, consequently, this is the ratio. If we divide 1.8284 into 1. we obtain the reciprocal. This process is equivalent to changing unity or 1. from AQ to BQ. If we consider BQ or 1.8284 as unity then AQ becomes .5468 plus. The reciprocal of a shape is similar to the whole, hence we may call the rectangle AB a 1.8284 or a .5468 shape depending upon whether AQ or BQ is considered as unity.

AJ is equal to CQ or .4142, hence the line JQ is equal to 1.4142 and the ratio of the rectangle JB is equal to 1.4142 divided into 1.8284 or 1.2929. The fraction .2929 equals the difference between .7071 and 1. This fraction represents the area JC, because the area CI is equal to a square or 1. The fraction .7071 equals the square root of two, 1.4142, divided by two. We reduce the area of the rectangle JC to recognizable terms by dividing .2929 into unity. This equals 3.4142. A root-two rectangle being represented by 1.4142, we recognize the area JC as being composed of a root-two shape plus two squares. The area AI is reduced to recognizable terms by dividing AJ or .4142 into JI or 1.8284. This equals 4.4142 or a root-two rectangle plus three squares. The area AC is reduced to recognizable terms by dividing CQ or .4142 into AQ or 1. This equals 2.4142, or a root-two rectangle plus one square. The line JI divides the containing rectangle of the overall shape of the design into two parts. JQ is equal to 1.4142, BQ equals 1.8284. The length 1.4142 multiplied by two equals 2.8284. The length 1.8284 divided into 2.8284 equals 1.5469. The reciprocal of this, *i. e.*, this number divided into unity equals .6464. This fragment of an example of Greek design, it is a fragment because the handle is missing, shows a symmetry theme in square and root-two and the above is a geometrical and an arithmetical analysis of that theme. We call the symmetry shape a 1.5469 or a .6464 rectangle. It will be noticed that the interior circle of the mirror disk is proportionally related to the triangle containing the decorative figures.

THE ELEMENTS OF DYNAMIC SYMMETRY

Much of the work upon which the Elements of Dynamic Symmetry is based, was done during the year 1919 under the Sachs Research Fellowship of Harvard University. This fellowship was founded by Samuel Sachs of New York City and, under the terms of the foundation, is assigned to scholars of proved ability, whether students, instructors or others, for the general purpose of advancing the Fine Arts. This acknowledgment is but a faint expression of the writer's appreciation of the benefits enjoyed from a fellowship of a type too rare in American Universities. Jay Hambidge.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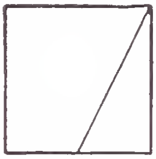
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THE BASES OF DYNAMIC SYMMETRY

The Square and its diagonal and the square and the diagonal to its half

METHODS OF MANIPULATING THE PLAN FORMS OF NATURE

THE square and its diagonal furnish the series of root rectangle. The square and the diagonal to its half furnish the series of remarkable shapes which constitute the architectural plan of the plant and the human figure. For the purpose of helping the student to advance as rapidly as possible a lesson on each of the two basic forms will be printed each month even at the risk of a slight repetition of ideas.

These lessons will deal entirely with the fundamental principles of symmetry as they are found in nature and in Greek art; no attempt will be made to show an application of these principles to specific examples of art or nature. The source of the process in nature and its employment in art will be handled exhaustively in other sections of The Diagonal.

The most distinctive shape which we derive from the architecture of the plant and the human figure, is a rectangle which has been given the name "root-five." It is so-called because the relationship between the end and side is as one to the square root of five, 1.: 2.2360 plus. As a length unit the end cannot be divided into the side of a root-five rectangle, because the square root of five is a never-ending fraction. We naturally think of such a relationship as irrational. The Greeks, however, said that such lines were not irrational, because they were commensurable or measurable in square. This is really the great secret of Greek design. In understanding this measurableness of area instead of line the Greek artists had command of an infinity of beautiful shapes which modern artists are unable to use. The relationship between the end and side of a root-five rectangle is a relationship of area and not line, because as lengths one cannot be divided into the other, but the square constructed on the end of a root-five rectangle is exactly one-fifth of the area of the square constructed on the side. The areas of rectangle which have this measurable relationship between end and side possess a natural property which enables us to divide them into many smaller shapes which are also measurable parts of the whole.

THE ROOT RECTANGLE AND THE RECTANGLE OF THE WHIRLING SQUARES

The construction of a root-five rectangle is simple. Draw a square as BC in the diagram (Fig. 1) and bisect one side as at A. Draw a line from A to B and use this line as a radius to describe the semicircle DBE. AE and AD are equal to AB, or the line DE is twice the length

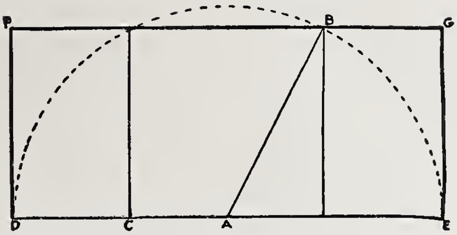

Fig. 1

of the line AB. Complete the rectangle by drawing the lines DF, EG and FG. It will be noticed that the rectangle FE is composed of the square CB and two rectangles FC and BE. This is the root-five rectangle and its end to side relationship is as one to the square root of five, 1:2.2360; the number 2.236 being the square root of five. Multiplied by itself this number equals 5.

This root-five rectangle is the basic shape of vegetable and animal architecture and is the form which has solved the mystery of the perfection of classical Greek art.

THE RECTANGLE OF THE WHIRLING SQUARES

The root-five rectangle produces a great number of other shapes which are measurable in area with themselves and with the parent shape. The principal one of these is that which is called the rectangle of the whirling squares. The relationship of this rectangle to the root-five rectangle is shown by its construction. Draw a square as CB in Fig. 2. Bisect one side as at A. Draw the line AB and make AE equal to AB. Complete the rectangle by drawing the lines BF, FE. This rectangle, DE, is the rectangle of the whirling squares. It is composed of the square CB and the rectangle BE. It is apparent that this shape is equal to the root-five rectangle minus one of the small rectangles FC or BE of Fig. 1.

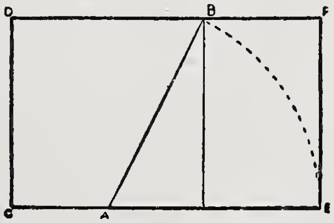

Fig. 2

As has been said above, the construction of the shapes of vegetable and animal architecture is simple and if we wished to begin using them in design little more would be necessary. Unfortunately we cannot afford to wait to discover all the wonderful properties possessed by these simple shapes by practically employing them in our design problems. Time is too short and our needs too great. But, fortunately, we have the use of a tool which the Greek artists did not possess. That implement is arithmetic. By the use of a little adding, multiplying, dividing and subtracting, we may expedite our progress enormously. This will be apparent if we consider the square of the root-five rectangle as representing one or unity; this may be 1. or 10. or 100. or 1000, but it will always be a square as an area of sides of equal length. Regarded thus it will be apparent that the area of any rectangle may be composed of one or more squares plus some fractional part of a square. The square root of five is 2.236; if one is subtracted from this number the result will be 1.236. In this case 1.236 represents the two small rectangles on either side of the square. If this number is divided by two, the result is .618. This number represents each one of the two small rectangles. We now see that the area of the root-five shape may be considered as 1. plus .618 multiplied by 2. Also it is now apparent that the area of the rectangle of the whirling squares may be considered numerically as 1. plus .618.

For the purpose of dividing up the areas of rectangles so that the divisions would be recognizable, the Greeks had recourse to a simple but ingenious method which is called the "application of areas." This idea was used by them both in science and in art. For a description of the process as used in science see any standard work on the history of Greek geometry. Classic design furnishes abundant examples of its use in art. The process in design may be illustrated by either of the rectangles described above. If, to a rectangle of the whirling squares, we apply a square on the end of that shape, the operation is equivalent to subtracting 1. from 1.618. A square on the end applied to the area of a whirling square rectangle leaves as a remainder a .618 area. If a square on the end of a rectangle is applied to its side, however, the operation is not so simple unless we use the Greek method. This process is shown in diagram Fig. 3.

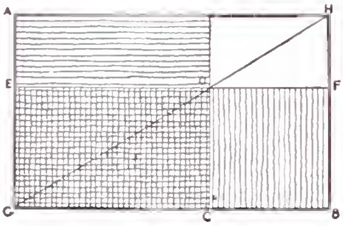

Fig. 3

To the rectangle AB the square on the end AC is applied. To apply this same square to a side as BC, we must first draw a diagonal to the whole shape as GA. This diagonal cuts the side of the square AC at D. Through the point D we draw the line EF. The area EB is equal to the area of the square AC. This process applies to any rectangular area whatever which may be applied to either the end or the side of a rectangle. It will be noticed that the square on the end of a rectangle when applied to a side changes its shape, *i. e.*, it is no longer a square, though equal in area to the area of a specific square. It is clear that now it is composed of a square plus some other area which may be composed of either a square or squares or some fractional part of a square. In the rectangles which we find in plant and animal architecture and in Greek art this area of a square applied to a side of such shapes is always understandable or measurable in terms of the whole.

A process connected with the science of plan making which was thoroughly understood by the Greeks, was that of determining the reciprocal of a rectangle. This conception of a reciprocal of a shape is most important. Briefly, the reciprocal of a rectangle is a similar shape, but the end of the rectangle becomes the side of the reciprocal. Greek art shows us several simple methods for determining reciprocals, but they all depend upon the fact that the diagonals of a reciprocal always cut the diagonals of the major shape at right angles. Fig. 4 is a rectangle

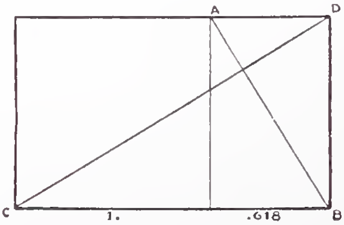

Fig. 4

of the whirling squares. AB is a reciprocal to the shape CD and the diagonals CD and AB cut each other at right angles at E. The rectangle AB is a similar shape to the whole, *i. e.*, the area AB is exactly like the area CD, the difference being one of size only. The line BD, which is the end of the larger rectangle, is the side of the smaller one. The name, the rectangle of the whirling squares, may now be explained. Because of the fact of similarity, the reciprocal of this shape, Fig. 4, is also a whirling square rectangle and the area AC, *i. e.*, the area in excess of the area AB is a square and if we define the continued reciprocals of this specific shape, the result would appear as Fig. 5. A diagonal of the whole and a diagonal of a reciprocal, enable us to draw readily the continued reciprocals. The area KB is a reciprocal of the area KL and the area KC is a reciprocal of the area KB, etc. These areas are all similar shapes. The numbered areas, 1, 2, 3, 4, 5, etc., are all squares. A rectangle of the whirling squares is so called because its continued reciprocals cut off squares and these squares arrange themselves in the form of a spiral whirling to infinity around a pole or eye. The appearance in nature of this rectangle is explained in the article on the phenomenon of leaf arrangement in this and succeeding issues of The Diagonal.

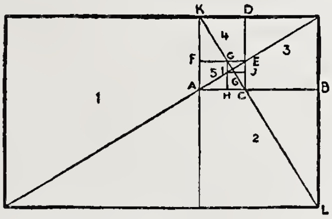

Fig. 5

The arithmetical statement of the reciprocal may now be considered. If unity or 1. represents the area of a square, then, in a whirling square rectangle, the fraction .618 must represent the reciprocal and the area .618 is also a whirling square rectangle. We obtain the reciprocal of any number by dividing that number into unity. 1.618 divided into 1. equals .618. The square root of five divided into unity, 2.236 into 1., equals .4472. The area represented by .618 is a whirling square rectangle, and that by .4472 is a root-five rectangle. As far as shape is concerned a root-five rectangle, numerically, may be 2.236 or .4472; this also applies to a rectangle of the whirling squares which may be 1.618 or .618. Any rectangle may be arithmetically expressed in the same manner. It will be noticed that a root-five rectangle may be considered as composed of a square plus two whirling square rectangles, or as a square plus two reciprocals of that shape. It may also be considered as two whirling square rectangles overlapping each other to the extent of a square. AB and CD are whirling square rectangles and they overlap to the extent of the square CE. Also, CE is a square and AD and CB are each whirling square rectangles. Each of these shapes, AD and CB, are reciprocals of the areas AB and CD.

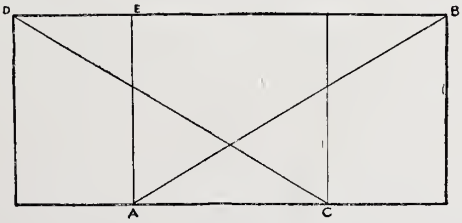

Fig. 6

It was stated at the beginning of this lesson that the relationship between the areas of the squares described on the end and side of a root-five rectangle was as one to five. The square described on the side of a whirling square rectangle is equal in area to the square described on the end plus the area of the rectangle itself.

[To be continued.]

THE ROOT RECTANGLES

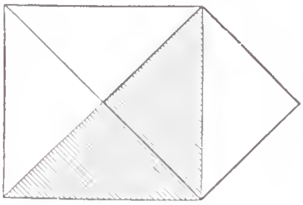

Fig. 7

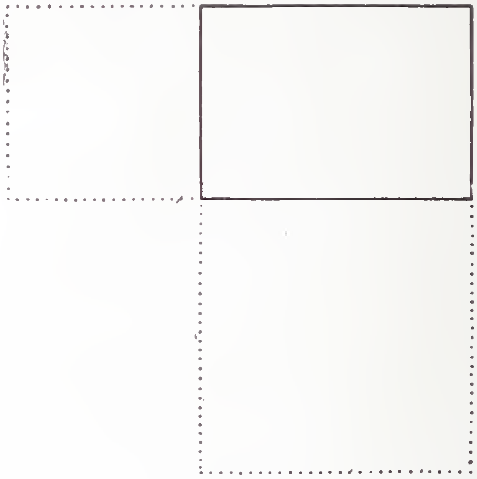

Fig. 8

THE area of a square described on the diagonal of a square is double the area of the smaller square, Fig. 7. The rectangle obtained from a square and its diagonal, Fig. 8. The square on the end of this rectangle is one-half the area of the square on the side.

This shape is called a root-two rectangle, because the relationship of its end to its side is that of one to the square root of two; 1 : 1.4142 plus. The end and side of a root-two rectangle are incommensurable, not measurable one by the other. They are, however, commensurable in square. (See Greek classification of incommensurables. Euclid, Book X.)

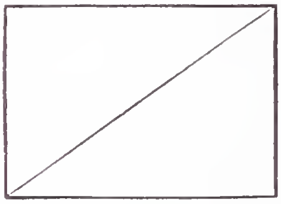

Fig. 9

For purposes of design the most important element of a rectangle is its diagonal. The area of the square upon the diagonal of any rectangle is equal to the areas of a square upon its end and a square upon its side. This important fact was supposedly discovered by the early Greeks. It is the 47th proposition of the first book of Euclid and has been picturesquely described by Kepler as "a measure of gold."

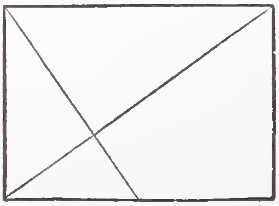

Fig. 10

The element of the rectangle second in importance to the diagonal, is the diagonal of a reciprocal. The reciprocal of a rectangle is obtained by dividing the side into the end. The reciprocal of a rectangle is a similar shape to the whole. The area of the reciprocal to a root-two rectangle is one-half the area of the whole. 1.4142, divided into 1. or unity, equals .7071.

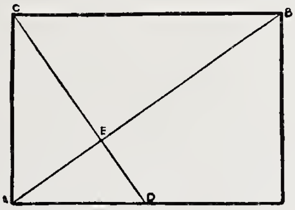

Fig. 11

The diagonal of the reciprocal of a rectangle cuts the diagonal of the whole at right angles. AB is the diagonal of a root-two rectangle and CD is the diagonal of its reciprocal. E is the right-angled intersection of the two lines.

The reciprocal of a rectangle may be established in various ways by simple geometrical construction.

(1) By "applying" the square on the end of the rectangle.

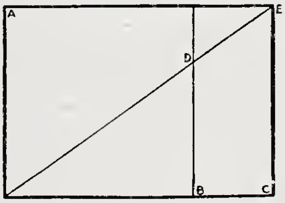

Fig. 12

The side of the square AB cuts the diagonal of the rectangle AC at D. The end of the reciprocal shape is the line BD, which is equal to one-half AE.

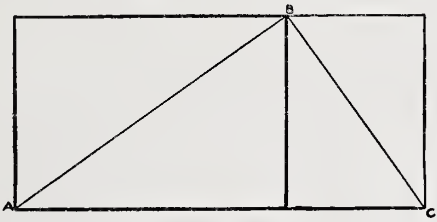

Fig. 13

(2) By a right-angled triangle, one side of which is the diagonal of the rectangle. AB is the diagonal of a rectangle; BC is a reciprocal. The triangle ABC is right-angled at B.

(3) Describe a semicircle on the end of a rectangle. The arc cuts a diagonal of the rectangle at C. A line from A through C to B is the diagonal of the reciprocal.

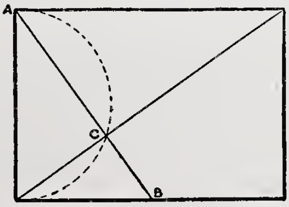

Fig. 14

The point of intersection of the diagonal of a reciprocal with the diagonal of the rectangle is the pole or eye of a rectangular spiral.

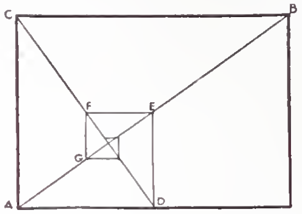

Fig. 15

AB is the diagonal of a rectangle, CD the diagonal of the reciprocal and BC, CA, AD, DE, EF, FG, etc., are the sides of a rectangular spiral whose pole is O. The spiral can never reach the point O.

The diagonal of a rectangle and a diagonal of its reciprocal cut each other to form lines in continued proportion.

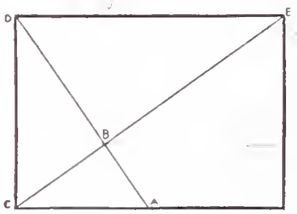

Fig. 16

AB, BC, BD, and BE are lines in continued proportion and BC, BD are two mean proportionals between the two extremes BA and BE. Also, AC, CD and DE, the sides of a rectangular spiral, are lines in continued proportion. Lines in continued proportion in a rectangle divide the area into proportional multiple and submultiple parts.

In a root-two rectangle diagonals of the whole and diagonals of the reciprocals divide the area into a never ending series of smaller root-two rectangles.

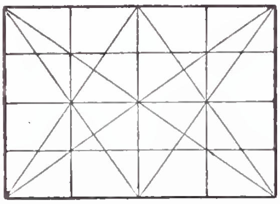

Fig. 17a

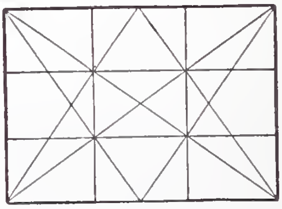

Fig. 17b

a. Continued subdivision of a root-two rectangle into similar figures. Ratio 2.
b. Continued subdivision of a root-two rectangle. Ratio 3.

[To be continued.]

WHAT INSTRUMENTS TO USE

THE implements necessary for a careful study of the shapes of Dynamic Symmetry are: a scale divided into millimeters, a small drawing board, one that is perfectly true, a T-square and a lead pencil. The student is advised, whenever possible, toconstruct the rectangles both geometrically and arithmetically. The diagrams of the lessons show how the geometrical constructions are made. For the arithmetical constructions the scale is used. The first step is the establishment of a right angle with the T-square. Measure off on one of the lines 100 millimeters or 10 centimeters and call this unity; if a whirling square rectangle is desired, measure off on the other line 161.8 millimeters or 16.18 centimeters. If 100 millimeters or 10 centimeters is assumed to be unity, then the reading is 1. or unity, plus .618 parts of unity, or 1.618. A little practice will enable the student to construct any rectangle, from its ratio, with great rapidity. A 1.809 rectangle is simply ten parts in one direction and 1.809 in the other, the shapes being completed with the T-square from the two measurements. A root-five rectangle is 10 parts one way and 2.236 the other; a 1.309 shape, 10 one way and 1.309 the other. Reciprocal ratios are always less than unity. The reciprocal of root five is .4472. To draw this by scale mark off 10 parts one way and .4472 the other; the reciprocal of a whirling square rectangle is .618; measure off 10 parts one way and .618 the other; the reciprocal of a 1.309 shape is a very small amount less than .764; scale off 10 parts one way and .764 the other; the reciprocal of a 1.809 shape is a little less than .528; scale off 10 points one way and .528 the other. Having a ratio the construction is always simple. To obtain a ratio measure the greatest height and the greatest width, whatever the object may be, and divide the lesser into the greater. Dividing the greater into the lesser has the same effect as dividing a ratio into unity to obtain a reciprocal. For example, if an object being examined measures 320 one way and 4188 plus the other, the greater measurement being divided into the lesser the answer is .764; if the lesser is divided into the greater it is 1.309; if the ratio 1.309 is divided into unity the answer is .764: in either case the result is a 1.309 rectangle.

The ratios mentioned in the article on the dynamic symmetry of the human figure in this number of The Diagonal may be scaled out by the student for practice and examined arithmetically. This is the best way to study the subject. This arithmetical method will help the student to understand the dynamic rectangles, their peculiar properties and their strange power of modulating or changing one into another and producing by this union other shapes equally strange and fascinating.

This strange power of modulation possessed by dynamic symmetry is shown in the article on the symmetry of the plant. Professor Church, in describing the phyllotaxis of the sunflower, speaks of the change in cross-curve numbers from the flowers lowest on the stalk to those highest and mentions the pairs 13-21, 21-34, 34-55, 55-89; these combinations all produce the same ratio, but no steps are observable between the pairs; they simply modulate perfectly within the system.

The student should memorize the following square roots of numbers:

Root two 1.4142
Root three 1.732
Root four 2.
Root five 2.236

A small school drawing board costs about 80 cents; a plain wooden metric scale may be purchased for 10 cents; a T-square is usually supplied with a school drawing board. For large work a metal tape, divided metrically, is desirable.

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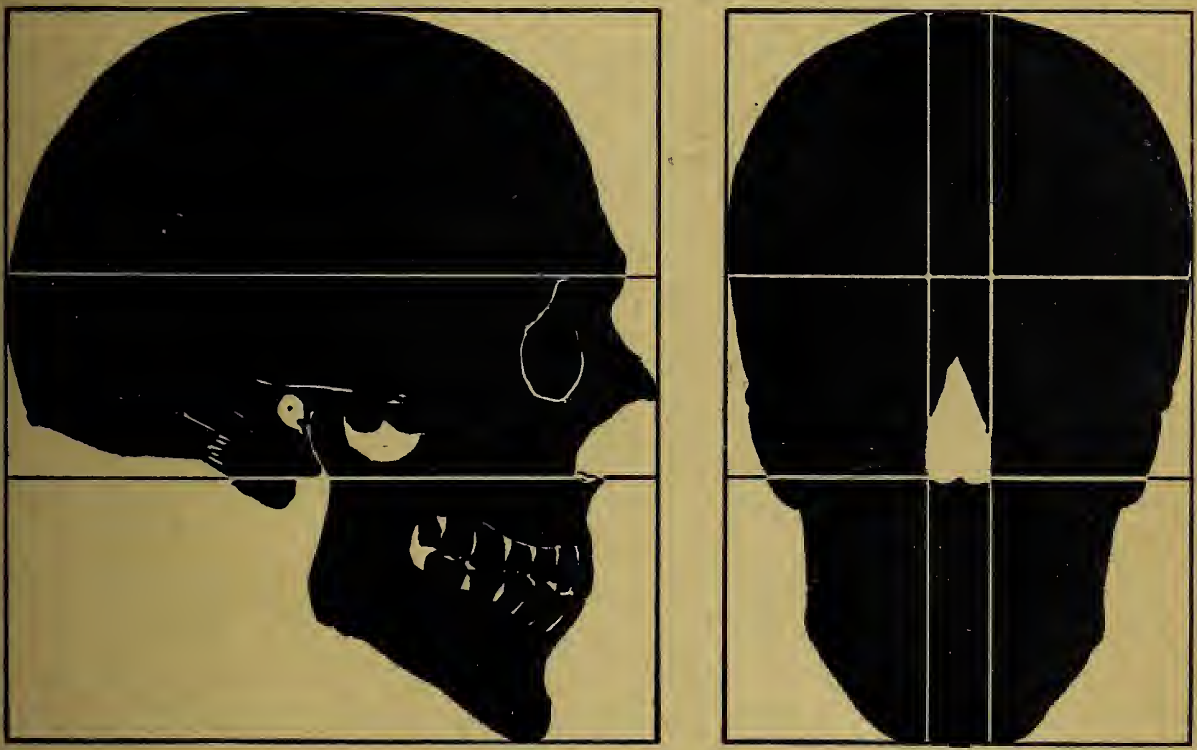
WITHDRA

THE DIAGONAL

AN ILLUSTRATED MONTHLY MAGAZINE DEVOTED TO THE EXPLANATION OF THE REDISCOVERED PRINCIPLES OF GREEK DESIGN, THEIR APPEARANCE IN NATURE AND THEIR APPLICATION TO THE NEEDS OF MODERN ART

EDITED BY JAY HAMBIDGE



See article on the Human Figure in this number of THE DIAGONAL

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THE DIAGONAL

THE DYNAMIC SYMMETRY OF THE PLANT

HOW DYNAMIC SYMMETRY WAS DISCOVERED

TO the end that the artist may understand the essential idea connected with the form rhythms observable in plant architecture and apply it to his immediate needs, it is advisable to digress at this point and explain how observation of this form rhythm led to the discovery of Dynamic Symmetry. Many years ago the writer became convinced that the spiral curve found in plant growth, which Professor Church describes in his work on the law of leaf distribution, and that of the curve of the shell, were identical, and must be the equiangular or logarithmic spiral curve of mathematics. It will not be necessary for the present purpose to enumerate all the evidence which justifies this assumption. For the benefit of those who may care to pursue the subject further, reference is made to a paper by the Rev. H. Moseley, "On the Geometrical Forms of Turbinated and Discoid Shells," (Phil. Trans. pp. 351-370, 1838) and to the able and clear treatment of the spiral by D'Arcy W. Thompson. ("Growth and Form," Cambridge, Eng., 1917.) Being convinced that the spiral was indeed the mathematical curve mentioned, the writer saw that, because of a certain property which it possessed, this spiral could be reduced from a curve form to one composed of straight lines and thereby be used by the artist to solve certain problems of composition and connect design closely with nature. This property of the curve is: between any three radii vectors of the curve, equal angular distance apart, the middle one is a mean proportional between the other two. The drawing, Fig. 1, explains this.

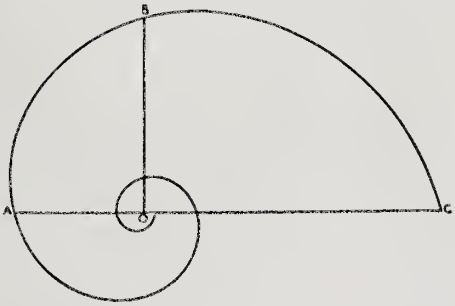

Fig. 1

O is the pole or eye of the curve and the lines OA, OB, OC, are radii vectors equal angular distance apart. In this case the angle is a right angle. According to the definition the line OB is a mean proportional between the other two, *i.e.*, OA and OC. This means, speaking in terms of area, that the line OB is the side of a square equal in area to a rectangle the end and side of which are the lines OA and OC. It necessarily follows, therefore, that if three lines stand in this relationship they constitute the essentials of a right angle. This is shown in Fig. 2.

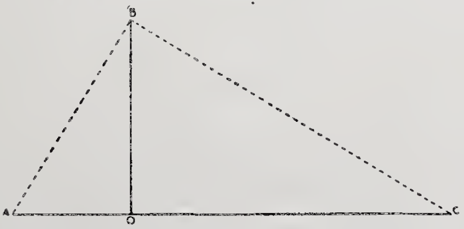

Fig. 2

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The line OB is a mean proportional between the lines OA and OC and the shape ABC is a triangle, right-angled at B, and the line AC is the hypotenuse. One of the earliest geometrical facts determined by the ancient Greeks was that in a right-angled triangle a line drawn from the intersection of the two sides to meet the hypotenuse at right angles was a mean proportional between the segments of the hypotenuse. Simply stated, this means that three lines situated like the three radii vectors mentioned constitute three terms of a continued proportion; OA is to OB as OB is to OC. The relation between these lines could be stated arithmetically as a ratio, this following from the fact that lines in continued proportion form a geometrical progression. If we assume, for example, that the ratio is 2 and the length of the line OA is two units then the line OB would be four units and the line OC eight units. 2, 4, and 8 constitute three terms of a simple proportion with a ratio of 2 and according to the rule of three, the square of the mean is equal to the product of the extremes. In this case 4 is the mean and 2 and 8 are the extremes. 4 multiplied by itself, or squared, equals 2 multiplied by 8. The mean is the side of a square equal in area to the area of a rectangle made by the extremes or 2 as end and 8 as side.

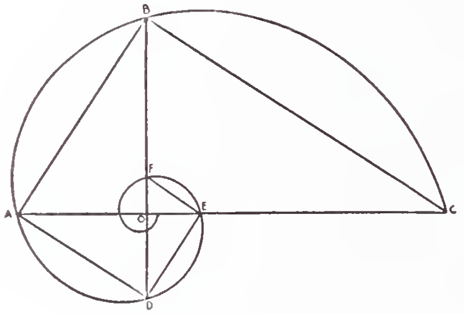


Fig. 3

But the line OB may be produced through the pole O of the spiral until it touches the curve at the point D and DAB will be a right angle and there will exist four lines in continued proportion, OD, OA, OB and OC. Also, the curve cuts the vector lines at E and F, etc., and other right angles are created. In short, the curve may be reduced from a curve to a rectangular spiral, and, as in the curve, the converging right angles wrap themselves to infinity around the pole. This particular curve never reaches the pole, it goes on forever. The essential point is that this simple construction enables the designer to introduce the law of proportion into any type of composition and that, too, in much the same way as it appears in the plant and the shell. The operation by which this is accomplished is the drawing of a diagonal to a rectangle and a line from one corner cutting this diagonal at right angles, Fig. 5.

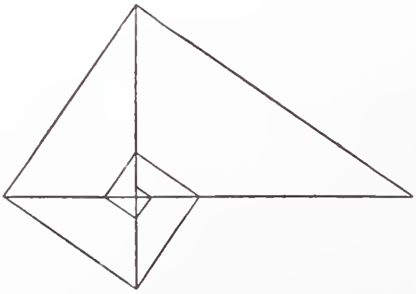


Fig. 4

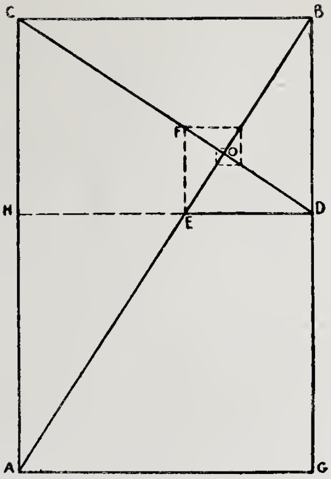


Fig. 5

Fig. 5 is any rectangle and AB is a diagonal. The line CD, drawn from the corner C, cuts the diagonal AB at right angles at O. It is apparent that the angular spiral formed by the lines AC, CB, BD, DE and EF is identical with the angular spiral derived from the shell and from the plant. Having established a condition of proportion within the area of a rectangle it becomes obvious that the line CD must perform some important function. Artists are well acquainted with at least one property of the diagonal to a rectangle. It is well known, for example, that any shape drawn within the area of a rectangle whose diagonal is common to the diagonal of the containing area is a similar shape to the whole. This geometrical fact is utilized in the arts, especially the reproductive arts, for the purpose of determining similar shapes on a larger or smaller scale. This, however, is about as far as modern artistic knowledge of proportion extends. The value of the line CD is unknown to modern art. That the value of this line was known at one time in the history of art and its power appreciated is abundantly proven by dynamic analysis of classical Greek design. This line CD determines the reciprocal of a rectangular shape and is itself the diagonal of that shape. Following the construction we appreciate the fact that, in a rectangular area, the diagonal of a reciprocal cuts the diagonal of the whole at right angles. (See lessons on the Elements of Dynamic Symmetry.) The line DH, Fig. 5, which is drawn parallel to the ends AG or CB, fixes the area of the reciprocal of the shape AB. With this notion of a reciprocal in mind it is apparent that the fatter or squatter the rectangle the larger will be the area of the reciprocal. For example, if instead of the rectangle AB of Fig. 5, one is constructed which much more closely approaches the shape of a square, the area of the reciprocal, because it is a similar shape to the whole, will also much more closely approach the shape of a square. The breadth of the reciprocal increases with the breadth of the parent form until it coincides with it as a square; or it decreases until both become a straight line. Obviously, also, there must be rectangles such that the reciprocal is some even multiple of the parent form, such as 1/2, 1/3, 1/4, 1/5 and so on. Perception of this fact led the writer to discover the root rectangles. It was found that a rectangle whose reciprocal equalled one-half the whole was a root-two rectangle; 1/3 a root-three, 1/4 a root-four and 1/5 a root-five rectangle and so on. When the root-five rectangle was defined and its commensurable area examined it was found that this shape was connected in a curious manner with the phenomena of leaf distribution. As has been explained, the ratio produced by the summation series of numbers which so persistently appears in the rhythmic arrangements of leaves and seeds in vegetable growth is 1.6180. When a rectangle was made wherein the relationship between the end and side was the ratio 1.6180 it was found that the end of the reciprocal of this area equalled .618. The side of a root-five rectangle, arithmetically expressed, is

2.2360 or the square root of five. If the ratio 1.6180 is subtracted from 2.2360 the remainder is .6180. The area of a root-five rectangle, therefore, is equal to a 1.6180 rectangle plus its reciprocal. The 1.6180 rectangle, because the end of its reciprocal equals .618, is a rectangle such that its *continued* reciprocals cut off squares and these squares form a spiral around a pole of the rectangle. This pole, of course, is the point where the diagonal of a reciprocal and a diagonal of the whole cross each other. Because of this property this shape was given the name "the rectangle of the whirling squares." (See lessons on the Elements of Dynamic Symmetry.)

[To be continued.]

THE DYNAMIC SYMMETRY OF THE HUMAN FIGURE FOR ADVANCED STUDENTS: LESSON II

ROMAN report of Greek tradition says that classic artists took extraordinary care to have their designs conform to the principles of a certain symmetry which they derived from the human figure. The Romans, however, assumed that this symmetry depended upon a modulus or length measurement which was multiplied or divided into even units to determine the proportions of the composing elements of a design, thus producing an ensemble the parts of which were commensurable with each other and with the whole. Dynamic Symmetry shows us that the Romans were in error in their interpretation of this tradition. No Greek design of the first class has been found wherein commensurability of line exists, and this is also true of the figure of man in nature. Design in nature and design in Greek art show conclusively that parts are invariably commensurable with the whole but not in line; the measurableness is that of area. When the human figure is measured and interpreted in terms of area the result is a revelation to art.

"Marcus Vitruvius Pollio was a famous Roman architect and engineer and was a military engineer under Caesar and Augustus. He wrote a treatise on architecture in ten books ('De Architectura'). It was dedicated to Augustus and is the only surviving Roman treatise on the subject. His book was well known to Pliny and on it was based almost all the earlier theory and practice of Renaissance and pseudo-classical architecture." *The Century Dictionary.*

Vitruvius gave elaborate rules for the construction of buildings of all kinds in the Greek style. He also said that the Greeks were careful to have their designs, especially of their temples, conform to the laws of symmetry. The principles of this symmetry, he said, were determined from the human figure, the Greeks having noticed that the members of the human structure were commensurable with the body as a whole. Modern research has shown that Vitruvius misinterpreted the Greek tradition, because no building has been found which in any particular conforms to his rules. In one of the ten books Vitruvius attempted to show the symmetry of man. Like the rules for the symmetry of architecture those advanced for the figure have been found to be without value, being in fact nothing more than a recipe. It has taken 2000 years to discredit these Roman recipes.

The entire Vitruvian scheme depends upon the use of a modulus. The composing elements of a building were to be made to conform to some unit of length which was derived from the plan, and were to be expressed as so many multiples or fractional parts of this modulus. The Roman rules for figure symmetry are somewhat the same as those for architecture.

Greek statuary and design in general, as well as Greek building, do not in any way bear out the Roman interpretation of the Greek tradition. In fact, the Roman rules, by their definition, prove their fallacy. The modulus idea, used in any manner, would necessarily produce static symmetry. It is not possible to express the human structure in even multiples or even fractional multiples of any member such as head, hand, foot, arm or leg. As lengths these members are incommensurable, unmeasurable one with the other.

The members of the human body are indeed measurable one by the other, but not in *line*, as the Romans thought. Both nature and Greek art show that the measurableness of symmetry is that of *area* and not *line*. The Romans were not given to scientific study or they would have heard of the Greek classification of irrationals, a subject which was taught at Alexandria in Roman times. This classification, familiar to us as the tenth book of Euclid's Elements, shows that lines incommensurable in length are not necessarily irrational; they may be commensurable in square. This is the secret. Dynamic Symmetry deals with commensurable areas. The symmetry of man and plant is dynamic; the symmetry of the entire fabric of classic art, including buildings, statuary and the products of the crafts, is dynamic. The symmetry of all art since Greek classic times is static.

Since the First Century B. C. many treatises have been written upon the human figure and its proportions. The list of authors includes the names of most of the best-known artists. Bertram Windle enumerates eighty-eight. The theories advanced in these works are developed upon the same general idea as the Roman scheme. There are slight variations occasionally, but nothing really new or true appears in any of them. In recent years scientists have measured an enormous number of men for the purpose of racial and type comparison and for military and economic statistics. If human figures were constructed in conformity with the canons of the artists or according to the measurements of the scientists the results would all be *static* and, consequently, untrue to nature. The measurements of the scientists, however, would result in figures truer to nature than those constructed according to the artist's canons, for the reason that the former, as a rule, have reduced the stature of man to a system of decimal fractions which approximates more closely to the actuality of the unmeasureable lengths than do the rules of the artists. In no case, however, have the commensurable areas of dynamic symmetry been recognized.

The statement was made in the previous lesson that it was essential for the purposes of design, wherein the human form was used, that the student should be able to construct a well-proportioned and rational figure without the use of the model, or, at least, after the rhythmic theme was defined, that the model be employed only for superficial corrections. The writer's anxiety to assist the student in acquiring this much-to-be-desired power as soon as possible induces him to digress slightly and for a short time put the cart before the horse, meaning by the simile that the logical process should be the reverse and that the rhythmic theme for a specific skeleton should be first learned with considerable thoroughness. One of the simple geometrical facts which will enable the student to transform a rhythmic theme obtained from the direct measurements of a figure in repose to a figure in any position or action, is furnished by a property of the triangle. This is: *lines drawn parallel to the base of a triangle cut the sides proportionally*.

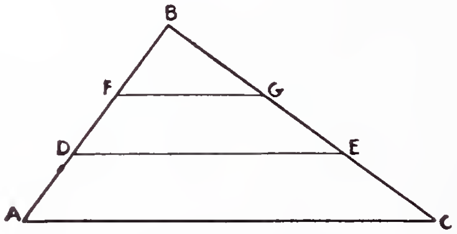


Fig. 1—Symmetry of Man

ABC, Fig. 1, is a triangle and AC is its base. DE and FG are lines drawn parallel to the base AC. These lines cut the sides AB and BC proportionally; that is, AD, DF and FB are proportional to CE, EG and GB.

For the present purpose a vertex-clavicle rectangle of an actual subject will sufficiently illustrate the method of using the triangle.

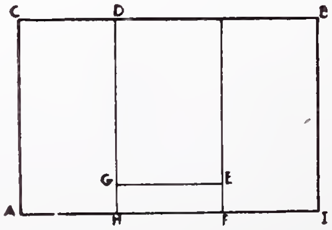


Fig. 2

We will assume that the rectangle AB, Fig. 2, is such a rectangle; that the end AC is the distance from the shoulder or clavicle to the top of the head, the vertex, and CB is the shoulder width from acromion point to acromion point (these points may be felt under the skin on the shoulders of a living model), GE is the width of the skull, DG its height and GH the height of the neck. It is desired to transform this rythmic arrangement of area from its present position to one of action of some kind. Suppose the position desired be that of an approximate three-quarter view. Obviously, in the new position, the line CB, as a two-dimensional rendering, will not be as long as it is in the plan. AH, HF and FI will also be shorter.

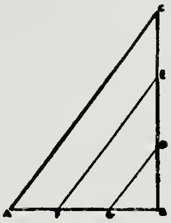


Fig. 3

Suppose AB, in Fig. 3, represents the new line of the shoulders. Parallel to the side of the paper or canvas draw the line BC, equal in length to the line AI, Fig. 2. Transfer to BC, Fig. 3, the lines AH, HF and FI of AI in Fig. 2. They will be D and E of Fig. 3. The line AC in this figure is the base of the triangle ABC. The lines EF and DC are parallel to the base AC. Consequently, the proportions BD, DE and EC are transferred to the line AB and are AF, FG and GB. Assume that the head has not moved, either forward or backward. The transformed rythmic theme would appear as in Fig. 4.

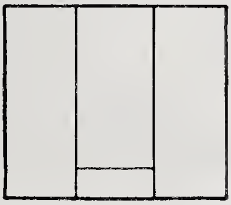


Fig. 4

The four composing rectangles of the major shape necessarily are also altered. When a figure is changed from a straight front or back view the three-dimensional element is introduced, and if an actual head were being drawn in this position its side elevation would have to be considered. With the shoulders in the present assumed position we may keep the head in its first position, twist it to the right or to the left or tip it forward or backward, but in all or any of these movements the original rhythmic theme may be rigidly preserved by the use of the simple property of the triangle as described. It should be remembered that the lines of reference, which are the actual measured lines from the skeleton, either life size or to scale, should always be drawn accurately either horizontal or perpendicular.

It is not advisable for the student to be overambitious too early in his study of the rhythmic themes of the human structure as the subject is entirely new and will, therefore, be to him more or less complicated, however simple it may be in its essentials. The determination of a theme from an original subject will not be possible until several examples, wherein the rhythm has been analyzed, are worked out and studied in detail. It should be remembered that the dynamic scheme, though basically simple, is nevertheless capable of as great variation as humanity itself. Something of the possibility of this variation is suggested by the piano keyboard. There are comparatively few keys, yet as the law of permutation shows, the possible combinations of these are almost inconceivable.

As an example for analysis we will take the rhythmic theme of the front elevation of the skull, with the shoulder breadth, of the so-called typical male skeleton of the Harvard Medical School which was mentioned in the previous lesson of this series. It will not be amiss to point out that before a skull, or indeed a head from a living model, can be measured and a rhythmic theme of area determined it is first necessary to establish a line across the skull or head which will definitely fix it at right angles to man's erect position. This line will represent what the anthropologists call a plane. Some fifteen different skull planes have been used at different times, but the writer has found that the best is that drawn from the center of the ear opening to the upper ridge of the bottom bone of the eye socket as shown in Fig. 5.

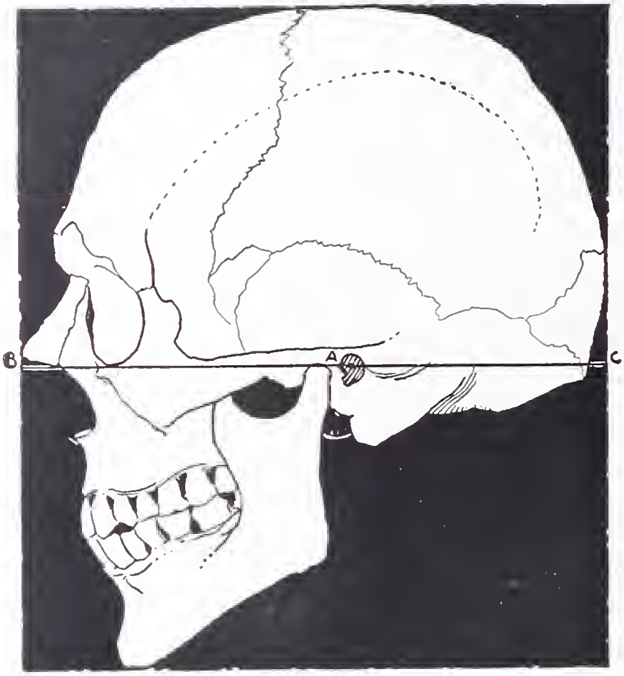


Fig. 5

AB is this line, and when it is fixed the rectangle of either the front or side elevation is easily determined. The importance of this line will be appreciated when the irregular nature of the head is considered. Without this line it is really impossible to fix a rectangle for a head because, as the chin is raised or lowered, the skull may be fitted into many rectangles. With this line settled, however, there is only one rectangle for the side and one for the front elevation. When we find that these two rectangles are logical elements in an encompassing rhythmic theme in commensurable area which takes in every detail of the human structure we may feel confident that we are proceeding correctly.

With the head plane fixed as described for the example being analyzed, the height of the skull reads from the metric scale 2236, the width 1427; the width of the shoulders 4045. The overall rectangle, therefore, has an end of 2236 and a side of 4045 mm. The end measurement divided into the side results in the ratio 1.809. The student must take his drawing board, T-square and scale and draw this shape. The 2236 line will be the end of the rectangle. He may make this line any length he pleases and call it unity, but it is suggested that he first construct the rectangle life size, as these readings were obtained on a metrically divided steel tape from a projection of an actual skeleton. (A steel tape divided as described may be purchased from most drawing instrument dealers.) The measurements given may easily be read from such a tape.

The width of the skull is 142.7 mm. (we cannot say exactly by measurement whether the last figure is 7 as this is seven-tenths of a millimeter; the figures 142 are unmistakable and 7 is closer to actuality than 5 or 9, but 7 is the figure necessary and when the general rhythmic theme is developed this figure is found to be correct). We must place this 1427 length in the center of the 1.809 shape. To do this we divide the 4045 length by 2 and find that the center is 2022.5 mm. from each end. Dividing 1427 by 2 the result 713.5 is scaled from either side of this center. Having established the 1427 by 2236 rectangle in the center of the 1.809 shape we are in a position to consider what is left of that area after the head elevation rectangle is applied to it, that is, the character of the rectangles A and B of Fig. 6.

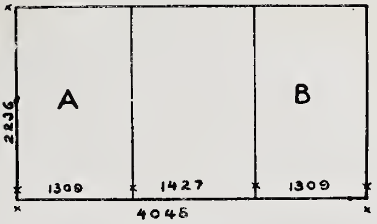


Fig. 6

1427 subtracted from 4045 leaves 2618. Therefore, A and B added have as end 2236 and as side 2618. The lesser divided into the greater supplies the ratio 1.1708 plus, and the reciprocal is .854 plus. The 1.1708 rectangle must be divided into two parts. Each part will then be represented by .5854. This new fraction is the reciprocal of 1.708 plus. This ratio represents either A or B when the short end is considered as unity. If the long side is unity then the ratio is that number of which .5854 is the reciprocal. If the ratio 1.708 is divided by two we see that either A or B is composed of two .854 rectangles.

The analysis may also be made as follows:—2618 divided by two equals 1309. Each of the shapes A and B actually measures 1309 by 2236. The lesser divided into the greater equals the ratio 1.708 plus. Both approaches are given, because analysis for rhythm themes is sometimes difficult and occasions will arise where known shapes will have to be added to or subtracted from unknown areas before they are reduced to recognizable terms.

The front elevation of the head will now be considered. This is a rectangle 1427 by 2236 and the ratio is 1.5669, the reciprocal being .6382. This reciprocal proves the construction, because .6382 subtracted from 1.809, the overall ratio, leaves 1.1708 plus, the ratio of A and B added. The point is again emphasized that before trying to understand the rhythmic character of all or any of these ratios the student must draw them, first life size, and then to different scales. To make a drawing of any of the rectangles on a reduced or enlarged scale, take any number, say unity or ten and give this value to the line which measures 2236, then make the actual ratios the measured lengths. For example, the overall ratio is 1.809. Draw this rectangle by having the end measure 10 and the side 1809; the 1427 measurement will be changed to .6382, etc.

THE AREA COMPOSITION OF THE RECTANGLE SO FAR CONSIDERED

The 1.809 rectangle, a frequently occurring shape in Greek design, is composed of a square and two whirling square rectangles, because .809 equals 1.618 divided by 2 (see lessons on the Elements of Dynamic Symmetry). The reciprocal of 1.809 is .5528. If this reciprocal is subtracted from unity or a square the part left over is a root-five rectangle, or .4472. (This fraction is the reciprocal of 2.236.) The reciprocal ratio .6382 will be identified in one way if it is considered as a compound ratio made by adding .191 to a root-five rectangle, .4472 plus .191 equals .6382. (For an explanation of the .191 reciprocal ratio and its actual use in a Greek design see the analysis of a bronze frying pan in this issue of THE DIAGONAL.) .6382 multiplied by 2 equals 1.2764. The fraction .2764 represents a very important shape constantly appearing in Greek art. For its actual appearance in this skeleton see the following paragraphs. Also note that .6382 subtracted from unity equals .3618. This fraction is the reciprocal of 2.764.

GEOMETRICAL ANALYSIS OF THE FRONT ELEVATION OF THE SKULL

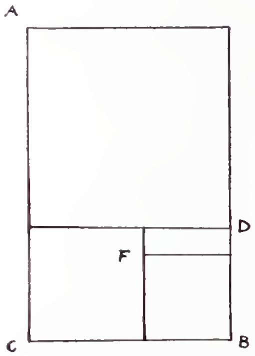


Fig. 7

To the rectangle of the front elevation of the skull AB, Fig. 7, apply the square AD. To the excess area CD apply the square CE. To the area BE apply the square BF, the area FD is composed of two whirling square rectangles. (See arithmetical proof.)

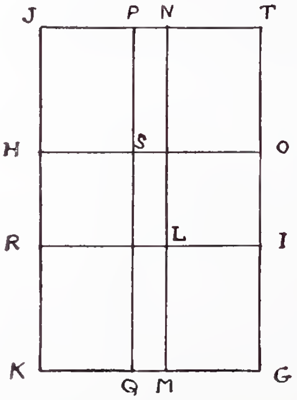


Fig. 8

In Fig. 8, the square GH is applied to the other end of the rectangle; it overlaps the first square JI to the extent of the area HI. Produce the side of the square KL to fix the line MN. The line PQ is the produced side of the square TS. The area of the elevation is now divided as follows:

JI, KO, OP, HN, KL, IQ, RS and LO are squares.
KI, JO, JL, PI, KS and SG are 1.764 rectangles.
RO, RP, HQ, OM and NI are 2.309 rectangles.
HL, IS, JS, NO, LG and RQ are 1.309 rectangles.

ARITHMETICAL PROOF

The rectangle of the elevation measures 1427 by 2236; the ratio is 1.5669; the reciprocal .6382. JK, Fig. 9, measures 2236 and JR 1427, therefore, the difference RK is 809. KR is 809, and KG 1427 and the ratio for this rectangle, RG, is 1.764; RM is a square and GL the .764 area. GL is a 1.309 rectangle, because .764 is the reciprocal of this shape. RK measures 809 as also does HJ. 809 multiplied by 2 = 1618 and this subtracted from 2236, the line JK, equals .618 or the line HR. .618 divided into 1427 equals the ratio 2.309, but HI, IN, RP, KS, and MO are similar figures, consequently, all are 2.309 rectangles. The other similar figures are apparent from the geometrical construction.

The reciprocal of the elevation rectangle is .6382 and this is compounded of the reciprocal ratios .3618 and .2764. KM measures 809 and KJ 2236, the lesser divided into the greater equals 2.764 the reciprocal of which is .3618, JM, therefore, is a .3618 or a 2.764 rectangle. The measure of MG is the measure of HR or .618 and this divided into 2236 equals 3.618 the reciprocal of which is .2764. The rectangle NG, therefore, is either a 3.618 or a .2764 rectangle. (Any ratio with a point before it is a reciprocal ratio.) We are now in a position to appreciate the effect of the four lines which subdivide the front elevation of the skull of this skeleton. It must be remembered that the complicated form in which these ratios are now presented is intentional. They are obtained directly from measurements. In their final form they will be much simplified, because they will then be taken from the stature of the skeleton as a unit. As a matter of fact the entire rhythmic scheme of Dynamic Symmetry may be obtained from a normal skeleton. The variation of the individual is simply the variation of a fundamental plan.

The point to bear in mind for the present is that we are considering a definite theme of areas as disclosed by the projection of an actual skeleton of a man and that no attempt is being made to draw that skeleton. This will be done later after a sufficient number of these themes are mastered. So far we have seen a plan unfolding which, beginning with a 1.809 rectangle, has advanced through certain stages of analysis and brought out the fact that as each succeeding rectangle has been developed it has been apparent that it modulated perfectly with the general or parent shape. Indeed, the modulating power of these shapes is one of the remarkable things about them. The student cannot be too familiar with the ratios so far developed, as they will appear and reappear not only in human structures, but in Greek masterpieces. It is recommended that the curious relationship existing between the ratios and reciprocal ratios 3.618, .2764, 2.764 and .3618 be carefully scrutinized. The student must not feel discouraged if the full significance of this extraordinary relationship is not immediately apparent as there is material here which will tax the power of concentration to its utmost. Several months is not too much time to give to the subject.

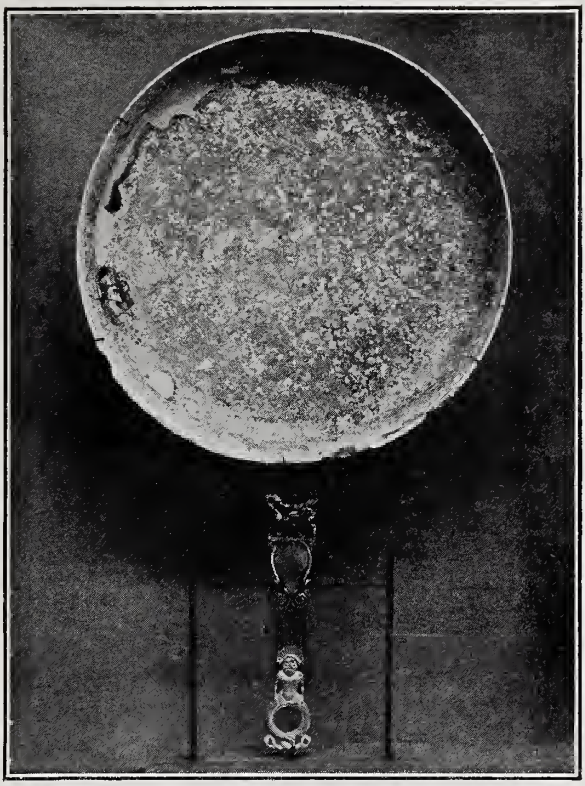
It should also be remembered that we have a great advantage over the Greek artists of the Sixth Century B. C. Every modern school boy understands the simple arithmetical operations necessary to express these area relationships in terms of numbers. To the average Greek arithmetic was an unsolvable mystery. The division of one number by another was an impossible feat except for certain highly trained specialists. But the Greek had certain advantages over us; as a rule the craftsman of the classic period was a master of figure dissection. During the very early days the Greeks regarded the Egyptians as masters of figure dissection, but the pupils soon outstripped their teachers and ended by developing out of the subject the perfect science of Euclidean Geometry.

SOURCES FOR THE STUDY OF DYNAMIC SYMMETRY

Material for the study of Dynamic Symmetry is obtained from three sources: from the symmetry of man and plant, from Egyptian and Greek art and from the five regular geometrical solids. We study Greek art for the purpose of learning now the greatest artists have used the rhythmic themes of area in actual masterpieces. The human skeleton shows us how nature employs these same themes in an organism. The five regular solids, the tetrahedron, the cube, the octahedron, the icosahedron and the dodecahedron furnish abstract geometrical material for study. The skeleton, however, is the source *par excellence* for the artist.

The discovery of Dynamic Symmetry places the human skeleton in a new position in relation to art. Heretofore it has been but slightly considered—being regarded by artists chiefly as a framework supporting a muscular system of more or less value to them from the anatomical viewpoint. We must now regard this framework of bone as the principal source of the most vital principles of design.

[To be continued.]



Bronze Patera, Sixth Century B.C., in the classical collection of the Metropolitan Museum, New York.
With ten vases said to have been found together in a tomb at Civitȁ Castellana.
As exhibited the handle is detached from the pan.

Front view of the detached handle

A SIMPLE COOKING UTENSIL AS A GREEK MASTERPIECE

A SIXTH Century B. C. bronze frying pan in the Metropolitan Museum of Art, New York Citv furnishes a design theme in a perfect whirling square rectangle.

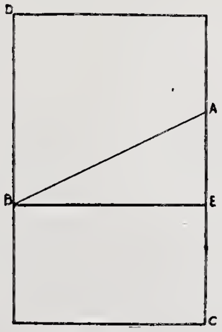
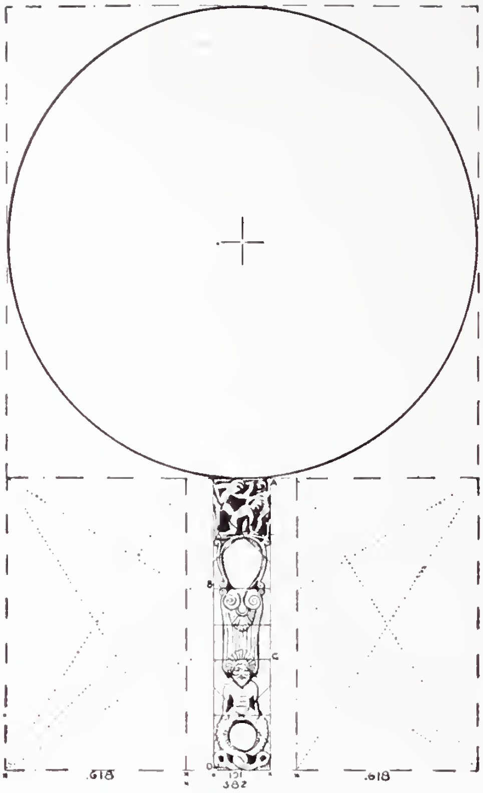


Fig. 1

Draw a square, DE, and bisect one side as at A, Fig. 1. Make the line AC equal to AB. The area CD is a rectangle of the whirling squares and



Drawing of Bronze Patera

the area CB is its reciprocal. Consider the area CB as a rectangle of the whirling squares, that is, a similar shape to the whole, and draw its diagonals and the diagonals of its reciprocals as in Fig. 2.

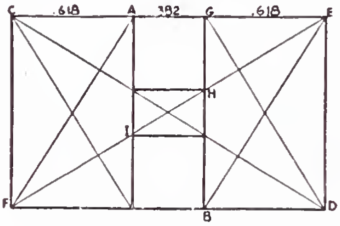


Fig. 2

The areas AF and BE are the two reciprocals of the area CD. These reciprocals exhaust the area CD except the area AB. This last area is important for the present purpose. Considering the area CD as represented by the ratio 1.618 then the line CA represents .618, the line GE .618 and the line AG .382; these added equal 1.618. The area AB which we term a .382 area is composed of the square AH, the whirling square rectangle HI and the square IB. This fraction .382 is the reciprocal of 2.618. Therefore, the area is composed of the whirling square rectangle GI plus the square IB.

The width of the handle of the frying pan divided into its height furnishes the ratio 5.236 and its reciprocal is .191. But 5.236 equals 2.618 multiplied by 2 and .191 equals .382 divided by 2. Consequently, the area

Back view of the detached handle. The sleeping deer at the top is under the pan when the handle is attached and, owing to the uncertainty of the angle, the proportions are difficult to determine.

of the handle of the frying pan occupies one-half the area AB, Fig. 2. The handle design is placed in the center of the .382 area. Reference to the analysis of the design shows that AB equals two squares, CD two squares and BC two whirling square rectangles. The disk of the frying pan of course is a circle escribed by the major square of the major whirling square rectangle.

This is one of the most beautiful bronze utensils of the Sixth Century B. C. which has survived. It is in excellent condition and is covered by a green patina of extraordinary beauty. It must have belonged to a personage of some importance, because with it in a grave at Cività Castellana, the ancient city Faleri, were found a number of fine bronze and silver jugs, all of which are now in the New York Museum. Further comment seems unnecessary. The utensil as an exquisite design to utilitarian purpose speaks for itself.

In the catalogue of the classical collection of the Metropolitan Museum of art Miss G. M. A. Richter, curator of the department, says of the handle of this pan, "The handle of the patera in particular, with its elaborate decorations in relief and open-work, may be regarded as a masterpiece of Greek decorative art."

THE ELEMENTS OF DYNAMIC SYMMETRY

Much of the work upon which the Elements of Dynamic Symmetry is based was done during the year 1919 under the Sachs Research Fellowship of Harvard University. This fellowship was founded by Samuel Sachs of New York City and, under the terms of the foundation, is assigned to scholars of proved ability, whether students, instructors or others, for the general purpose of advancing the Fine Arts. This acknowledgment is but a faint expression of the writer's appreciation of the benefits enjoyed from a fellowship of a type too rare in American Universities. Jay Hambidge.

RHYTHMIC THEMES OF THE WHIRLING SQUARE RECTANGLE

In the preceding lesson we learned how to construct the two most important shapes of Dynamic Symmetry, the root-five rectangle and the rectangle of the whirling squares. It is now necessary to study the methods of making logical subdivisions of these rectangles so as to form themes in area for design purposes. The area themes found in Greek art teach us that the great majority of classic designs were made to conform to certain compound shapes. These shapes may all be reduced to arrangement themes composed of root-five or the whirling square rectangle or some clearly defined subdivision or multiple of these shapes. Moreover, these compound shapes have a base in nature, but for the purpose of avoiding confusion the explanation of this base is reserved for a future lesson.

ROOT-FIVE RECTANGLES WITHIN THE RECTANGLE OF THE WHIRLING SQUARES

Draw a whirling square rectangle (see lesson in the November Diagonal), the diagonals to the whole and the diagonals to the two reciprocals. These diagonals cross at the points A, B, C and D. These points are called the poles or eyes of a rectangle, Fig. 1. Through the eyes AB, Fig. 2, draw the line CD, parallel to the sides of the rectangle. The area CE is a root-five rectangle. AF is a square and AG and BE are whirling square rectangles.

ARITHMETICAL ANALYSIS

If the line HG represents unity or 1. and GE 1.618 then CG equals .7236. The student is advised to construct this shape with a metric scale and study closely the subdivided area.

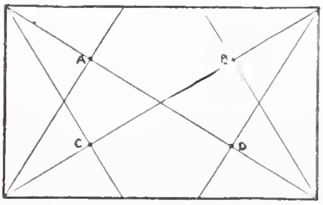


Fig. 1

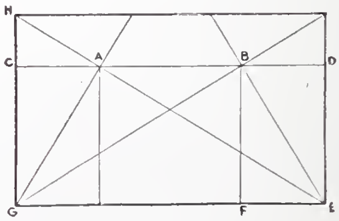


Fig. 2

The reciprocal of the square root of five is .4472. The line GE being the side of a whirling square rectangle, is numerically expressed as 1.6180. This number multiplied by the reciprocal of the square root of five, that is, .4472, gives us the numerical value of the line CG, or .7236. This fraction .7236 divided into 1.618 equals 2.236, or the square root of five. The area CE, therefore, is a root-five rectangle.

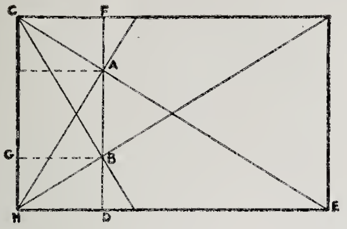


Fig. 3

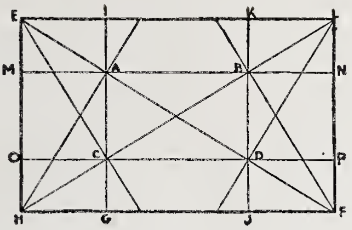


Fig. 4

If, through the eyes AB of the whirling square rectangle CE, Fig. 3, the line DF is drawn parallel to the ends of the rectangle it determines the area CD. This area is that of a root-five rectangle and is composed of the square AG and the two whirling square rectangles CA and BH. The student should remember that we are dealing with similarity of shapes of areas and not sizes.

ARITHMETICAL ANALYSIS

If the line CH of Fig. E is unity or 1. and the line HE is 1.618, and the area is a root-five rectangle, then the line HD is represented by .4472. HD or .4472 divided into CH or 1. equals 2.236 or the square root of five.

Obviously, from the above construction, we may draw lines through the eyes A, B, C and D, Fig. 4, and divide the area of the whirling square rectangle EF into four overlapping root-five rectangles, MF, EP, EG and KF; AJ, DI, AO and BP are squares, and AE, AH, CH, OI, BL, DF, KP and DL are whirling square rectangles.

OTHER SUBDIVISIONS OF THE WHIRLING SQUARE RECTANGLE

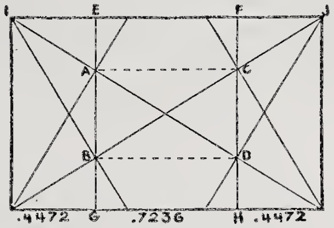


Fig. 5

If the lines EG and FH are drawn through the eyes A, B, C and D of Fig. 5, and the areas GI and HJ are root-five rectangles, then the area EH is a .7236 area. The total length IJ equals 1.618. IE equals .4472 and FJ .4472. These two fractions added equal .8944; this subtracted from 1.618 equals .7236. The fraction .7236 divided into unity or 1. equals 1.382. The area EH, therefore, may be represented by the ratio 1.382 or its reciprocal .7236. AH is a square or 1. and EF is a .382 rectangle. This fraction divided into unity or 1. proves to be the reciprocal of 2.618. Therefore, the area AF may be expressed by the ratio 2.618 or its reciprocal .382. The ratio 2.618 equals 1.618 plus 1. Therefore, the area AF equals a whirling square rectangle plus a square. The area is similar and equal to the area AF. Consequently, the line BG is of equal length with the line AE or is .382. BG added to AE is equivalent to multiplying .382 by 2. It produces .764. The whole line EG equals 1.382. We are now speaking of the rectangle EH and considering EF or GH as unity. If from EG we subtract BG and AE, or .764 from 1.382, the remainder, .618, represents the line AB. The rectangle AD, therefore, is a whirling square rectangle; AB equals .618 and BD equals 1.

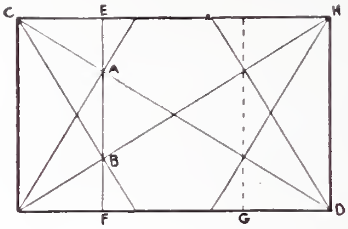


Fig. 6

If the line EF is drawn through the eyes A, B of the whirling square rectangle CD, Fig. 6, it defines the root-five rectangle CF. The line CE equals .4472, and the line EH equals 1.618 minus .4472 or 1.1708. This ratio divided into unity or 1. produces its reciprocal .854. The area ED represented by the ratio 1.1708 or its reciprocal .854, is composed of the root-five rectangle GH plus the 1.382 area EG. If a 1.1708 were used for the purposes of design, one way of subdividing that area would be as described.

Summing up this lesson the following ratios appear other than those of the root-five and the whirling square rectangles:

Ratios	Reciprocals
1.382	.7236
2.618	.382
1.118	.8944
1.309	.764
1.1708	.854

The ratios 1.118 and 1.309 appear indirectly through their reciprocals, .8944 and .764. The ratio 1.118 may be found by dividing the square root of five, 2.236, by 2. The 1.309 shape will be recognized as a square plus two whirling square rectangles. .618 divided by 2 equals .309. The fraction .309 is the reciprocal of 3.236 or 1.618 multiplied by 2.

[To be continued.]

CONTINUATION OF THE ELEMENTS OF THE ROOT-TWO RECTANGLE

SIDES of the rectangular spirals in a root-two rectangle touch.

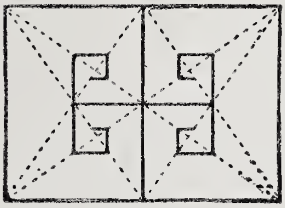


Fig. 1

THE APPLICATION OF AREAS

When the square on the end of a root-two rectangle is "applied" to the area of that rectangle the excess area is composed of a square and a root-two rectangle.

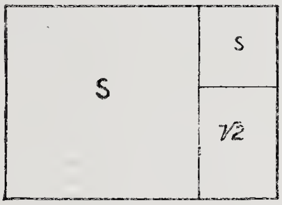


Fig. 2

The application of areas to other areas may be made by geometrical construction or scale. For this purpose a metric scale is used. The square root of two is 1.4142 plus.

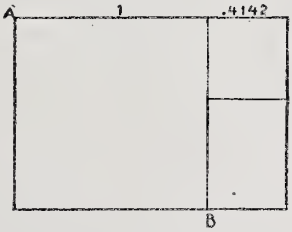


Fig. 3

The end of a rectangle is always regarded as 1. or unity. In the case under consideration AB is the square and the area BC is a square plus a root-two rectangle. The nature of the area BC is determined arithmetically by dividing the fraction .4142 into unity. $\frac{1}{.4142}$ equals 2.4142 and 2.4142 × .4142 equals 1. The ratio 2.4142 plus is readily recognized as 1.4142 plus 1., *i.e.*, a square plus a root-two rectangle.

When a square is applied to each end of a root-two rectangle they overlap and the area of the rectangle is divided into three squares and three root-two rectangles.

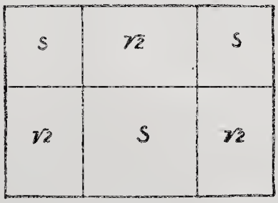


Fig. 4

The area BC is explained above and AD is equal to BC. The area BE is represented by the fraction .5858, and this is obtained by subtracting .4142 plus from 1. The fraction .5858 divided into 1. equals 1.7071. Of this ratio 1. represents a square and .7071 the reciprocal of 1.4142 plus. In a root-two rectangle there are two reciprocals and the end of each is obtained by dividing 1.4142 plus by 2.

The diagonal of a root-two rectangle cuts the side of an applied square and divides the area of that square into two squares and two root-two rectangles.

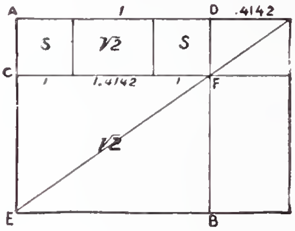


Fig. 5

The area BC of the square AB is a root-two rectangle, because its diagonal is the diagonal of a root-two rectangle. Its side, *i. e.*, AD or CF, is equal to unity, and its end CE equals .7071. The area CD is 1. on the side and the difference between .7071 and unity on the end. .7071 subtracted from 1. equals .2929. This fraction divided into 1. equals 3.4142, *i. e.*, 1.4142 plus 2, or two squares plus a root-two rectangle.

When the square on the end of a root-two rectangle is applied to the side of that rectangle the area of the applied square is divided into two squares.

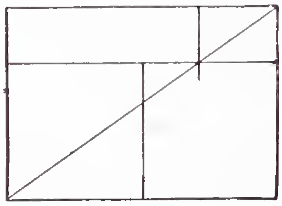


Fig. 6

The application of a square on an end to a side of a rectangle is effected by a simple geometrical construction. The diagonal of a rectangle cuts the side of an applied square on the end as

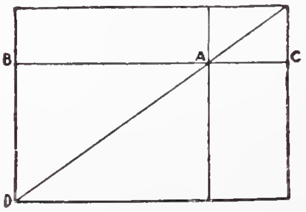


Fig. 7

at the point A. The line BC through A determines the area CD. In a root-two rectangle BD is equal to .7071 and BC 1.4142 plus and .7071 multiplied by 2 equals 1.4142. This is a geometrical fact of great value to design and should be carefully considered. The geometrical proof that the area DE is equal to the area CD is furnished by similarity of figure.

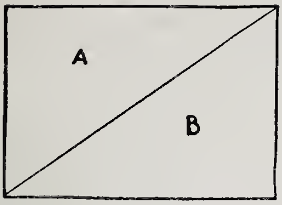


Fig. 8*a*

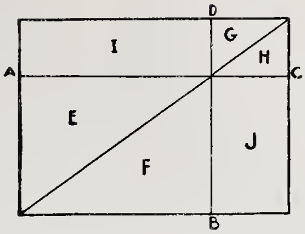


Fig. 8*b*

A diagonal divides the rectangle into two equal and similar triangles, as AB, in *a*, Fig. 8. In *b* the area AB is divided into two equal and similar triangles E, F; G, H are also two equal and similar triangles and the area I must be equal to the area J, because (in *a*) area A equals area B. The triangles E, F are common to the square DK and the rectangle CK, and J of the area CK, is equal to the area I of the square DK.

A root-two rectangle plus a square on the end is expressed by the ratio 2.4142.

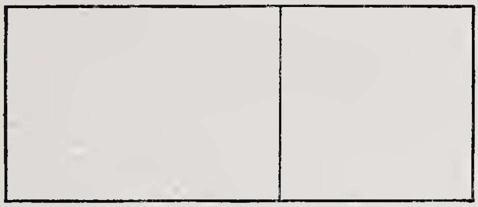


Fig. 9

The reciprocal of a root-two rectangle, .7071, plus a square is expressed by the ratio 1.7071.

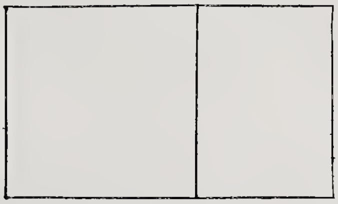


Fig. 10

The ratio 2.4142, as an area, may be shown as a square plus the reciprocal of a root-two rectangle on either end.

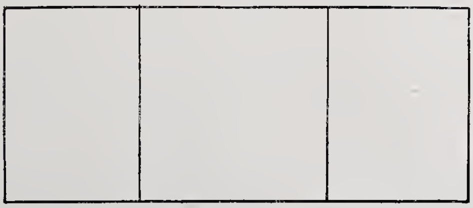


Fig. 11

A root-two rectangle is described within a square by half the diagonal of that square. Fig. 12.

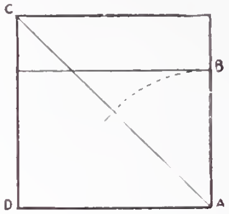


Fig. 12

AB is equal to half the line AC. DB is a root-two rectangle. When root-two rectangles are described on the four sides of a square the area of the square is divided into five squares and four root-two rectangles. Fig. 13.

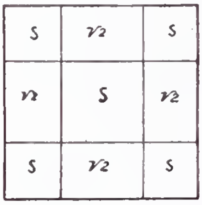


Fig. 13

When a root-two rectangle is applied to the square of a 2.4142 shape the side of this rectangle produced through the major root-two rectangle determines two squares within that shape. Fig. 14.

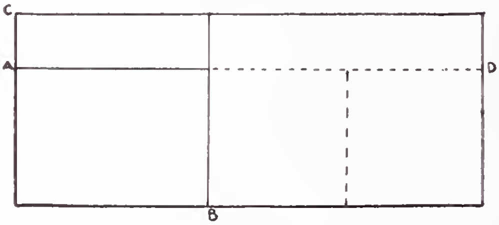


Fig. 14

AB is a root-two rectangle applied to the square BC; the continuation of its side is the line AD; this line fixes the area DB, which is composed of two squares.

The side of a square applied to the .7071 part of a 1.7071 rectangle projected through the major square defines a root-two rectangle in that square. Fig. 15.

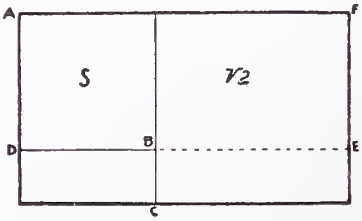


Fig. 15

AB is a square applied to the root-two rectangle AC; the side of this square DB, projected to E defines the root-two rectangle BF in the square FC

[To be continued.]

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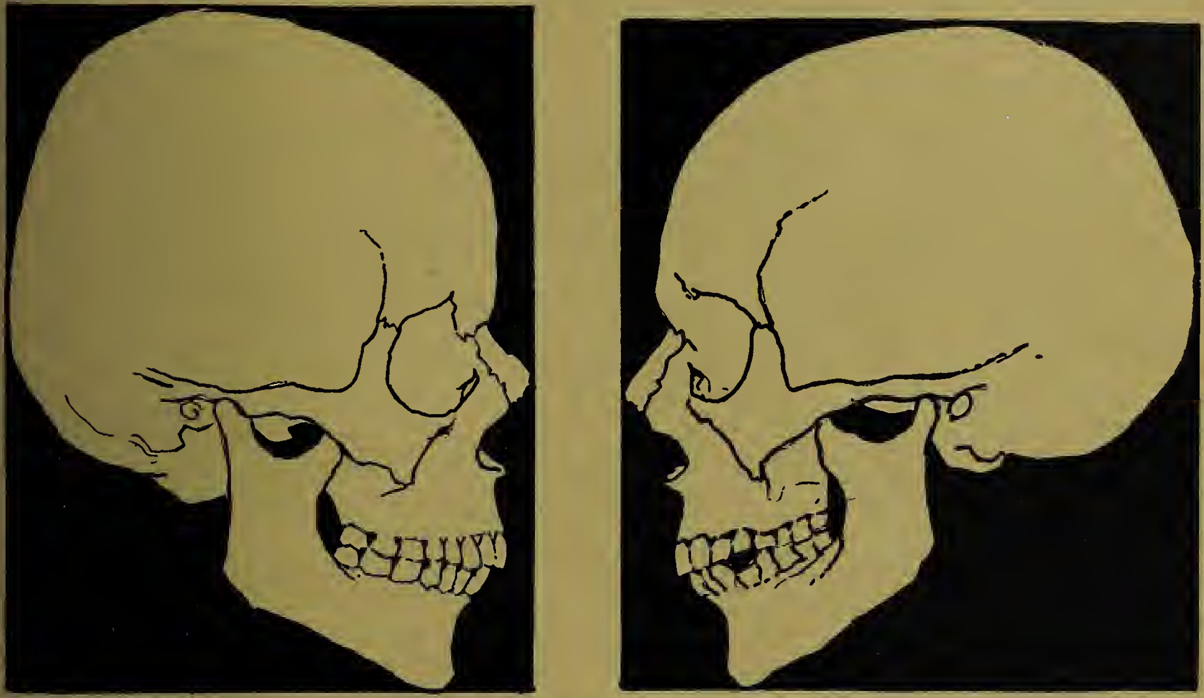
THE DIAGONAL

AN ILLUSTRATED MONTHLY MAGAZINE DEVOTED TO THE EXPLANATION OF THE REDISCOVERED PRINCIPLES OF GREEK DESIGN, THEIR APPEARANCE IN NATURE AND THEIR APPLICATION TO THE NEEDS OF MODERN ART

EDITED BY JAY HAMBIDGE



A Round Head—Individuality emphasized. Actual projections

A Long Head.—Worn condition of a tooth in the lower jaw shows the subject was a pipe smoker

THE CHARACTER OF THE SKULL

Brain capacity of each head approximately the same

See article on the Human Figure in this number of The Diagonal

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APPLICATION FOR ENTRY AS SECOND-CLASS MATTER PENDING AT THE NEW HAVEN POST OFFICE

THE DIAGONAL

THE DYNAMIC SYMMETRY OF THE PLANT

THE importance to design of the curve-cross pattern arrangements found in the leaf distribution phenomena lies in the fact that the normal scheme represented by the sunflower disk is clearly connected with commensurability of area. And, because an area may represent the projection of a solid, the measureableness of the two-dimensional plan is also that of the three-dimensional plant example. The dynamic rectangles, which we obtain from the growth phenomena, are distinguished by this property of area measureableness. It is this characteristic which lies at the base of the rhythmic·theme conception and gives the dynamic scheme its greatest design value. Because of the persistence of the normal ratios of phyllotaxis the conclusion is inevitable that the measurable area themes possess life and all the qualities that go with it, while areas which do not have this peculiar property do not have life. They are "static" or dead areas, at least as far as design is concerned. If the testimony of Greek art has value this would seem to be so. We know that one characteristic of Greek design is just this life-suggesting quality. Many have noticed it. We know also that Roman art, by comparison, is lifeless. Many have noticed and commented upon this fact. Indeed, we need not go further than Roman sculpture, with its surface commonplaceness and stodgy, uninspired design quality, to see how true this is.

It is recognized that the substitute summation series which has been suggested to take the place of the whole number series used by botanists to express the curve-cross phenomena of distribution (see November DIAGONAL), can have little if any value in checking an actual plant structure. The fractions necessary to make the ratio 1.618 complete for the system are so small

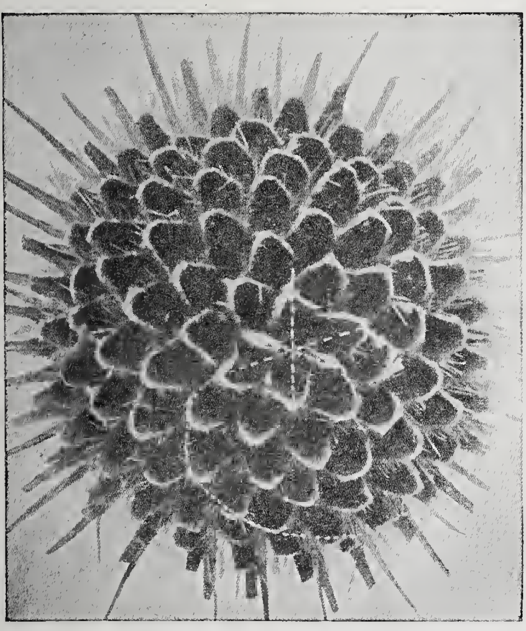

Fig. 1*a*—*Pinus pumilis,* bijugate cone of type 6/10/16

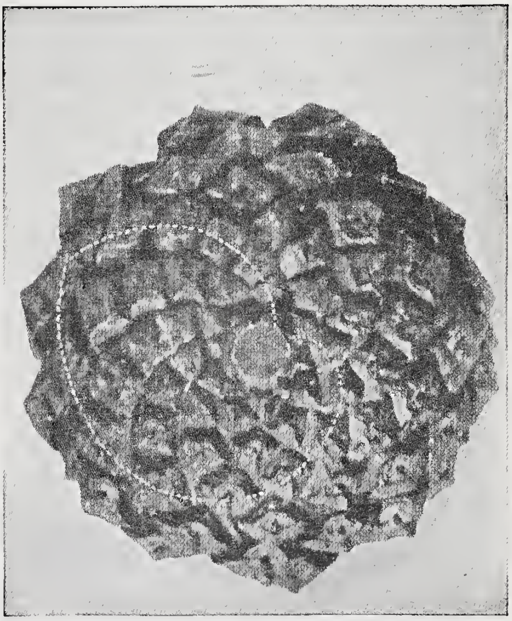

Fig. 1*b*—*Dipsacus pilosus,* L. Inflorescence of the type 10/16

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that their presence in a specimen could not be noted. We are using the phyllotaxis phenomena for purposes of design and are not so much interested in botanical research. The plant structure is so delicate and the skeletal residue so slight after an average plant withers that direct observation for commensurable area themes is out of the question. We may leave the field to the botanist and let him retain the conventional series of whole numbers. To the artist, however, the numbers of the substitute series, 118, 191, 309, 500, 809, 1309, 2118, 3427, 5545, etc., are of much interest, because they not only furnish the exact ratio of 1.6180, but each member of the series is an actual ratio of the dynamic commensurable area scheme and is found abundantly not only in the human skeleton, but throughout classic Greek design. Nature provides many plant examples wherein the curve-cross system varies from the normal whole number series. These less common arrangements of curve-cross pattern form are mentioned by Church in the following terms:

"When the type of normal asymmetrical phyllotaxis is thus completely isolated as consisting of systems mapped out by log spiral curves in the ratio-series of Braun and Fibonacci, 2, 3, 5, 8, etc.; and the type of normal symmetrical phyllotaxis is equally clearly delimited as a secondary construction, physiologically independent of the ratio-series, though connected with it phylogenetically, the greatest interest attaches to all other phyllotaxis phenomena, which, though less common, may throw light on the causes which tend to induce symmetry, before postulating, as a last resource, some hypothetical inherent tendency in the protoplasm itself.

"These may be included under two series: first, the multijugate systems of Bravais, and, second, systems in which the parastichy ratios belong to series other than that of Braun and Fibonacci, e.g., the 3, 4, 7, 11———, 4, 5, 9, 14———, or still higher series (these latter Church classifies as belonging to anomalous series. It will be noticed that they also are summation series).

"The term *multijugate* was applied by the brothers Bravais to types of phyllotaxis in which the numbers expressing the parastichy ratios are divisible by a common factor; so that 2 (13/21) = (26/42), a bijugate system; while 3 (13/21) = (39/63) would be a trijugate one."

The fact that they may occur in the plant (i.e., multijugate systems) which has already been found to exhibit normal phyllotaxis phenomena most completely, lends additional interest to these constructions. Thus, out of a batch of heads, collected at haphazard by E. G. Broome, two were bijugate, 26/42 (read 26 crossing 42) and 42/68, respectively; the others were quite normal; while out of a total crop of 130 cones on a plant of *pinus pumilo* one cone only was 6/10/16—see illustration—the rest being normal, "Phyllotaxis in Relation to Mechanical Law," or 5/8/13.

These so-called multijugate types are interesting, particularly when a specimen shows a variation from the normal scheme to one of multiples of that ratio by 2 or 3. Using the suggested substitute series and considering the commensurable area themes of dynamic symmetry, the bijugate scheme as 26 crossing 42 is 1309 crossing 2118 wherein both members are multiplied by 2. The trijugate arrangement, 39 crossing 63, becomes 1309 and 2118 multiplied by three, and the schemes would be, as measurable area arrangements, 2618, 4236, 3927, 6354, respectively. A 6/10/16 plan, as a commensurable area theme of Dynamic Symmetry would be .618, 1. and 1.618.

This leads us to consider somewhat in detail the substitute summation series. Its properties are curiously interesting. Any two members of the series added equals a third member.

Every term divided into its successor equals the ratio 1.618.

Every term divided into a third term equals 1.618 squared or 2.618.

Every term divided into a fourth term equals 1.618 cubed or 4.236.

Every term divided into a fifth term equals 1.618 raised to the 4th power or 6.854.

Every term divided into a sixth term equals 1.618 raised to the 5th power or 11.090, etc.

Powers of 1.618 divided by 2 produce the summation series of numbers.

$\frac{2618}{2}$ equals 1309

$\frac{4236}{2}$ equals 2118

$\frac{6854}{2}$ equals 3427

$\frac{11090}{2}$ equals 5545

$\frac{17944}{2}$ equals 8972

$\frac{29034}{2}$ equals 14517

$\frac{46918}{2}$ equals 23459, etc.

1.618 divided by 2 equals .809.
Unity divided by 2 equals .5.

.618 squared equals .382

.618 cubed equals .236

.618 4th power equals .146, etc.

$\frac{.618}{2}$ equals .309

$\frac{.382}{2}$ equals .191

$\frac{.236}{2}$ equals .118, etc.

The square of 1.618 equals 1.618 plus 1. or 2.618.
.618 plus .618 squared equals 1.

The student will recognize all these numbers as ratios representing the dynamic measurable areas. The few relationships mentioned are merely suggestive; an extraordinary number of curiously fascinating interdependencies may be worked out of this summation series. When the series is charted as the expansion of a scheme of squares and subdivisions of squares it appears as in Fig. 2.

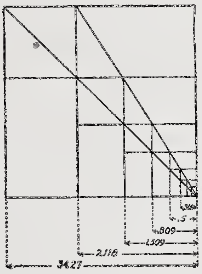


Fig. 2

It will be clear to those who have studied dynamic symmetry that the basic pattern plan of leaf arrangement is but one phase of an architectural scheme connected with the phenomena of growth in general. Fortunately, as artists, we need not depend upon plant forms for detailed knowledge of the mysteries of dynamic form. The human skeleton furnishes all that the plant does and more. It is practically impossible to measure the withered plant structure. Comparatively, the bone network of man's frame is more stable than the skeleton of the plant, and, what is of much greater artistic importance, man is and always has been recognized as the most perfect type of design in nature.

It is felt that the foregoing rough outline of the dynamic symmetry of the plant is all that is necessary to show the artist that the dynamic scheme has a general basis in plant growth.

[The End.]

THE DYNAMIC SYMMETRY OF MAN

ROMAN report of Greek tradition says that classic artists took extraordinary care to have their designs conform to the principles of a certain symmetry which they derived from the human figure. The Romans, however, assumed that this symmetry depended upon a modulus or length measurement which was multiplied or divided into even units to determine the proportions of the composing elements of a design, thus producing an ensemble the parts of which were commensurable with each other and with the whole. Dynamic Symmetry shows us that the Romans were in error in their interpretation of this tradition. No Greek design of the first class has been found wherein commensurability of line exists, and this is also true of the figure of man in nature. Design in nature and design in Greek art show conclusively that parts are invariably commensurable with the whole but not in line; the measurableness is that of area. When the human figure is measured and interpreted in terms of area the result is a revelation to art.

"Marcus Vitruvius Pollio was a famous Roman architect and engineer and was a military engineer under Caesar and Augustus. He wrote a treatise on architecture in ten books ('De Architectura'). It was dedicated to Augustus and is the only surviving Roman treatise on the subject. His book was well known to Pliny and on it was based almost all the earlier theory and practice of Renaissance and pseudo-classical architecture." *The Century Dictionary.*

Vitruvius gave elaborate rules for the construction of buildings of all kinds in the Greek style. He also said that the Greeks were careful to have their designs, especially of their temples, conform to the laws of symmetry. The principles of this symmetry, he said, were determined from the human figure, the Greeks having noticed that the members of the human structure were commensurable with the body as a whole. Modern research has shown that Vitruvius misinterpreted the Greek tradition, because no building has been found which in any particular conforms to his rules. In one of the ten books Vitruvius attempted to show the symmetry of man. Like the rules for the symmetry of architecture those advanced for the figure have been found to be without value, being in fact nothing more than a recipe. It has taken 2000 years to discredit these Roman recipes.

The entire Vitruvian scheme depends upon the use of a modulus. The composing elements of a building were to be made to conform to some unit of length which was derived from the plan, and were to be expressed as so many multiples or fractional parts of this modulus. The Roman rules for figure symmetry are somewhat the same as those for architecture.

Greek statuary and design in general, as well as Greek building, do not in any way bear out the Roman interpretation of the Greek tradition. In fact, the Roman rules, by their definition, prove their fallacy. The modulus idea, used in any manner, would necessarily produce static symmetry. It is not possible to express the human structure in even multiples or even fractional multiples of any member such as head, hand, foot, arm or leg. As lengths these members are incommensurable, unmeasurable one with the other.

The members of the human body are indeed measurable one by the other, but not in *line*, as the Romans thought. Both nature and Greek art show that the measurableness of symmetry is that of *area* and not *line*. The Romans were not given to scientific study or they would have heard of the Greek classification of irrationals, a subject which was taught at Alexandria in Roman times. This classification, familiar to us as the tenth book of Euclid's Elements, shows that lines incommensurable in length are not necessarily irrational; they may be commensurable in square. This is the secret. Dynamic Symmetry deals with commensurable areas. The symmetry of man and plant is dynamic; the symmetry of the entire fabric of classic art, including buildings, statuary and the products of the crafts, is dynamic. The symmetry of all art since Greek classic times is static.

Dynamic analysis of the human structure discloses an aesthetic situation of the greatest promise to modern art. The high standard of perfection in Greek art has always had a depressing effect upon artists who have studied it. The standard, indeed, is so high that it would seem impossible of rivalry; in fact very few, if any, have made an attempt. It has come to be accepted as beyond human power and, for comfort of conscience and solace of pride, most dismiss it as a freak of nature and the men who produced it as superhuman, demi-gods, with whom it is impossible to compete. Yet Greek art was not a freak. It was the logical development and outcome of a sound attitude toward nature and toward art assisted by a happy combination of circumstance which placed in the hands of alert and eager artists the proper tool at the most propitious time for its use.

Almost the only criticism which can be made of Greek art is that possibly its creators overstressed ideality of type. The rapidity with which classic artists mastered the technique of human figure construction (it was an accomplishment of less than a hundred years), probably blinded them so that they could not appreciate how immensely superior they had become. The easy power of execution they so soon acquired seems to have made them indifferent to the charm of individual character. At any rate this is a field they neglected,—fortunately for us. The complaint of most who contemplate Greek art is that it is too austere, too far removed from actuality of life, too perfect for human association. To an extent this is true, otherwise taste for Greek art would not be so much a matter of acquisition. Since classic Greek days humanity has had to go through great emotional experiences. These have emphasized the importance of human brotherhood and aroused the interest of man in his fellow. In art this awakened interest, as a rule, has taken the form of an intensive examination and treatment of the superficial aspects of the individual. Interest in portraiture is an illustration of this. The larger abstract aspects of design have become almost completely lost through the diversion of artistic interest from the general to the particular. Humanism, with all of its valuable contributions toward the advancement of human good, had just this reaction on art. The work of the great masters of the Middle Ages and the Renaissance apart from its emotionally moving compositional power, emphasized the individual by stress of superficial appearance. The real spirit of the individual was seldom, if ever, realized. And when these artists attempted the creation of a great symbol by the human figure the result was nothing but a symbol; the human medium became impossible, neither god nor human. The Moses of Michael Angelo cannot exist as a human; it is simply a pawn symbol created by the artist to express an emotional idea. The artist was compelled to use such a pawn to avoid the triviality of individuality when it is expressed by its superficial and accidental markings. He knew no method of attaining nobility of individual character in design. Yet nobility of individual character can be realized in design. Its essentials lie in the structure of every man and it can be brought out and given its due position when the artist understands human symmetry. Every individual has within him, apart from his mind or soul, something of the godlike and this something is the character perfection contained in his symmetry.

It will be noticed that these articles treat of the structure of man by considering individual example in detail. This would not be possible without Dynamic Symmetry. Heretofore the artistic structure and anatomy of the human figure have been exemplified by an impossible generalized manikin. Experience with life has taught the artist that a manikin is useless for this purpose. It shows nothing whatever of the artistic wonders of the individual. It is in the just artistic development of the individual character in art and in design that the promise given by the dynamic analysis of man will be fulfilled. This is an untilled field.

In the preceding article the front elevation rectangle of the skull, of the skeleton under analysis, was explained as a simple subdivision of an area. The student will find it profitable to review the ratios therein explained and compare them with those developed by Dr. Caskey's analysis of the bronze Greek mirror in this number of THE DIAGONAL. These ratios are all basic and may be used as rhythmic area themes for any design purposes.

THE SIDE ELEVATION OF THE SKULL

The height of the skull measures 2236, the width 2000. The lesser measurement divided into the greater gives the ratio 1.118 (see examples of design created within this rectangle in the writer's book on the Greek Vase); this ratio is equal to the square root of five, 2.236, divided by 2. As an area it may be regarded as half of a root-five rectangle. Fig. 1 is a root-five rectangle divided in half and the heavy lines show the area shape of the skull. It will be noticed that this area is composed of two root-five rectangles. We must now divide the area of the side elevation of the skull by the same lines of subdivision used on the front. The first line of subdivision is 1427 mm. from the top; the second 1427 mm. from the bottom. These two lines divide the side elevation into three rectangles, two of which measure 809 by 2 and one 618 by 2. The measure 809 divided into 2 equals the ratio 2.472 and 618 into 2 produces 3.236. These two ratios show us that the rectangle of the side elevation of the skull is divided into ten whirling square rectangles by the subdivisional lines of the front elevation. The ratio 2.472 equals .618 multiplied by 4 and 3.236 equals 1.618 multiplied by 2. Fig. 2 shows the arrangement. It will be interesting practice for the student to prove how and why five whirling square rectangles can fill up the area of a root-five rectangle.

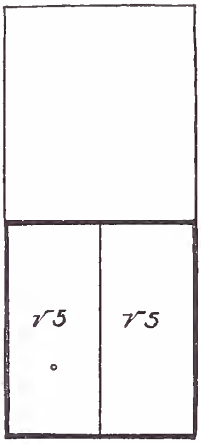


Fig. 1

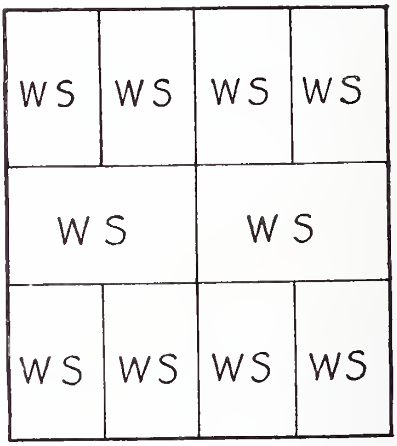


Fig. 2—Symmetry of Man

No attempt is as yet made to consider the facial or head details of the two elevations. This will be done after the rhythmic theme of the figure has been developed.

THE TORSO AND THE HEAD

The width of the shoulders, acromion point to acromion point, is 4045 mm., the height, from the base of the pelvis to the top of the skull, is 9045. The lesser divided into the greater gives the ratio 2.236; the greater into the lesser .4472. The body and head of the skeleton example, therefore, are contained, accurately, within a root-five rectangle. The height of the pelvis measures 2382, the distance from the top of the skull to the shoulders or clavicle 2618. These two measurements added equal 5000. This compound measurement subtracted from 9045 equals 4045. But this is the shoulder width; consequently, when the vertex-clavicle and the pelvic rectangle are subtracted from the root-five rectangle, 4045 × 9045, the remaining area is a square. But a root-five rectangle equals a square plus two whirling square rectangles: 4045 divided into 5 equals 1.236 or two whirling square rectangles, .618 multiplied by 2.

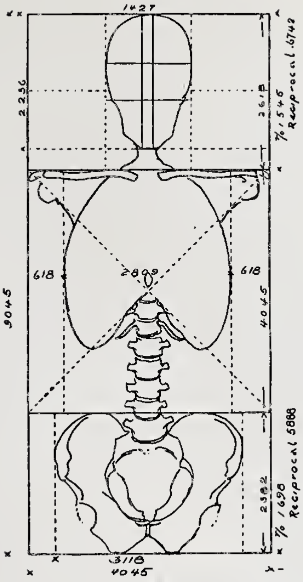


Fig. 3—Dynamic Symmetry of Man

THE ACTUAL PELVIC RECTANGLE

The height of the pelvis measures 2382 mm.; the width, 3118. The lesser divided into the greater gives the ratio 1.309. This ratio has been found to be that of every male pelvis so far

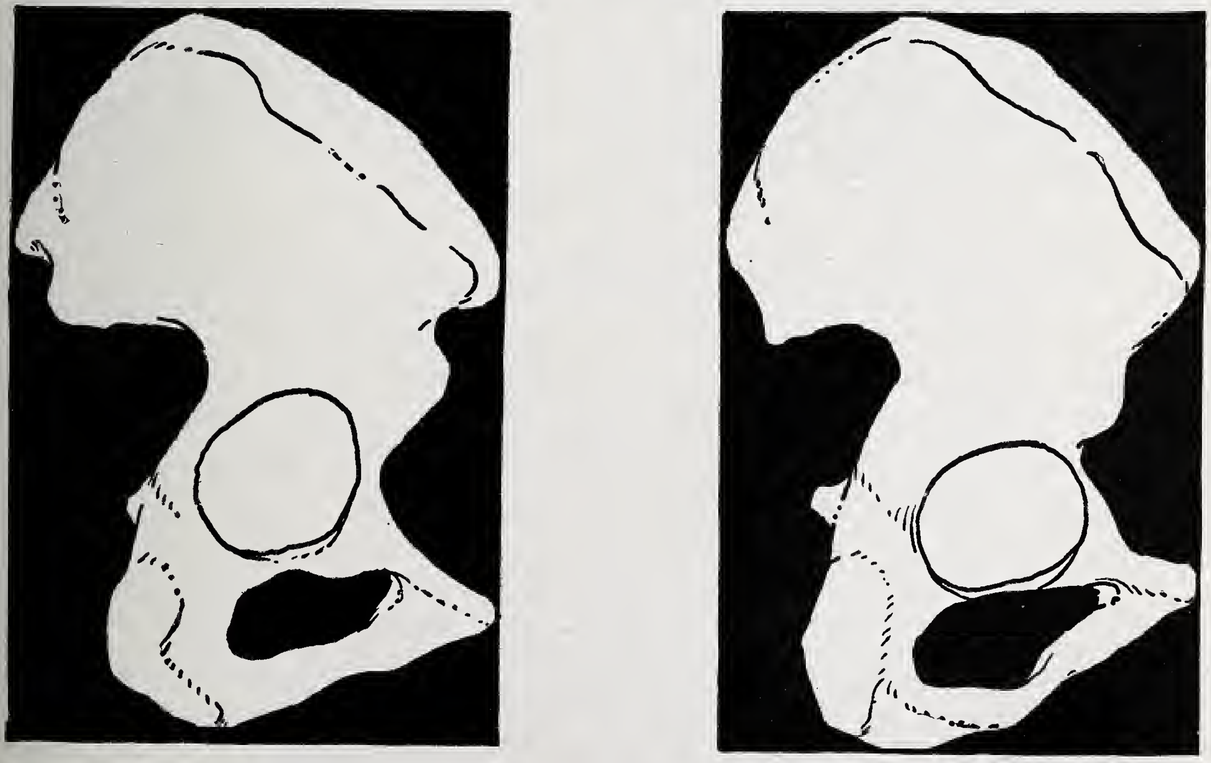


Fig. 4*a*—Female Pelvis

Fig. 4*b*—Male Pelvis

Side elevations of the human pelvis showing the points which touch a perpendicular plane measured. Every female pelvis, so far measured, supplies a 1.382 rectangle (the error of measurement and that of parallelism is astonishingly small), and the side elevation of every pelvis, male or female, is one-half of the rectangle of the front elevation, except the side elevation of the pelvis of the skeleton now being examined. This is a whirling square rectangle. It, however, belongs to an abnormal man. Half of 1.309 is .6545 and this is the reciprocal of 1.528 minus. This is a shape wherein a root-five rectangle plays an important part. Half of the ratio 1.382 equals .691, the reciprocal of 1.4472, a square and a root-five rectangle. If these pelves were cut transversely the plan would be that of two squares. We must be very close to the normal for the white race pelvis. The surgical index for the white pelvis, irrespective of sex, is .73 (Cunningham, "Surgical Anatomy," p. 288). The reciprocal of 1.382 is .7236, that for 1.309 is .764 minus. Surgical measurements are well known to be approximations made for rough comparison of racial types, usually. They vary greatly, depending often on the position assumed for the pelvis in the erect standing position of the skeleton. The writer finds that the correct position for the pelvis is that where four points on the front touch a plane perpendicular to the erect position. This position enables us to obtain easily a reliable projection.

[To be continued.]

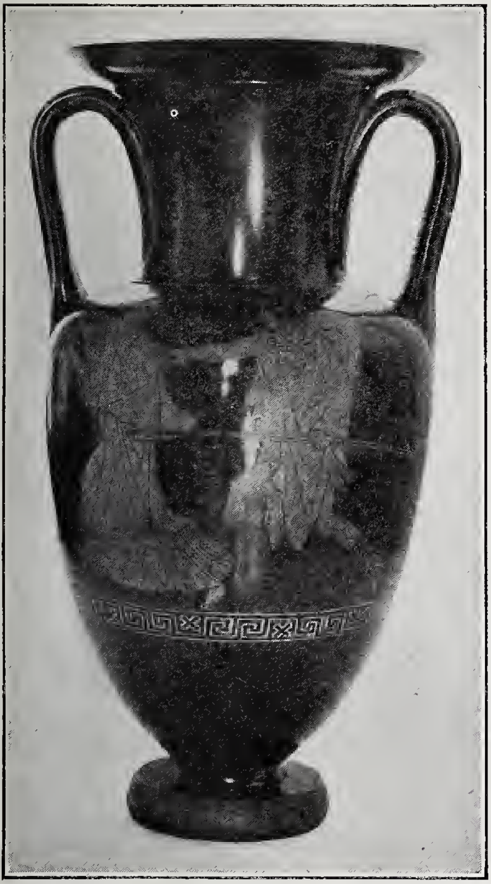


Fig. 5*a*

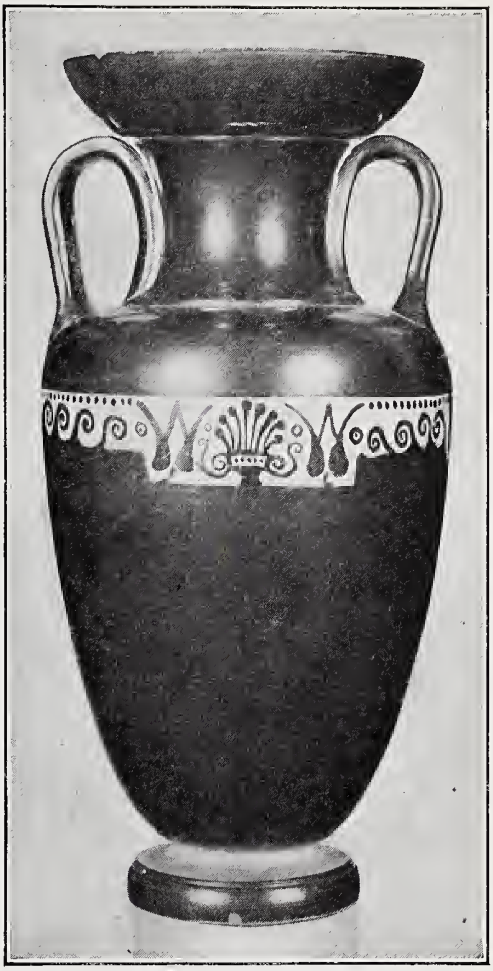


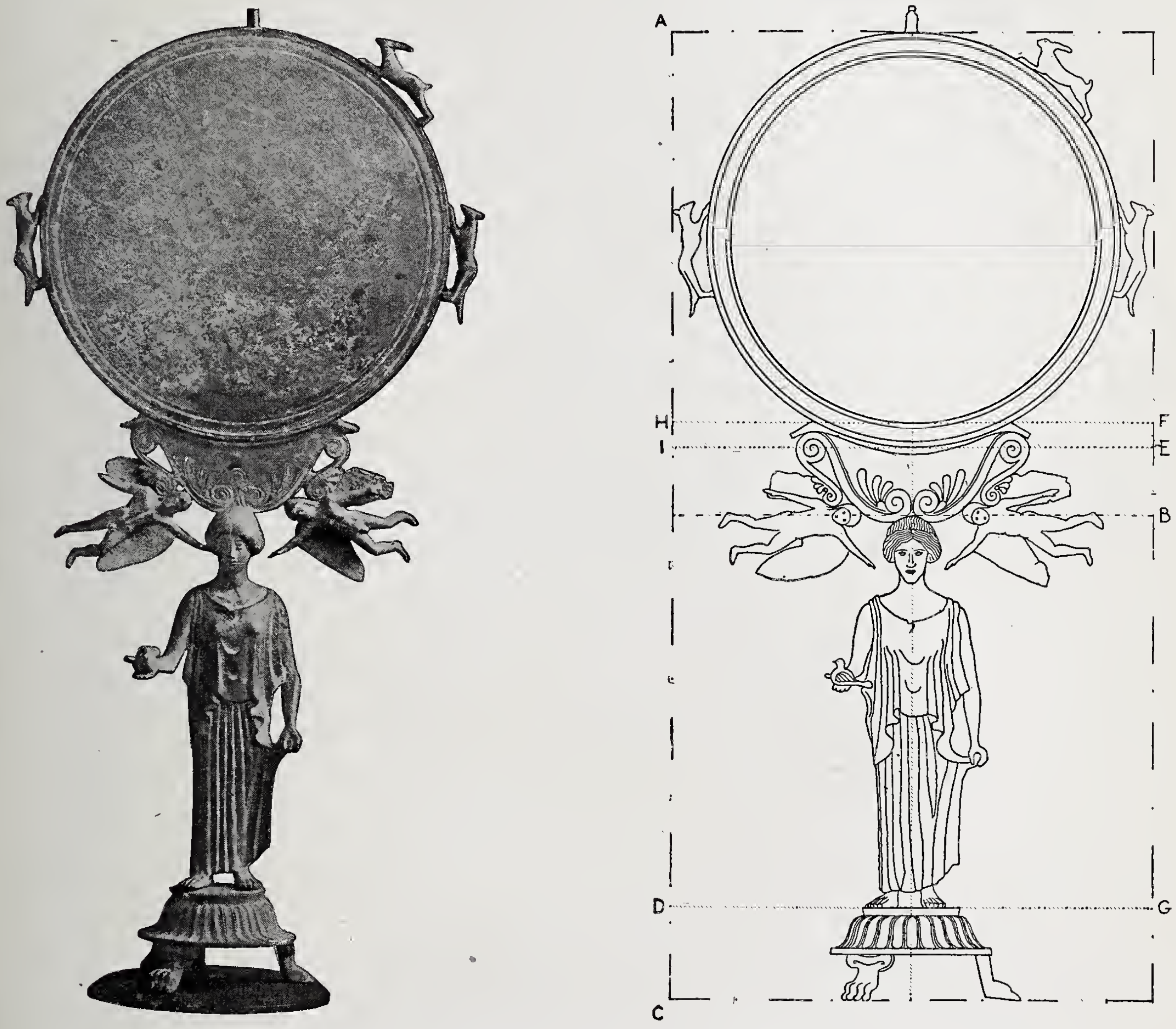
Fig. 5*b*

AN EXAMPLE OF REAL GREEK DESIGN AND AN ITALIAN IMITATION

THE difference between a real example of Greek pottery and an imitation is shown by the two photographs above. The one on the left is a genuine Nolan Amphora of the best type. This is Number 136 in the Stoddard Collection at Yale University. The one on the right is an Italian imitation from Campania, now in the Museum of Fine Arts at Boston. The genuine has an overall ratio of 1.809 (see article on the dynamic symmetry of man in the December Diagonal) and the details of the wonderful shape form a charming theme in rhythmic area. The width of the bowl is exactly one-half the height. The meander band at the base of the pictorial composition passes through the center of the major square, that is, the square which represents 1. of the 1.809 ratio. In the theme arrangement two whirling square rectangles stand side by side on top of this square. The fraction .809 represents these two shapes because it is the reciprocal of 1.236 or .618 multiplied by 2. It is also equal to 1.618 divided by 2. The juncture of the neck with the bowl is a line passing through the intersection of a diagonal to one of these whirling square rectangles with a diagonal to the two shapes taken together. The theme arrangement throughout is simple.

These two photographs show more clearly than pages of description the difference between a design pulsating with the proportions of growth symmetry and one in which no symmetry of value appears and, consequently, is leaden and dead. The mechanical craftmanship of the Italian example is good and the black glaze excellent, but the design, even the decoration, is unclassical and commonplace.

DYNAMIC SYMMETRY OF A GREEK MIRROR ALL THE PROPORTIONS OF WHICH ARE FOUND IN AN ACTUAL SKELETON



Fifth Century Greek Bronze Mirror, Museum of Fine Arts, Boston. No. 98.667.

A Fifth Century B.C. Greek bronze mirror, 98.667 in the Museum of Fine Arts, Boston, is a dynamic design within the area of two squares. AB is one of these squares and BC the other. The area AG, that is the overall shape minus the height of the pedestal, is a 1.809 rectangle. The overall ratio is 2. or the square root of 4; the height of the pedestal equals .191. This subtracted from 2. equals 1.809. CG subtracted from the square BC leaves the .809 area BD or two whirling squares. This equals the height of the figure. The area BH is equal to the area CG. AF is .809 or two whirling square rectangles. The area AE, which incloses the disk of the mirror, is a 1.1708 shape, the reciprocal of which is .854. The ratio 1.1708 may be analyzed as a compound form composed of the reciprocal ratios .618 plus .5528; this last is the reciprocal of 1.809. The area BI is represented by the difference between .854 and unity or .146. BH equals .191, therefore, EH is .045. It is interesting to find this fraction identified with an actual Greek design, because it appears in the ratios 1.045 and 2.045, both of which seem to have been considered quite important by the Greeks. The ratio 1.045 is composed of the reciprocal ratios .618 and .427, this latter being equal to .854 divided by two. A beautiful bronze oinochoe in the Boston Museum has an overall ratio of 2.045. This example will be pictured and described in THE DIAGONAL in the near future. Every ratio found in the rhythmic theme of this mirror appears in the area theme arrangement of the vertex-clavicle rectangle of the human skeleton described in the December number of THE DIAGONAL. The student should study the two comparatively. The projection at the top of the mirror disk contained originally a ring or a small modelled siren with wings, to which the ring was fastened. This detail, however, has not survived. Probably the original plan was a root-five theme. It will furnish good practice for the student to assume that this was the case and to figure out arithmetically the relation of the two squares, with their dynamic sub-divisions, to the root-five area. Originally the mirror had two hares on the disk, but one of these is now missing.

The pedestal for the figure stands on three feet, but contrary to what might be expected, and what would be the case in 999 modern examples out of a thousand, these feet are not placed on the points of an equilateral triangle described in the circle of the bottom of the pedestal. This mirror was carefully measured and the drawing made by Dr. L. D. Caskey, Curator of Greek Antiquities in the Boston Museum.

GENERAL CONSTRUCTIONS FOR SIMILARITY OF FIGURE

THE average modern artist knows that rectangular shapes are enlarged or reduced by a diagonal. If the painter or designer desires to make a shape similar to the shape of his canvas, or any other rectangular area he may be using, he draws a diagonal. Any rectangular figure with a diagonal common to the diagonal of the whole is similar. Fig. 1 shows this.

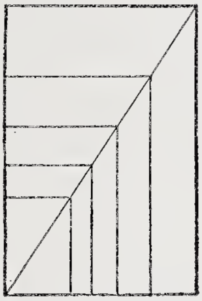


Fig. 1

AB is a diagonal to any rectangular area. The areas AC, AD, AE, AF are similar figures to the whole, because their diagonals are common to the shape AB. The modern artist also knows that similar figures to the whole may be constructed as in Fig. 2.

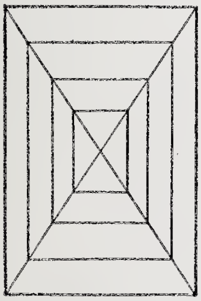


Fig. 2*a*

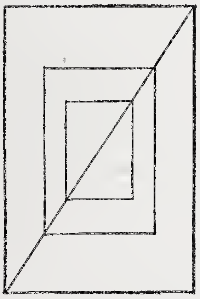


Fig. 2*b*

This property of the diagonal is used extensively in the arts, particularly the reproductive arts, to enlarge or reduce drawings. This knowledge proves its value every day in the arts, but it is insufficient. There are many cases wherein similarity is not only desirable but neces-

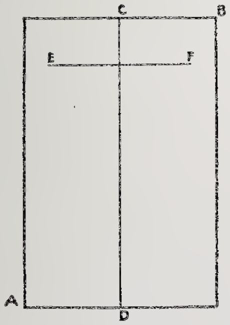


Fig. 3*a*

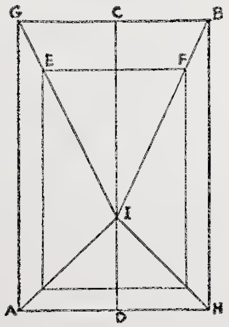


Fig. 3*b*

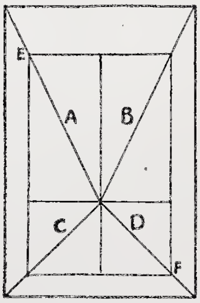


Fig. 3*c*

sary, and for such the method described does not apply. Figs. 1 and 2, show similar shapes either centered in the parent form or moving up or down on either one of the two diagonals. But suppose a similar figure is desired which may be moved up or down on a medial line and, therefore, must be unsymmetrical with the top and bottom line of the enclosing space. Fig. 3, *a*, *b* and *c*, shows how this may be accomplished.

Suppose it is desired to construct a similar figure to the whole upon the line EF in the rectangle AB of Fig. 3*a*. CD is a medial line. From the corners BG of Fig. 3*b* draw the lines BI and GI, passing through the points E and F. Also draw the lines AI, HI and complete the rectangle. Fig. 3*c* furnishes the proof that the rectangle EF is a similar shape to the whole constructed on a medial line. The lines BI, GI, AI and HI of *b* are diagonals to areas which exhaust the area AB. These diagonals are also common to diagonals of areas which exhaust the area EF of *c*.

Suppose, for example, it is desired to construct a similar figure to the whole from any points in a given rectangle. Fig. 4 shows the process.

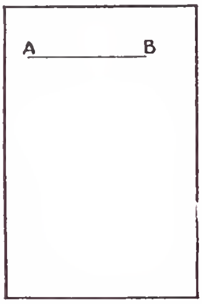


Fig. 4*a*

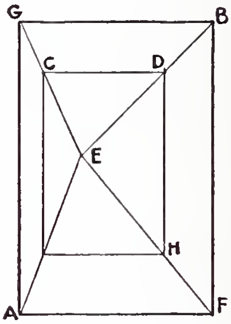


Fig. 4*b*

We shall assume that the line upon which the similar figure is to be constructed is AB of Fig. 4*a*. Draw the lines BE, GE, as in *b*, passing through the points C and D; also the lines AE, FE. Complete the rectangle. The area CH is a similar shape to the figure AB for the same reasons given for Fig. 3. The constructions for the definition of similar figures, of course, apply to any rectangle. It is advisable, however, to use only the rectangles of Dynamic Symmetry, because of their measurable properties. Unless rectangles are used which possess this dynamic property of modulation and measurableness it will be impossible to give design the vital qualities of life and growth. Design created within rectangles which do not possess this function of commensurable area are always flat and dead.

Using dynamic rectangles we may construct within them either a similar figure to the whole in any position, where sides and ends are parallel to those of the parent figure, or a figure which is similar to any dynamic subdivision of that area. Assume, for example, it is required to draw a square in some place, within the major square of a whirling square rectangle. In that case the asymmetric diagonals will appear as in Fig. 5.

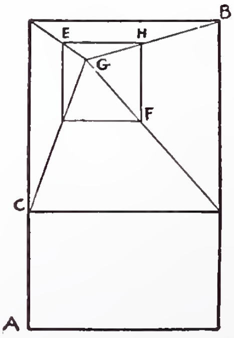


Fig. 5

AB is a whirling square rectangle and BC is its major square. EF is a square in some position within the area of the major square. Again, it is better to select a point, as G, which is a dynamic point, or line, as EH, which is a dynamic line. The arrangement of Fig. 5 applies to any dynamic area other than the square and the points for these areas may be either on the side or end of the parent rectangle. Fig. 6 will make this clear.

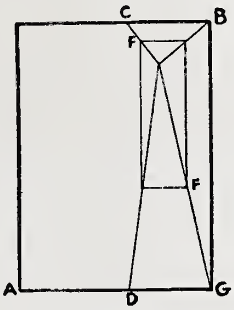


Fig. 6

AB is a dynamic rectangle and C is a dynamic point of subdivision on one end. EF is a dynamic rectangle similar to the area CG.

Any dynamic subdivision of a dynamic rectangle is like a seed endowed with the eternal principle of growth. It possesses the property of expanding or dividing until it includes the entire dynamic system. Like an osier twig planted in congenial soil which soon develops into a beautiful tree, dynamic shapes have in them the life impulse which causes design to grow.

THE ELEMENTS OF DYNAMIC SYMMETRY

Much of the work upon which the Elements of Dynamic Symmetry is based was done during the year 1919 under the Sachs Research Fellowship of Harvard University. This fellowship was founded by Samuel Sachs of New York City and, under the terms of the foundation, is assigned to scholars of proved ability, whether students, instructors or others, for the general purpose of advancing the Fine Arts. This acknowledgment is but a faint expression of the writer's appreciation of the benefits enjoyed from a fellowship of a type too rare in American Universities. JAY HAMBIDGE.

HAVING considered the root-five and whirling square rectangles in a general way and indicated the manner by which the simple operations of arithmetic may be used to prove geometric construction and reduce areas to understandable forms, we are in a position to take up the great class of compound shapes.

THE SQUARE PLUS ROOT-FIVE RECTANGLE

One of the most important of these is represented by the ratio 1.4472. This is the key ratio for the Parthenon plan and appears as an area motif many times in Greek design. We find a natural source for this rectangle in the regular pentagon. The ratio of the radii of the inscribed and escribed circles of a regular pentagon is 1 : .809. The fraction .809 is the reciprocal of 1.236 (.618 multiplied by 2). It is also equal to 1.618 divided by 2. The relation of this ratio to the whirling square scheme is, therefore, apparent. The reciprocal of the square root of five is ·4472, consequently the ratio 1.4472 represents a square plus a root-five rectangle. If a circle is drawn within a square and within that circle a regular pentagon is drawn, as Fig. 1, the area fixed by the intersection of two diagonals of the pentagon is a 1.4472 shape.

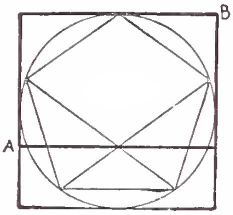


Fig. 1*a*

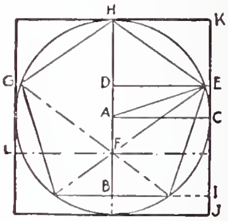


Fig. 1*b*

AB (*a*) is a square plus a root-five rectangle for the following reasons:

In Fig. 1*b* the line AB is the radius of an inscribed circle to a regular pentagon; AC is the radius of the escribed circle. If AC equals unity or 1. then AB is .809; that is, CI equals .809. IJ is the difference between .809 and 1. or .191. The line AE has an angular relationship to the line AC of 18 degrees. The sine of 18 degrees is .309. The line KJ equals twice unity or 2. EC equals .309; CI, .809 and IJ, .191; added these lengths equal 1.309. This number subtracted from 2 leaves .691. This fraction is the reciprocal of 1.4472. The area HE, therefore, is composed of a square plus a root-five rectangle, and the line HE is a diagonal. The angle at H is similar to the angle at E, therefore the area KL is also composed of a square plus a root-five rectangle.

HOW TO DRAW A SQUARE PLUS A ROOT-FIVE RECTANGLE

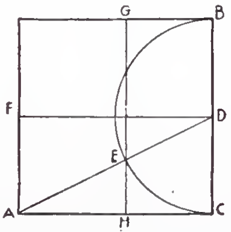


Fig. 2*a*

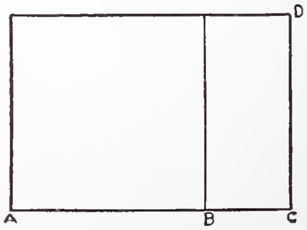


Fig. 2*b*

Draw a square as AB, Fig. 2*a*, and bisect its area by the line DF. Draw AD (this being the diagonal to two squares) and with the point D as center describe the semicircle BEC. The circle arc cuts the line AD at E. Through E draw the line GH. The area BH is a root-five rectangle. In Fig. 2*b*, BC is equal to HC of *a*. The area AD is a square plus a root-five rectangle. The ratio 1.4472 divided by 2 equals the reciprocal ratio .7236. This is the reciprocal of 1.382. There is a definite connection between the ratio 1.4472 and 1.382 which is not apparent by geometrical construction. Indeed, every ratio of the dynamic planning scheme is connected with some other ratio by division and by reciprocal. The ratio 1.382 divided by 2 equals .691, the reciprocal of 1.4472. Every 1.382 rectangle, therefore, may be interpreted as composed of two squares plus two root-five rectangles and every 1.4472 shape may be considered as two 1.382 figures. The fraction .382 is the reciprocal of 2.618; the fraction .4472 is the reciprocal of 2.236 or the square root of five. 2.236 subtracted from 2.618 equals .382. The 2.618 shape minus its reciprocal .382 equals 2.236. Fig. 3 shows the geometric picture of this relationship of area.

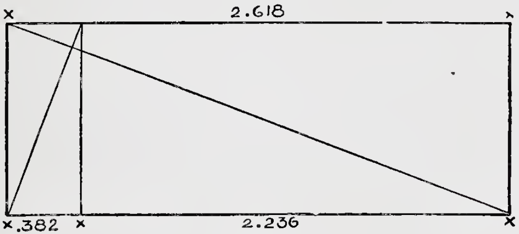


Fig. 3

The fraction .382 is the difference between unity or 1. and .618. When a whirling square rectangle is applied to a square the excess area is equal to a square plus a whirling square rectangle; 1.618 plus 1. or in reciprocal form .382. Fig. 4.

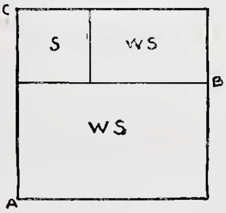


Fig. 4

A whirling square rectangle is applied to a square by the following method:

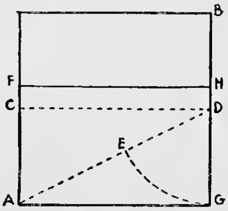


Fig. 5

Draw a square, as AB, Fig. 5, and bisect the area by the line CD. Draw AD and make DE equal to DG. Make AF equal to AE and draw the line FH. The area AH is a whirling square rectangle. AG = 1. GH = .618 and HB = .382.

When diagonals to the whole and diagonals to the reciprocals are drawn to a whirling square rectangle comprehended within a square, a small square is defined whose center coincides with the center of the major square. See Fig. 6.

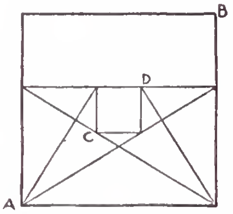


Fig. 6*a*

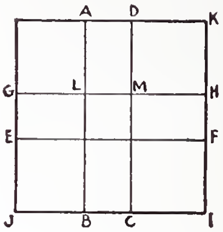


Fig. 6*b*

If the sides of this square are produced to the sides of the major square as shown by the lines AB, CD, EF and GH, Fig. 6*b*, the area of the major square is subdivided into a nest of whirling square rectangles, as JH, KE, AI, JD, AE, DF, CH, BG, EL, FM, etc. JI equals 1. JB .382 and CI .382; these multiplied by 2 equal .764. The line BC is the difference between .764 and 1. or .236. BA equals 1. Unity divided by .236 equals 4.236; BD is a 4.236 rectangle; that is, it is a shape composed of a root-five rectangle, 2.236, plus two squares; or two whirling square rectangles, 1.618, multiplied by 2 plus one square, 3.236 plus 1. GF is a similar area.

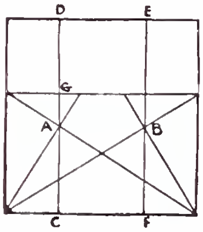


Fig. 7

If lines are drawn through the eyes of a whirling square rectangle comprehended within a square and produced to the opposite sides of that square they define the area of a root-five rectangle. Fig. 7. A, B are the eyes mentioned, and CD, EF are the two lines, CE is a root-five rectangle, AF is a square and A, E two whirling square rectangles.

[To be continued.]

THE ROOT RECTANGLES

TWO root-two rectangles added together, end to end, equal a root-eight rectangle. The square root of two equals 1.4142, the square root of eight equals 2.8284. The square on the side of a root-eight rectangle is eight times the area of a square on the end. When root-eight rectangles are found in Greek design they are almost invariably treated as two root-two rectangles, and the rhythmic theme is in terms of root-two.

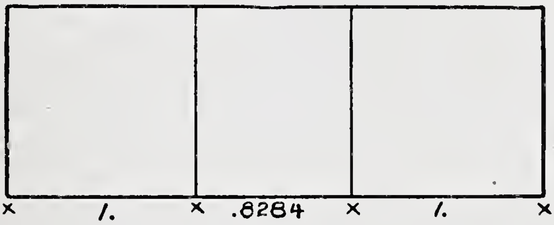


Fig. 1

When squares are applied to either end of a root-eight rectangle the excess area in the middle has the reciprocal ratio .8284. When a square is applied to the end of a root-two rectangle the excess area is the reciprocal ratio .4142; this multiplied by 2 equals .8284.

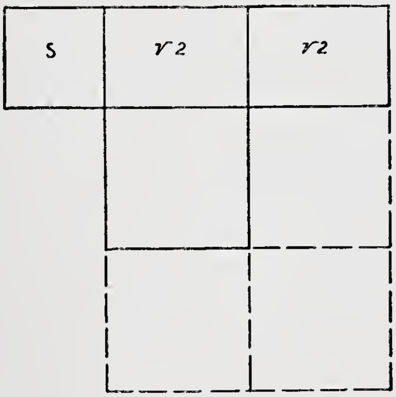


Fig. 2

Fig. 2, by the large square on the side shows why two root-two rectangles, added end to end equal root eight. The large square is four times the area of the square on the side of a root-two rectangle.

.8284 is the reciprocal of 1.2071; the fraction .2071 is equal to .4142 divided by 2. The ratio 1.2071 is best considered as compounded of .7071 plus .5. The reciprocal of a root-two rectangle is .7071 and is also a root-two rectangle. A root-four rectangle is composed of two squares and the ratio is 2. This multiplied by itself equals 4. The reciprocal of root-four is .5 and is always equal to two squares. The ratio 1.2071, therefore, is an area composed of a root-two rectangle plus two squares. See Fig. 3.

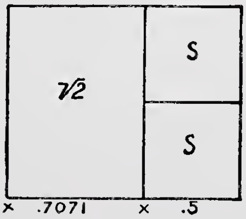


Fig. 3

This figure supplies a good illustration of a Greek method of combining two shapes and using the diagonals of the compound forms to divide the areas of the composing shapes into sub-areas such as could not be obtained by the logical subdivision of either shape by itself.

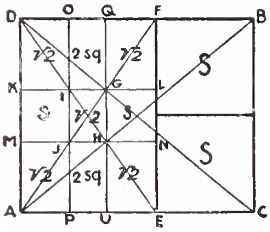


Fig. 4

AB, Fig. 4, is a 1.2071 area and AF, DE are diagonals to the root-two rectangles. Diagonals of the whole, AB, DC, cut the diagonals of the root-two rectangle at G and H. The lines KL, MN, OP and QU divide the area of the root-two rectangle into squares, double squares and root-two rectangles. This is a subtle design idea worthy of the master craftsmen who used it.

For practice the student is advised to draw diagonals to the .5 shape of the 1.2071 area, and to the two composing squares, then to determine the effect of the diagonals of the whole upon the areas.

A sufficient number of the root-two rectangle areas have now been analyzed to enable the student to work out such rhythmic themes for himself.

THE ROOT-THREE RECTANGLE

The square on the diagonal of a rectangle is equal in area to the squares on the end and side. Euclid I, 47.

The square on the side of a root-two rectangle is double the square on the end. (See above.)

The square on the diagonal of a root-two rectangle is equal in area to the squares on the end and side or is three times the area of the square on the end. If the diagonal of a root-two rectangle is used as the side of a rectangle, the end of which is equal to the end of the root-two rectangle, this new rectangle is called a root-three rectangle. The end and side relationship of this rectangle is one to the square root of three, 1 : 1.732 plus.

There are three reciprocals in a root-three rectangle.

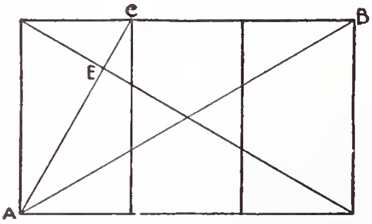


Fig. 5

AC is a reciprocal of the root-three rectangle AB, and AC is one-third AB. The diagonal of a reciprocal and the diagonal of the whole of the root-three rectangle cut each other at right angles at E, a pole or eye. Fig. 5.

If a square on the end is applied to a root-three rectangle the excess area is composed of two squares and two root-three rectangles.

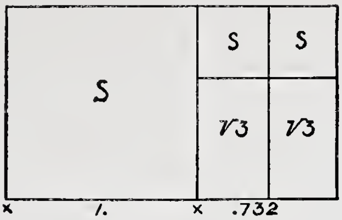


Fig. 6

The fraction .732 divided into 1. equals 1.366. Fig. 6. Two root-three rectangles standing side by side would be represented by the fraction .866 or one-half of 1.7236. Two squares in this position are designated by .5 or half of unity. Fig. 7.

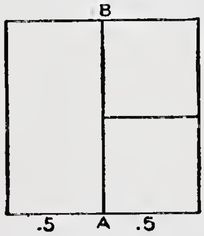


Fig. 7

AB is a line dividing this square into two equal parts. Each of these halves consists of two squares and each is represented by .5. The relation of .5 and .866 is thus made clear. The reciprocal of 1.366 is .732 and this fraction plus unity equals 1.732 or root-three. The difference between .732 and unity is .268 and .268 is the reciprocal of 3.723 or two squares plus a root-three rectangle; consequently if squares on both ends of a root-three rectangle are applied to that area it is divided into six squares and five root-three rectangles.

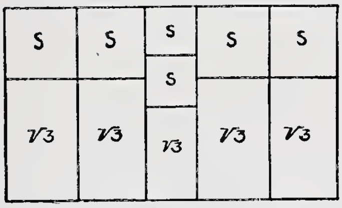


Fig. 8

If a square on the end is applied to the side of a root-three rectangle the area of the square changes from the form of a square to that of three squares.

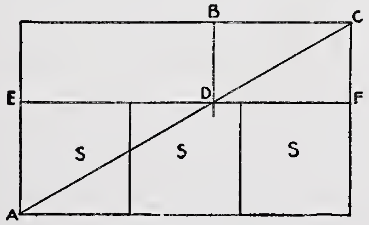


Fig. 9

AB is a square on the end of the root three rectangle AC. A side of this square is cut by the diagonal of the whole at D; the line EF determines the area AE, which is equal to the area of the square AB.

[To be continued.]

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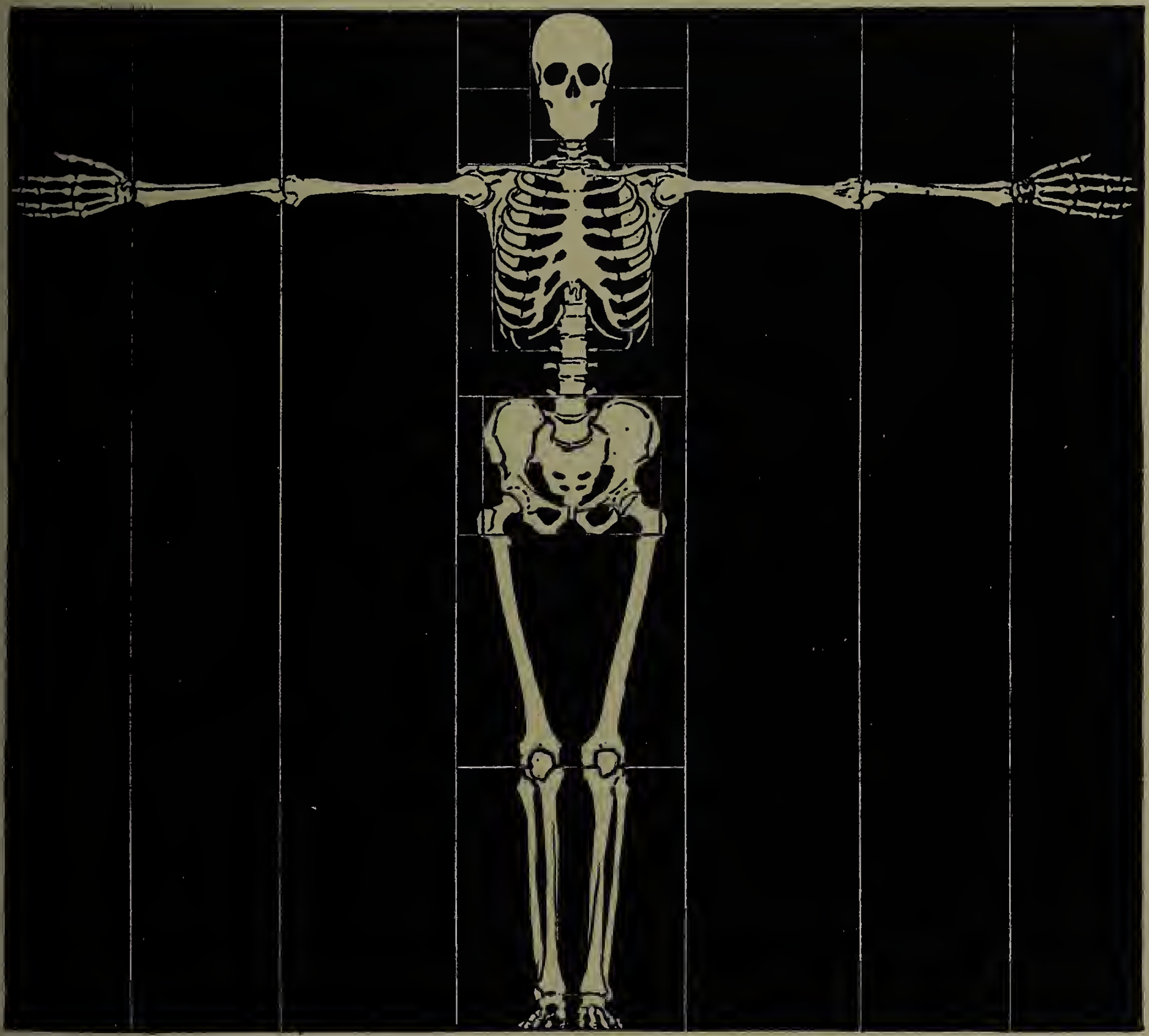
THE DIAGONAL

AN ILLUSTRATED MONTHLY MAGAZINE DEVOTED TO THE EXPLANATION OF THE REDISCOVERED PRINCIPLES OF GREEK DESIGN, THEIR APPEARANCE IN NATURE AND THEIR APPLICATION TO THE NEEDS OF MODERN ART

EDITED BY JAY HAMBIDGE



See article on the Human Figure in this number of THE DIAGONAL

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THE DIAGONAL

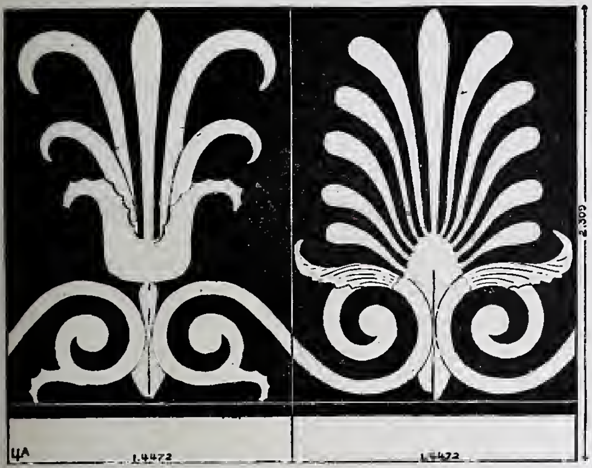

Fig. 1*a*—First pattern Unit on Slab 4
Ratio from center to center 1.4472

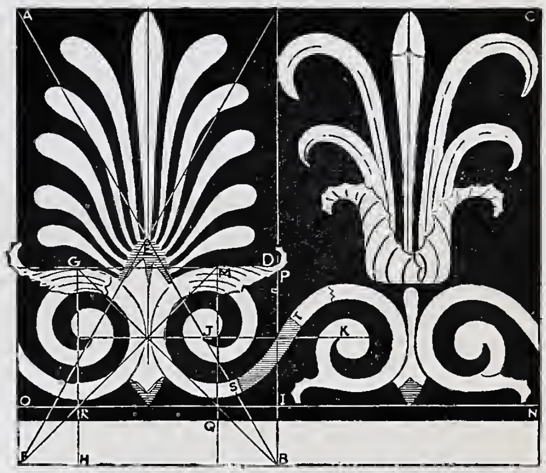

Fig. 1*b*—Pattern Unit for Slabs 1, 2 and 3
Ratio from center to center 1.764

A SECTION OF THE ERECHTHEUM WALL MOULDING AND A PILASTER CAPITAL IN THE BRITISH MUSEUM

AMONG the exquisite works in marble which form the Elgin Collection in the British Museum is a large section of the frieze or ornamented wall band, including one of the pilaster capitals, from the Erechtheum at Athens.

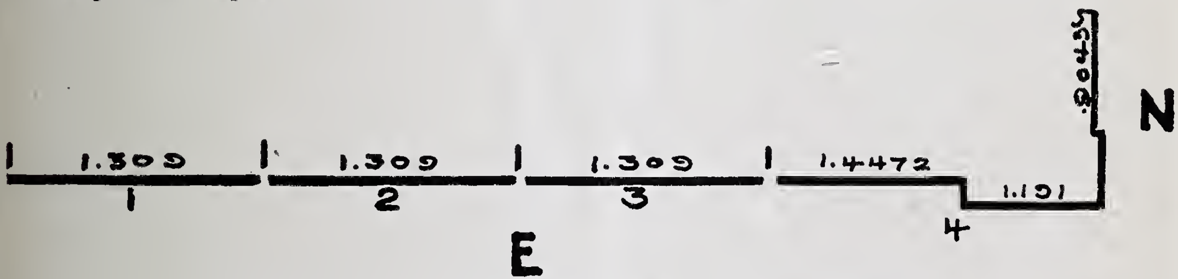

Fig. 1. Plan of the Erechtheum Moulding Section in the British Museum

This moulding will interest readers of The Diagonal because it is the first architectural material which the writer has been able to secure that contains a well-marked design theme on a large scale.

The museum catalogue description of the frieze reads:

"Capital of one of the pilasters (antae) and part of the necking or wall-band from the East wall of the Erechtheion. It is to be observed that the frieze on the pilaster, though analogous to that on the walls, differs from it in details which heighten the richness of the effect, and which assimilate the pilaster to the columns of the East portico, while the walls resemble the capitals of the North portico. Moreover, on the pilaster, the carving of the

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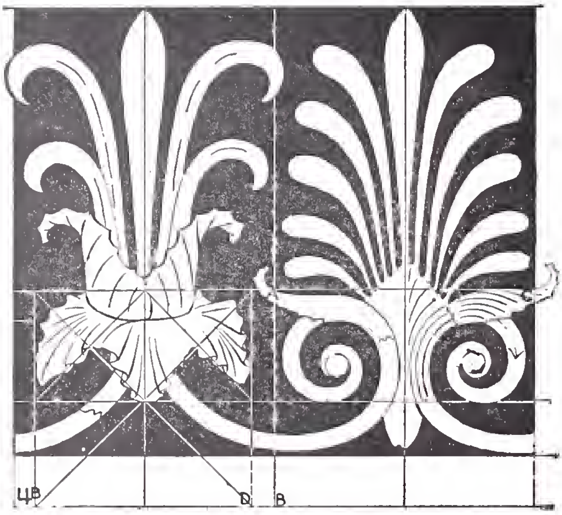

Fig. 2*a*

Pattern Unit for Slabs 1, 2 and 3. Ratio from center to center 1.764

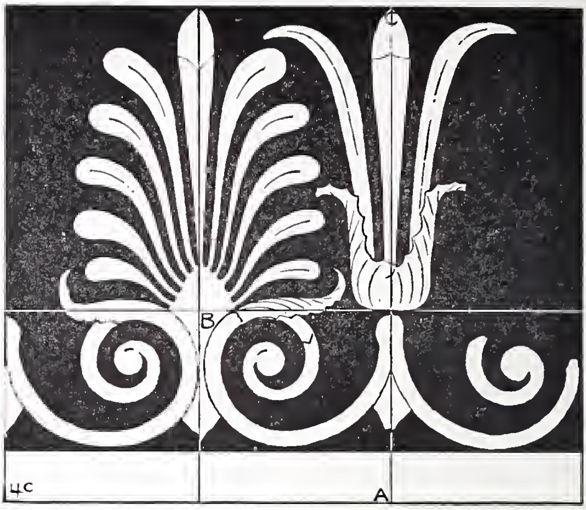

Fig. 2*b*

Fourth Pattern Units, North Face of the Pilaster. Ratio from center to center .9045

frieze is raised above the surface of the courses, while on the wall it is set back from the wall face.

"The slab on the right (shown by very heavy lines on the plan and marked 4, Fig. 1) must be from the North-East angle of the Erechtheion. The three slabs next to it might belong to the East, North or South sides, as regards the form and design. But the excellent preservation of the surface, as compared with that of the unprotected North-East angle, seems to show that these slabs are derived from the East wall, where they were protected by the portico.

"It is interesting to note the numerous repairs in the series of slabs. They probably date from the time of the construction of the building and were meant to make good what was broken by accident in the course of construction. On the North side of the pilaster seven inches of the bead and reel moulding, immediately surmounting the anthemia, have been skillfully inserted in a groove and fastened with lead. On the East side of the pilaster one of the beads of the lower bead and reel moulding was attached by a plug of which the hole remains. On the next slab on the left, one bead of the upper bead and reel moulding was similarly added. On the second slab from the left, one tongue of the egg and tongue, one piece of spiral connecting the anthemia and one bead were let into the marble. Traces of red color remain on the upper part of the frieze on the slabs.

"Height 1 foot 7½ inches. The slab containing the capital of the pilaster is 6 feet in length. Of the other slabs two are each 4 feet 3 inches in length. The fourth is broken on the right hand joint and measures 4 feet 1½ inches."

The interesting feature of this example of classic design is that the width of the anthemia pattern on the slab containing the pilaster capital changes three times. On the three slabs on the left the widths of the anthemia pattern units are the same but they are different in width from any of those on the pilaster slab. All of the pattern units, of course, are of the same height. Moreover, on slabs one, two and four the pattern units of the egg and dart and tongue and groove, which enrich the courses above the anthemia, do not register with the anthemia units. On slab number three there is perfect registry. In the center of each slab containing non-registering units there is registry in the middle of the slab. Were these patterns laid out on either side of the center of the slab and if so was it a general Greek craft practice so to plan ornament? Unfortunately the example does not furnish a satisfactory answer. The important point, however, is that this apparent pattern complex discloses a well-ordered theme in design arrangement in Dynamic Symmetry and from it we can determine the nearly exact difference between the measuring unit employed by the Greek architect and the metric scale. From whatever source the Greeks derived their unit it seems to have been divided decimally. Leonardo da Vinci made a translation of that part of Vitruvius, the Roman architectural writer on Greek design, wherein he undertakes to show the Greek canon of proportion for the human figure. As has been mentioned, in an earlier number of The Diagonal, the Greek tradition of the method of procedure in determining proportion in temples, statues and general design may have been handed down accurately enough. The Roman may have failed in interpretation. The part referring to units of length, according to Leonardo is:

"The standards according to which all measurements are wont to be made are likewise deduced from the members of the body; such as the digit, the palm, the foot and the cubit, all of which are subdivided by the perfect number which the Greeks called TELEOIS." This word means perfect or complete. It seems to have been understood that the perfect number was ten. If this be true we have a very interesting situation. The measuring unit, at least for this temple, may have been taken from some man. If any member of the body, such as a finger, a hand, a foot, or hand and forearm, *i.e.*, the cubit, be divided by ten, a decimal scale would be the result and from it could be read directly the terms of Dynamic Symmetry. The present investigation of the symmetry of man shows that such a decimal system would be astonishingly close to the modern metric scale. This is exactly what these pattern units from the Erechtheum disclose. The actual widths of the four different pattern units are:

	Metric Scale	Should be on Metric Scale
Slab 1	1.3009	1.309
Slab 2	1.3009	1.309
Slab 3	1.3009	1.309
Slab 4	1.438	1.4472
Slab 4	1.1840	1.191
Slab 4	.8989	.9045

The third or fourth decimal point in these figures is justifiable because of the design arrangement on slab 3 or 2. Each of these two slabs contains exactly ten pattern units. Undoubtedly the same number existed on slab 1 before it was broken. Each of the two slabs measures metrically, 1 meter, 3 decimeters, no centimeters and almost 1 millimeter, or 13.009 decimeters. This divided by ten equals 1.3009. If we divide 1.3009 into 1.309 we have for the Greek unit .99381 to 1. decimeter. The units are so close that we may calculate all the ratios directly from metric scale readings, exactly as we do with the skeleton, and reduce them to the actual thing by .99381 as a multiplier or divider. Assuming that the pattern width, that is, that the measurement from center to center of each unit, is 1309, then the course on which the palmette is cut has for height 2.0652; the height of the palmette pattern itself is 2.; the height, including the narrow course at the base of the pattern, is 2.309. On the pilaster faces this course is cut into bead-and-reel pattern. Otherwise it is plain. On slabs 1, 2 and 3, from the base to the top of the palmette or anthemia pattern, we have:

1.309 divided into 2.309 producing the ratio 1.7639
1.309 divided into 2.0652 producing the ratio 1.5776
1.309 divided into 2. producing the ratio 1.5278

The ratio 1.7639 represents a square plus a 1.309 shape, .7639 or .764, for short, being the reciprocal of the ratio 1.309.

The ratio 1.5776 is connected with an arrangement of root-five. The reciprocal of root-five, .4472, multiplied by 4 equals 1.7888. If a square be subtracted from this the remainder, .7888, multiplied by 2 equals 1.5776. The fraction .7888 is the reciprocal of 1.2677.

The ratio 1.5278, or 1.528 for short, is equal to two 1.309 shapes added. The reciprocal of 1.309 is .764. This multiplied by 2 equals 1.528. The fraction .528 is the reciprocal of 1.8944 or a square plus two root-five rectangles. .4472 multiplied by 2 equals .8944.

The total height of the frieze is 4.854 decimeters, actual measurement 4.823, according to the unit .99381. The measurement 1.309 divided into 4.854 equals the ratio 3.708 or six whirling square rectangles, .618 multiplied by 6. But there are 10 units of anthemia pattern on this slab. If the marble were trimmed to fix the top edge of the moulding, the blank face of the slab would be an area equal to 60 whirling square rectangles. On this slab the egg-and-dart and tongue-and-groove patterns are exactly half the width of the anthemia patterns. There are 20 of each and consequently they measure .6545 wide. As a ratio this fraction is the reciprocal of 1.528 and also equal to 1.309 divided by 2. The ratio of the ground plan of the Erechtheum, as worked out by the writer from Steven's recent survey of the building, and verified by Dr. Caskey, Curator of Greek and Roman Antiquities in the Museum of Fine Arts, Boston, is 1.1618. This ratio represents an area of a square plus ten whirling square rectangles. The pattern division of slab No. 3 of this section of the frieze, therefore, shows a direct connection with the ground plan of the temple.

[To be continued.]

DYNAMIC SYMMETRY OF THE HUMAN FIGURE FOR ADVANCED STUDENTS. LESSON IV.

IF the Polykleiton tradition that "beauty was produced from a small unit through a long chain of numbers," is correct, then the so-called canon for the figure, in the light of dynamic analysis of the human structure, is understandable and is artistically sound. It is also clear that the canon tradition connected with Greek sculpture, as it has been interpreted by literary men, is quite a different affair from what the Greek sculptor had in mind.

An archaeological authority has suggested that the Greek sculptor in early times planned the figure to be cut from a rectangular piece of marble by drawing on the marble a front and side elevation. The finding of unfinished statues wherein the torse and limbs were evidently first cut as rectangular forms and then chamfered until the shape approached closer and closer to the form desired, seems to bear this out.

"There are several unfinished statues now in the National Museum at Athens which seem not to have attracted as yet the attention they deserve. Whatever be the reason which has led the ancient sculptor to leave them unfinished, they are full of instruction to the modern student. In them we almost seem to see the artist at his work and to be admitted to his studio. Even if they were given up because of a flaw or mistake, that very mistake may teach us more as to the methods of the artist than many a completed statue. Fortunately, also, these unfinished statues in Athens illustrate various periods from the archaic to one that is certainly later than the finest; and thus we are able to see what changes, if any, took place in the technique of sculpture during this interval.

"Our first example is a statue about two thirds of life size, which was seen by Ross (Inselbreise I. p. 41) lying just below the quarries at Naxos, where he saw also the well-known colossal unfinished statue. Therefore there can be little doubt as to the place where it was made; it was evidently never finished, perhaps because the sculptor saw his proportions would not come right, and so remained where it was until transported to the National Museum at Athens. It happens, most fortunately, that this statue is, or was going to be, a typical example of the first period of Greek sculpture. It clearly represents what is commonly called the archaic 'Apollo' type, a nude male figure standing up stiffly, with the left leg advanced, and with both arms pinned down to the sides. It was also intended to have long hair. There is no necessity to discuss here this well-known type or its various meanings or applications, whether to represent a god or a man. All we are now concerned with is to notice the manner in which the artist sets to work, when he intends to make a statue of this type.

"The statue is nearly perfect in preservation, only the legs from above the knees being lost. Though the treatment of the knees and feet might have further exemplified the sculptor's methods, I think enough has been left for us to be able to see clearly what those methods were.

"The first thing that we notice is the extreme flatness of the surface at the front, back and sides as far as the original outline is left. In the back this is clearest; taken vertically, there is a marked curve; but a rule held horizontally against the back at any height would touch every point in the whole breadth from shoulder to shoulder, or side to side. In front we see almost the same thing. At the sides there is a similar flat surface of the breadth of the arm, but the outline of the arm has been cut in parallels to the back and front planes from the side and parallel to the side planes from the front so that almost rectangular pieces are cut out. The result may best be realized, if one imagines the statue cut through horizontally at almost any height; the section resulting will be contained by lines parallel to the base and front of the statue, and others at right angles to them, parallel to the sides. In fact, at most parts of the body the section will represent a parallelogram with a small rectangle attached at each side for the arms. The corners are not of course left quite sharp, but they are not rounded off enough to obscure the rectangular shape. Now when one considers the freeness of the outlines of the figure taken vertically—that is to say, viewed from the front or side—and compares it with the two sets of straight lines at right angles to one another taken in horizontal section—that is, viewed from above or below—the conclusion is obvious. The outline of the figure from the front or side must be drawn freely; the horizontal section at any point is dependent for its outline on two parallel systems of lines at right angles to one another. That is to say, the process followed in making the statue is precisely that followed by a beginner in sculpture now—or at any time—when he has to set to work on a rectangular block of marble and hew a statue out of it. First he draws the outline of the statue in full face and in profile on the front and the side of the block. Then he carries these outlines straight through, working from the front parallel to the side and from the side parallel to the original front plane. When this process is completed the statue from the front or side has the required outline; but in horizontal section, it is at any point perfectly rectangular. When the arms and legs have been similarly outlined, and cut in to the required depth, and the face a little shaped, the result is a statue in precisely the condition in which we see the Naxian statue now before us." E. N. Gardiner, *Journal of Hellenic Studies,* Vol. XI, p. 129.

In these articles on the human figure we have, so far, considered the general proportions of certain parts of a specific skeleton example. We are now prepared to examine the interrelation of the composing units of this skeleton in three dimensions, as it is brought out by the proportions of a front and side elevation. The major proportions of the front elevation of the trunk and head have already been explained. The principal width measurement for the side elevation is furnished by the thorax, Fig. 1. This is .2236, exactly the height of the head. The full height is 1.7664, consequently we have for the side elevation a rectangle measuring .2236 by 1.7664 and the resulting ratio is 7.899. This shows that, with the height of the head measuring .2236, the figure is 7.899 heads high. The measurement from the bottom of the pelvis to the top of the head is .9045. The side elevation width of .2236 divided into .9045 produces the rectangle 4.045. It will be remembered that the width of the front elevation measured .4045. The rectangle 4.045 will be recognized as a compound shape consisting of a 2.236 or root-five rectangle plus a 1.809 rectangle. A 1.809 rectangle appeared in the front proportion of the head. The height of the pelvis measures .2382. The width .2236 divided into .2382 produces the shape 1.0652. This area is equal to .618 plus .4472. The distance from the shoulders to the top of the head is .2618. Divided by .2236 (see December Diagonal, page 40) the ratio 1.1708 plus is produced. This is a very interesting ratio. The two ratios 1.0652 and 1.1708 equal 2.236 or a root-five rectangle, that is, the rectangles of the side elevation of the pelvis and shoulder-vertex represent the area of a root-five rectangle cut in these proportions. We are now able to place the division of the 4.045 rectangle into its two composing areas, 2.236 and 1.809. When the pelvis height .2382 is added to the shoulder-vertex height .2618, the result is .5. When this is subtracted from .9045, the torse and head height, the result is .4045, the distance from the top of the pelvis to the shoulders. .2236 divided into .4045 fixes the ratio 1.809. The distance from the base of the ribs to the top of the shoulders measures .3236. This divided by .2236 fixes the ratio 1.4472 or a rectangle composed of a square plus a root-five shape, .4472 being the reciprocal of 2.236 and itself a root-five rectangle. It will be noticed that in Fig. 1, AB is a square, that is, an area .2236 by .2236. AC is the 1.1708 rectangle, CG 1.4472, CD 1.809 and DE 1.0652 shapes. JB is an area similar to the grand span area or a double root-five rectangle, 2. divided into 2.236. All of these areas appear frequently in Greek design.

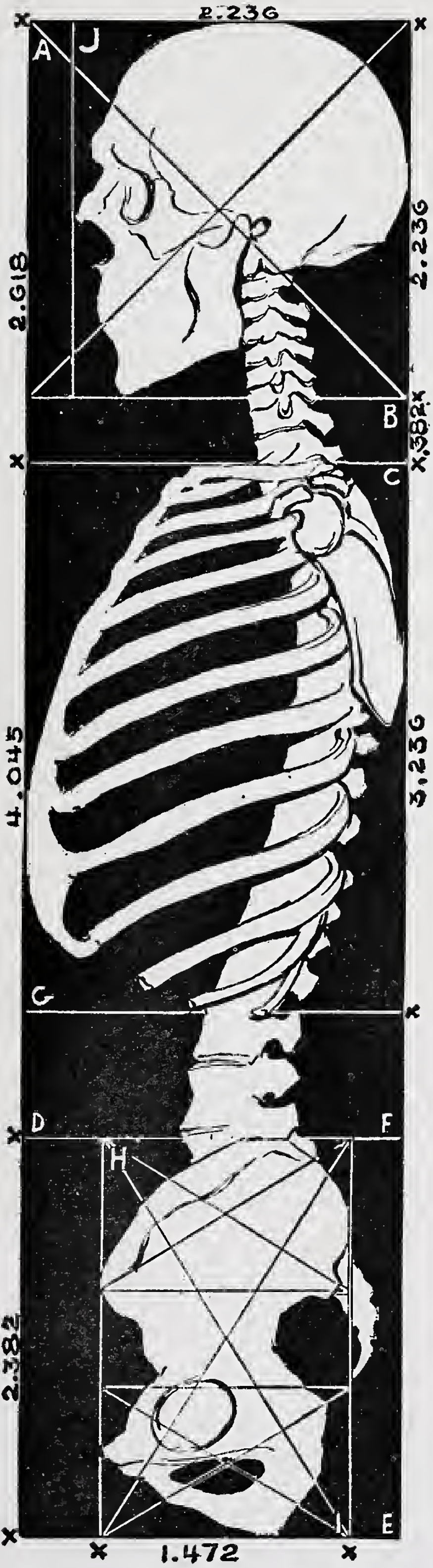

Fig. 1

The rectangle HI, containing the side elevation of the pelvis, measures 1.472 by 2.382, the resulting ratio being 1.618 or a whirling square rectangle. This area is contained within the rectangle DE but its exact position therein is difficult to fix.

The total height of the skeleton is 1.7664, the distance from the base of the pelvis to the top of the head is .9045; the difference is the measurement .8618, the distance from the base of the pelvis to the ground. .2236 divided into .8618 fixes the ratio 3.854 plus. This is interesting because the fraction .854 plus is the reciprocal of 1.1708 plus, the rectangle AC. The distance from the base of the pelvis to the knees is .4045. The measurement .2236 divided into this shows another 1.809 rectangle. The side elevation rectangle from the knees to the top of the head would be 1.809 plus 4.045 or 5.854. The distance from the knees to the ground is .4573. This divided by .2236 establishes the ratio 2.045. The difference between 1.809 and 2.045 is .236. The difference between 2.045 and 4.045 is 2. or two squares.

The student by this time probably realizes that this commonplace skeleton of an unnamed man is existing evidence of an extraordinary arrangement of rhythmical pattern form. Any one of the area units which have so far been described by itself or in union with its fellows in this skeleton is an example of a wonderful modulating scheme. These curious shapes unite with each other or individually split up to form systems upon systems.

Suppose we now draw a solid, such as a block of marble that an archaic Greek might have used, and indicate on it the proportions of the skeleton without its arms. One side of this solid must measure .4045 by 1.7664; the other .2236 by 1.7664, see Fig. 2. We must first measure from the top of the .4045 side the distance .2618. The next measure downward on this slab is .4045, then .2382. We have now carried the measurements to the cross-section line marking the base of the pelvis. Another measurement of .4045 units takes us to the knees, and another one of .4573 to the ground. These section measurements are contin-

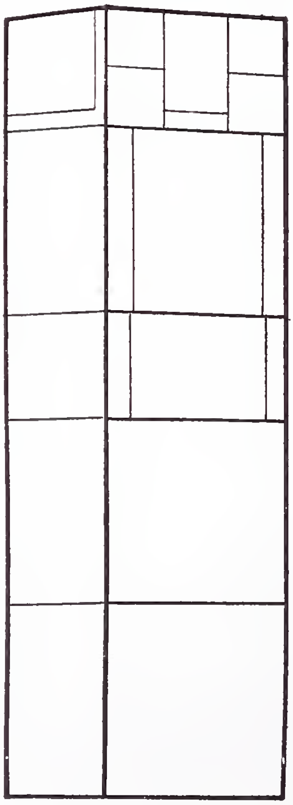

Fig. 2

ued to the .2236 side. In the upper right-hand corner of the .2236 side, which is .2 by .2236, we draw this rectangle to contain the side elevation of the head. Within the .2618 by .4045 area at the top of the .4045 side of the slab, we draw the rectangle of the front elevation of the skull, .1427 by .2236. (See November number, 1919, of THE DIAGONAL.) Within the .4045 by .4045 shape we place the .2809 by .4045 front area of the rib-basket down to the top of the hips. In the center of the .2382 by .4045 section we place the .2382 by .3118 shape which fixes the 1.309 rectangle of the front elevation of the pelvis.

We have seen that the side elevation of the pelvis divided by the side elevation width, *i.e.,* .2382 divided by .2236, produced a 1.0652 area, DE of Fig. 1. The width of the front elevation is .4045 and the distance from the base of the pelvis to the ground is .8618. The width .4045 divided into this fixes the ratio 2.1304, or an area twice that of DE of Fig. 1. 1.0652 multiplied by 2 equals 2.1304. The distance from the knees to the ground is .4573. This divided by .4045 fixes the ratio 1.1304 or 2.1304 minus 1. We are now in a position to understand the compound nature of the rectangle made by the entire front elevation of the skeleton, .4045 by 1.7663, the ratio being 4.3664. The root-five rectangle of the torse and head, .4045 by .9045, *i.e.,* 2.236 subtracted from 4.3664 leaves 2.1304, the rectangle of the legs from the base of the pelvis to the ground. The ratio 1.0652 is equal to .618 plus .4472, *i.e.,* a whirling square plus a root-five rectangle. The entire rectangle of the front elevation of the skeleton therefore is equal to a root-five rectangle plus two whirling square and two root-five rectangles.

The overall side elevation rectangle ratio is 7.899, produced by .2236 divided into 1.7664. The side elevation of the torse and head is 4.045; this is equal to root-five, 2.236, plus 1.809. Also, 4.045 subtracted from 7.899 leaves the rectangle 3.854 for the side elevation of the legs, from the pelvis to the ground. 3.854 is equal to a root-five plus a whirling square rectangle, 2.236 plus 1.618.

The writer has gone into the matter of these figures at the risk of confusing and tiring his readers because careful analysis is necessary for the understanding of the modulating power of the rectangles of Dynamic Symmetry. For this particular skeleton there remain the hands, feet and long bones. The proportions of these will be considered next month.

[To be continued.]

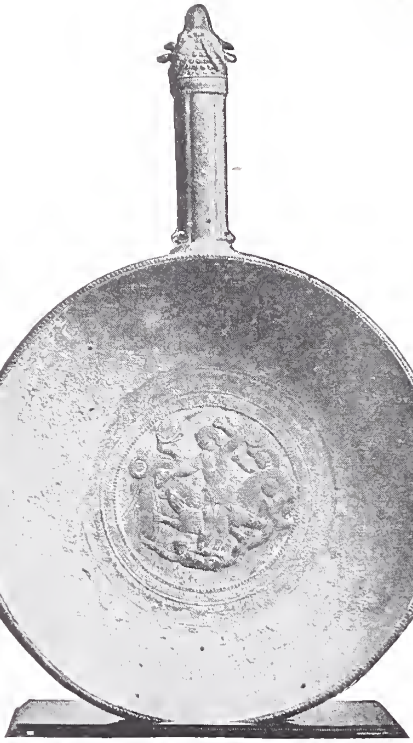

Fig. 1

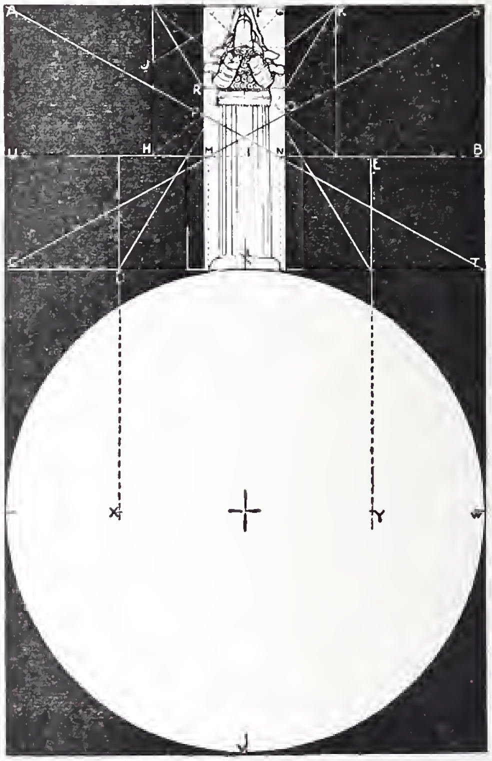

Fig. 2

A BRONZE PATERA IN THE BRITISH MUSEUM

READERS of The Diagonal who have closely followed the length measurements of the members of the human skeleton, as read from the metric scale, have undoubtedly been struck by the remarkable coincidence between these measurements and the ratios of Dynamic Symmetry. The Greek symmetry tradition, as it came to the Roman writer Vitruvius, was to the effect that this symmetry was derived from the human figure. If this is true, and also, if the length measurements from human bones read from the metric scale in terms of this symmetry are exact, then the conclusion is inevitable that the metric scale, as now used, is indeed an actual part of a quadrant meridian of the earth, as a true sphere, and human growth proportions are apparently rigidly governed by the earth's proportions. This of course is merely a suggestion. It would, probably, be impossible to prove.

In the Greek department of the British Museum there is a handsome bronze patera, a sort of skillet, which, in all probability, dates from the second century B. C. It is placed among the Graeco-Roman bronzes and numbered 882. The description reads:

"The handle ends in a ram's head, finely modelled, with curly hair between the horns; it is fluted and ends in an inverted calyx spreading out into a flat piece on which are volutes in low relief; round the edge of the patera are bead and egg patterns. In the interior is a medallion with borders of bead net-work and egg-and-dart (with palmettes on the eggs); on it is a group in low relief of Scylla Destroying the Companions of Odysseus." Then follows a description of the action of the composition and the attack of Scylla and her three dogs on the struggling men. "Diameter 10¾ inches, length 16¾; acquired 1897, from Bosco Reale, near Pompeii. Monumenti Antichi VII, p. 513, fig. 75. For the subject compare Od.

XII. 245-259; Overbeck, Her. Vilder, p. 796; and Miss Harrison, Myths of the Odyssey, p. 183 ff."

The design plan of this patera is interesting for the following reasons:

(1) The connection of its proportions with proportions found in the first human skeleton analyzed in THE DIAGONAL.

(2) The closeness of the actual length measurements with dynamic ratio terms read from the metric scale.

(3) The unmistakable evidence of design decay. There is a curious combination of dynamic and static symmetry in the example which may be explained as design laziness; the mind of the designer is not working at the same high pitch of efficiency as that of the artist during the classic period.

The measurement 10¾ inches, given by the catalogue, is not correct; it is nearly 3 mm. short. The length, 16¾ inches, is more nearly correct. Measurements read on the metric scale from a projection are:

Bowl width 2757 mm. plus; length, including handle, 4260, the lesser into the greater being the ratio 1.545.

The human skeleton would furnish 2764 and 4270, respectively. The measuring unit used in the patera was a very little shorter than the metric scale. The difference is so slight that we may disregard the actual measurements and use the metric system to determine the ratios. 2.764 minus and 4.270 plus are two standard ratios of Dynamic Symmetry. 2764 is one of the Parthenon ratios and is equal to 1.382 multiplied by 2; that is, it is a square and root-five arrangement. .4270 appears in the Harvard Medical School skeleton as 1.4270. And .4270 is a fundamental shape in the regular pentagon and appears in Greek design many times.

The top of the bowl of the patera is slightly irregular as the following height measurements show:

.710, .680, .715, .700, .670, .670, .700, .690. The actual height, as shown by construction, is .691, or one-fourth of the width. The side elevation of the bowl, therefore, is an area composed of 4 squares. The total length is 4.270, and this divided by .691 equals 6.18. The side elevation of the patera, including the handle, therefore, is an area composed of 10 whirling square rectangles. (See Fig 3.)

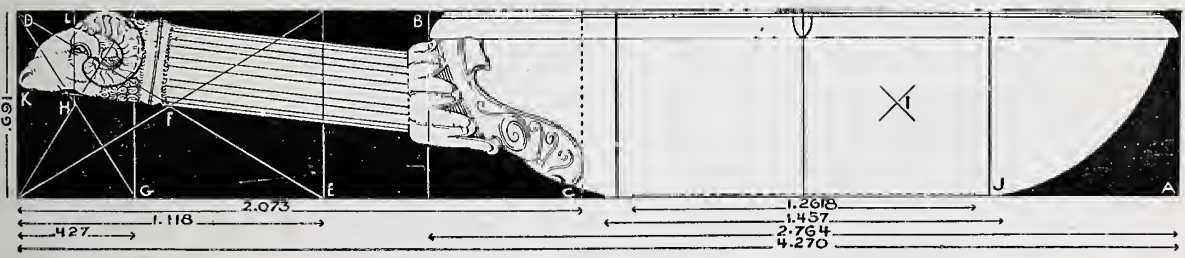

Fig. 3

The length of the handle, including the calyx, is 2.073 or .691 multiplied by 3. The side elevation of the handle, therefore, shows a rectangle of 3 squares. The ground plan of the bowl is an area 4 by 4; the ground plan of the handle and attachment, considering the width of the bowl, and the calyx attachment is an area 3 by 4.

The ratio 1.545 is equal to .618 multiplied by 2½ or 1.236 plus .309.

The ratio 1.545 will be remembered as the vertex-clavicle ratio of the skeleton described in the November number of THE DIAGONAL, the measurements there being .2618 by .4045.

The width of the skull of this skeleton was 1.4270. The width 1.427 divided into .2618 equals the ratio 1.8345 and the reciprocal of this ratio is .545. The area of the ground plan of the patera is a 1.545 rectangle. Subtracting unity or 1, which is the square escribing the circle of the bowl, the area remaining is a 1.8345 or a .545 rectangle. This .545 shape may be regarded as composed of .309 plus .236. The .309 shape is the reciprocal of 3.236 or two whirling square rectangles, while .236 is the reciprocal of .4236 or a root-five rectangle plus two squares. AB, Fig. 2, is the .309 area while BC is that of .236. HK is an area of two whirling square rectangles. AH is a square. HF is a whirling square rectangle. Diagonals of the .545 area, CS or AT, cut the line UB at M and N to fix the width of the handle at the bowl rim. These diagonals cut the diagonals of the double whirling square rectangle HK at P and O to fix the 1.309 rectangle PQ. This last shape is the rectangle for the details of the ram's head. If the entire area of the patera plan is divided into two parts by the line FV, the areas AV, VS, will each be composed of five whirling square rectangles or the overall ground plan will contain ten whirling square rectangles. The side elevation, .691 by .4270, is also an area composed of ten whirling square rectangles. Thus the side elevation rectangle echoes the ground plan rectangle.

The height of the side elevation of the patera is .691 and length .4270, the ratio 6.18, or ten whirling square rectangles. The reciprocal of 1.545, the ratio of the overall plan, is .6472. This divided by 4 (the side elevation is one-fourth of the ground plan), equals .1618 or ten whirling square rectangles. A casual comparison of the side elevation of this patera with the ground plan of the Erechtheum at Athens is interesting. This ground plan ratio is 1.1618, that is, a square plus ten whirling square rectangles. The side elevation of the patera, therefore, is an area similar in shape to that part of the Erechtheum ground plan which exceeds a square.

AB, of the side elevation, Fig. 3, represents the area of four squares. The bowl width is 2.764, the total length 4.270, the lesser subtracted from the greater is 1.506 plus, the length of the handle in excess of the bowl. The height, .691, divided into 1.506 plus, equals 2.180, or .545 multiplied by 4. A line from L to I, Fig. 3, fixes the top of the handle; a line from F to J, the bottom of the handle. DE is a whirling square rectangle measuring .691 by 1.118; F is its center. DG is a whirling square rectangle measuring .427 by .691; H is its center. J is a corner of a square of the bowl elevation, and I is the center of another square. The handle, figuratively, lifts from the extreme width of the base of the bowl. This is a design idea appearing frequently in the best work of the classic period.

The full outside diameter of the ring on the outside base of the bowl is equal to the width of the root-five rectangle of the ground plan of the handle excess, or 1.457. The diameter of the ring immediately escribing the medallion of the inside, from center to center of beads, is 1.090, or .545 multiplied by two. The diameter of the extreme outer ring around the medallion measures 1.545. The area DC, Fig. 3, is composed of three squares, .691 multiplied by 3 or 2.073.

In spite of the fact that an important Greek shape with all its modulating components is utilized in this patera design, there is, nevertheless, a stolid or somewhat bulky quality in the bowl and handle. The ram's head is the best element of the design. The static or heavy quality mentioned is probably due to the presence of so many squares, resulting from the 3 and 4 multiples used so lavishly in the ground and elevation plans. In the photograph the patera is not perpendicular, hence the somewhat shortened effect of the ram's head.

THE ELEMENTS OF DYNAMIC SYMMETRY

Much of the work upon which the Elements of Dynamic Symmetry is based was done during the year 1919 under the Sachs Research Fellowship of Harvard University. This fellowship was founded by Samuel Sachs of New York City and, under the terms of the foundation, is assigned to scholars of proved ability, whether students, instructors or others, for the general purpose of advancing the Fine Arts. This acknowledgment is but a faint expression of the writer's appreciation of the benefits enjoyed from a fellowship of a type too rare in American Universities. JAY HAMBIDGE

COMPOUND OF RECTANGLES AND THEIR RELATIONSHIPS

A ROOT-FIVE rectangle is drawn within the area of a square as shown in Fig. 2 of the last lesson. When this root-five area is placed within a square it becomes necessary to define the nature of the area of the square in excess of the root-five shape.

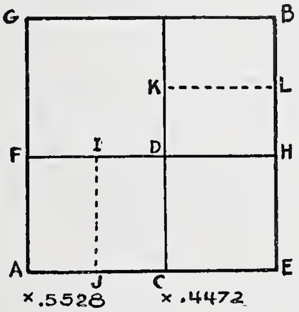

Fig. 1

AB, Fig. 1, is a square and BC is a root-five rectangle. This shape is composed of the square DE and the two whirling square rectangles DB. If the side of the square DE, as HD, is produced to F the area FE is also a root-five rectangle. When the root-five area BC is applied to a square the excess area is GC or GH. Because the area DE is a square and its sides HE, EC are common to the sides of the major square EB, EA, the diagonal DE is common to the diagonal of the major square EG; consequently GD is a square. And the area GC or GH is composed of the square GD plus the shape FC or BD. But the areas BD or FC are each composed of two whirling square rectangles. GC or GH, therefore, is composed of a square plus two whirling square rectangles. But a root-five rectangle is composed of a square plus two whirling square rectangles. The difference between the shapes BC and GC lies in the fact that in BC the side of the square DE as DH is also the side of one of the whirling square rectangles, as KH. The side of the square GD, as FD, is also the ends of the whirling square rectangles FJ, JD. And the square GD is larger than the square DE. The difference between these two shapes is shown in Fig. 2, a and b.

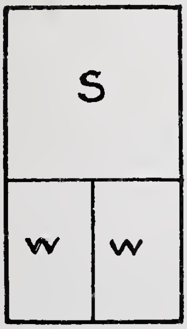

Fig. 2*a*

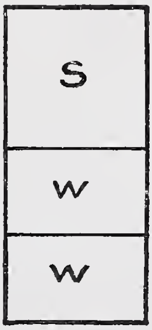

Fig. 2*b*

Diagram *a* is the GC shape and Diagram *b* is the BC shape of Fig. 1. The difference between the two shapes consists in the manner in which two whirling square rectangles are joined to a square.

If the side of the major square, as AE, Fig. 1, represents unity, or 1., then the line CE is equal to .4472 and CA is equal to .5528. These two numbers, added, equal 1, or the line AE. GC, therefore, if AG equals 1, is a .5528 rectangle and BC a .4472 rectangle. These numbers are less than unity, however, consequently are reciprocals of ratios greater than unity. .5528 divided into 1 equals 1.809 and .4472 divided into 1 equals 2.236. GC may be identified by either 1.809 or .5528, and BC by 2.236 or .4472. A 1.809 rectangle frequently occurs in Greek design. .809 is equal to 1.618 divided by two. One of its natural sources is shown in Fig. 3.

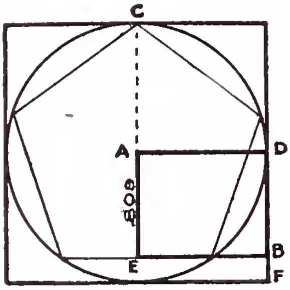

Fig. 3

In this diagram a regular pentagon is inscribed in a circle (see last lesson). The point A is the center of the circle. The line AD equals 1., the line AE equals .809. The area AB, therefore, is an .809 rectangle. CD is a square and CB is a 1.809 rectangle.

.809 divided into unity equals 1.236. It is therefore the reciprocal of 1.236 or .618 multiplied by two. A mathematician would describe a .618 rectangle as root-five minus one divided by two. Root $\frac{5-1}{2}$ *i.e.,* 2.236—1. equaling 1.236 and this divided by two producing .618. Fig. 4 shows the .809 and 1.236 shapes.

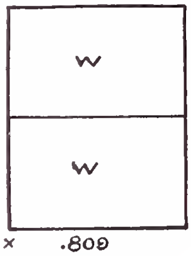

Fig. 4*a*

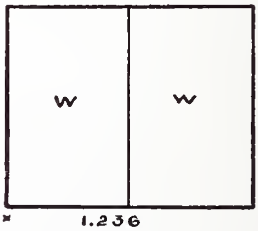

Fig. 4*b*

The mathematician would describe a 1.618 rectangle as root-five plus one divided by two, root $\frac{5+1}{2}$, *i.e.,* 2.236 plus 1 being 3.236 and this divided by two producing 1.618.

The reciprocal ratio .618 subtracted from .809 equals .191. The area which is represented by .191 is that of EF in Fig. 3. The character of this area will be apparent if .191 is multiplied by two, .191 multiplied by 2 equals .382. As has been explained in the last lesson, .382 is the reciprocal of 2.618, *i.e.,* it is a square plus a whirling square rectangle. Dividing .382 by 2 is equivalent to multiplying 2.618 by 2. An arithmetical operation applied to a reciprocal is always the reverse of the operation applied to a direct ratio. 2.618 multiplied by 2 equals 5.236, this equaling 1.618 multiplied by 2, or 3.236, plus 2. Fig. 5 shows the arrangement.

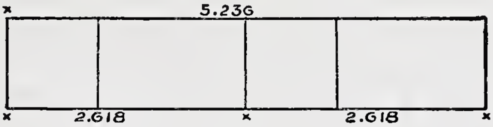

Fig. 5

The ratio 5.236 divided into unity or 1. equals .191. Both in nature and in Greek art the fraction .191 is frequently encountered as part of a compound shape. .809 plus .191 equals 1. The ratio 1.236 plus .191 equals 1.427 and this result plus .191 equals 1.618. 2.236 plus .191 equals 2.427, and this divided by 3 equals .809. The reciprocal ratio .427 plus .191 equals .618. The ratio 1.191 is an important ratio in Greek design. This area may be regarded as composed of an .809 plus a .382 area or as .691 plus .5. This last reciprocal ratio, .5, is half of unity, or 1., and is equal to two squares. The square root of 4 is 2 and the resultant area is equal to two squares. .5 is the reciprocal of root-four. .5 plus .191 equals .691 or a square plus a root-five rectangle. (.691 is the reciprocal of 1.4472.)

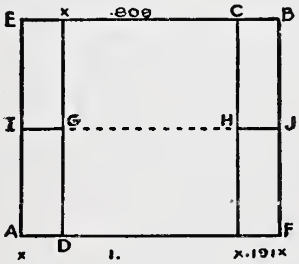

Fig. 6

AB in Fig. 6 is a 1.191 rectangle and AC is an applied square. AC equals 1 and CF equals .191. BD is an applied square to the other end of the rectangle. It overlaps the first square to the extent of the area CD. This overlap is equal to the square AC minus the area ED or .191. The fraction .191 subtracted from 1, or AC, equals .809. The area DC consequently is an .809 area and is composed of two whirling square rectangles DH and GC. If the line GH is produced to I and J, the areas ED and CF are bisected and EG, GA, CJ and HF are .382 rectangles.

[To be continued.]

THE ROOT RECTANGLES—*Continued*

THE ROOT-THREE RECTANGLE

THE side of a root-three rectangle is equal to the diagonal of a root-two rectangle on the same unit base.

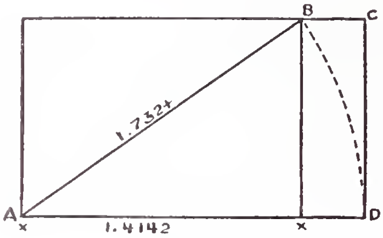

Fig. 7

AB, Fig. 7, is a root-two rectangle and its diagonal is equal to AD, the side of the root-three rectangle AC.

A simple method for constructing all the root rectangles is shown in Fig. 8.

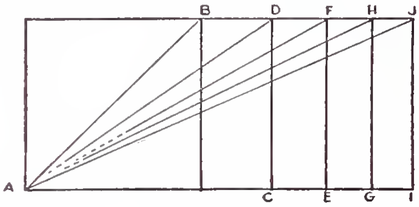

Fig. 8

In Fig. 8, AB is a square. Its diagonal is equal to AC, the side of the root-two rectangle AD. The diagonal of the root-two rectangle AD is equal to AE, the side of the root-three rectangle AF. The diagonal of the root-three rectangle AF is equal to the side of the root-four rectangle AG. The diagonal of the root-four rectangle AH is equal to the side of the root-five rectangle AJ. AJ is equal to the side of the root-six rectangle, and so on. In Greek art a rectangle higher than root-five is seldom found. When one does appear it is almost invariably a compound area composed of two smaller rectangles added together.

The root rectangles are constructed within a square by the following simple method:

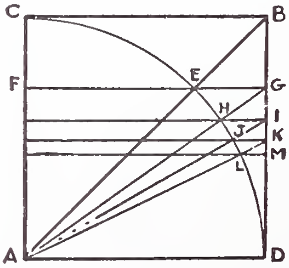

Fig. 9

In Fig. 9, AB is a square and CDE is a quadrant arc of a circle, the radius of which is AD. The diagonal of the square cuts the quadrant arc at E, and FG is a line through the point E drawn parallel to AD. AG is a root-two rectangle within the square AB. The diagonal of the root-two rectangle AG cuts the quadrant arc at H. A line through the point H similar to the line through E establishes the root-three rectangle AI. The diagonal of the root-three rectangle cuts the quadrant arc at J. The area AK is a root-four rectangle. The diagonal of the root-four rectangle cuts the quadrant arc at L. The area AM is a root-five rectangle, and so on.

There are many other methods for the construction of root rectangles, both outside and inside of a square, but, in the writer's opinion, the ones described are the simplest.

The reciprocal of a root-three rectangle is equal to one-third the area of that shape. Fig. 10 shows a root-three rectangle and its three reciprocals with all the diagonals in position.

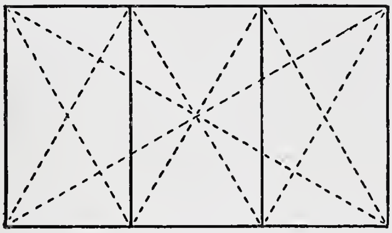

Fig. 10

It will be noticed that the diagonals of all the reciprocals cut the diagonals of the whole at right angles.

The area of a square on an end of a root-three rectangle is one-third the area of a square on a side.

Lines coinciding with the spiral wrappings of a root-three rectangle divide the area of that shape into an infinity of similar shapes to the whole with a ratio of three. Fig. 11, *a* and *b*, shows this process.

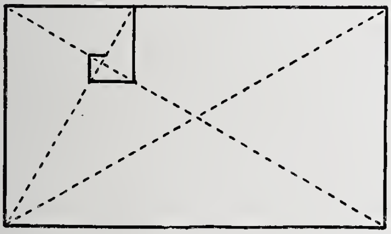

Fig. 11*a*

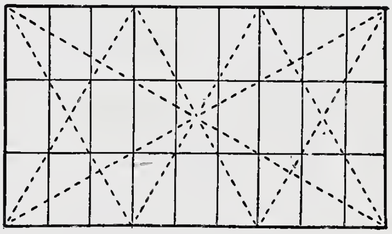

Fig. 11*b*

Lines drawn through the eyes of a root-three rectangle divide the area of that shape into similar figures to the whole, to infinity, with a ratio of four. (See Fig. 12.)

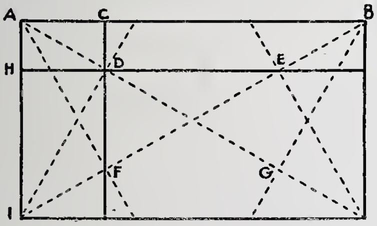

Fig. 12

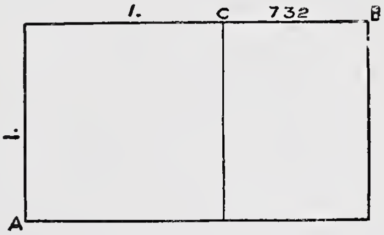

Fig. 13

The line AC of Fig. 12 is equal to one-fourth of the line AB, and AH is one-fourth of AJ. The points D E F G are "eyes" or poles.

When a square on the end is applied to the area of a root-three rectangle the excess area is composed of two squares and two root-three rectangles.

The square root of three is 1.732 plus. The square AC applied to the area of the root-three rectangle AB leaves an excess area measuring 1. on the side and .732 on the end. The lesser divided into the greater equals 1.366 plus. This ratio is composed of two shapes having the reciprocal ratios of .866 plus and .5. The fraction .866 is equal to 1.732 plus divided by two; hence .866 represents two root-three rectangles. This is made clear by Fig. 14.

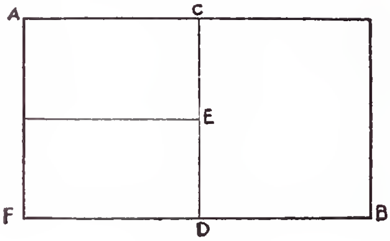

Fig. 14

In Fig. 14 AB is a root-three rectangle and CD is a line dividing it into two equal parts. The area AD is composed of the two root-three rectangles AE and EF. The fraction .5 is the reciprocal of a root-four rectangle. A root-four rectangle is composed of two squares; therefore its reciprocal is composed of two squares. The fraction .5 may also be considered as half a square or 1. Half a square is equal to two squares. The reader should bear in mind that we are now speaking of similarity of figure only. Two squares and two root-three rectangles placed together in the manner described form a 1.366 plus or a .732 plus shape. (See Fig. 15.)

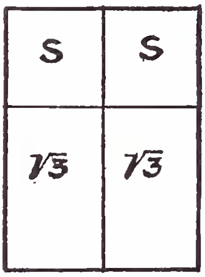

Fig. 15

If a square is applied to the other end of a root-three rectangle, the two applied squares overlap and the area of the root-three shape is divided into six squares and five root-three rectangles, as in Fig. 16.

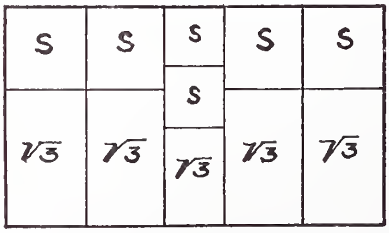

Fig. 16

[To be continued.]

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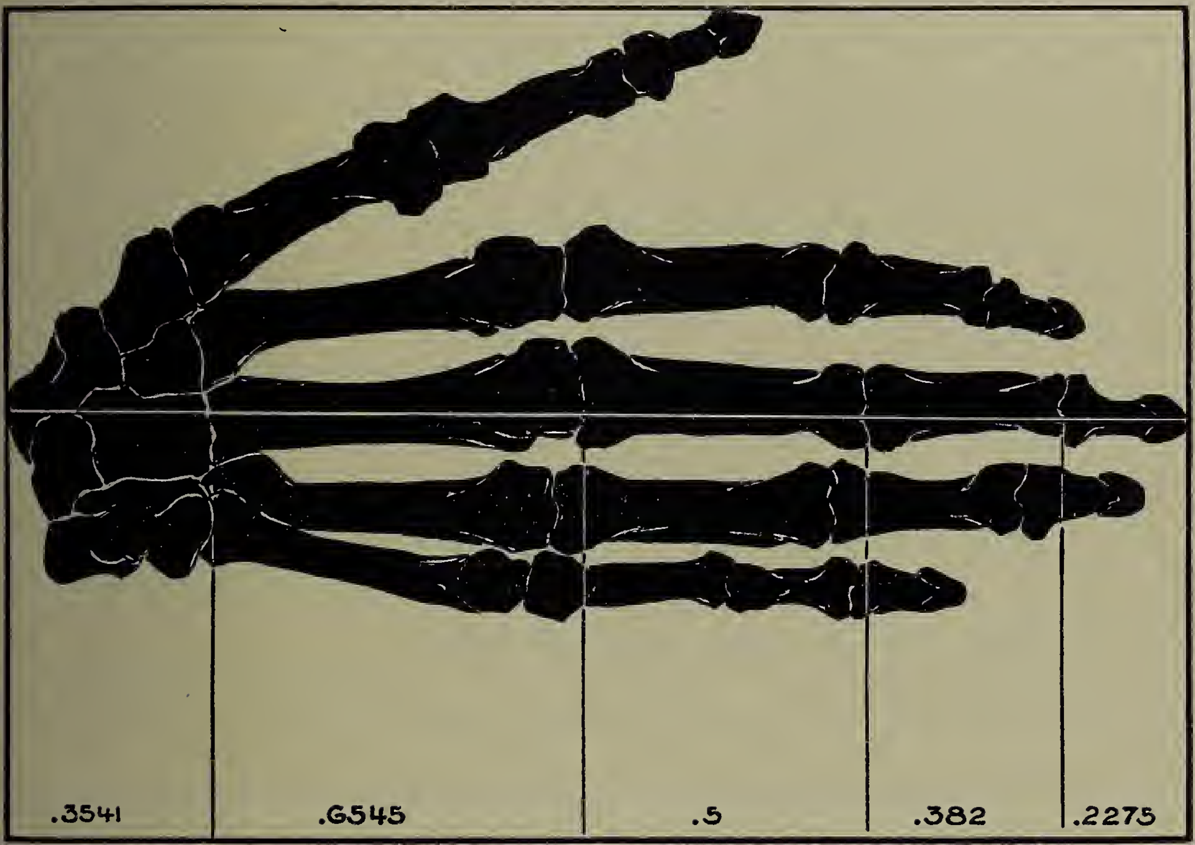
THE DIAGONAL

AN ILLUSTRATED MONTHLY MAGAZINE DEVOTED TO THE EXPLANATION OF THE REDISCOVERED PRINCIPLES OF GREEK DESIGN, THEIR APPEARANCE IN NATURE AND THEIR APPLICATION TO THE NEEDS OF MODERN ART

EDITED BY JAY HAMBIDGE



See article on the Human Figure in this number of The Diagonal

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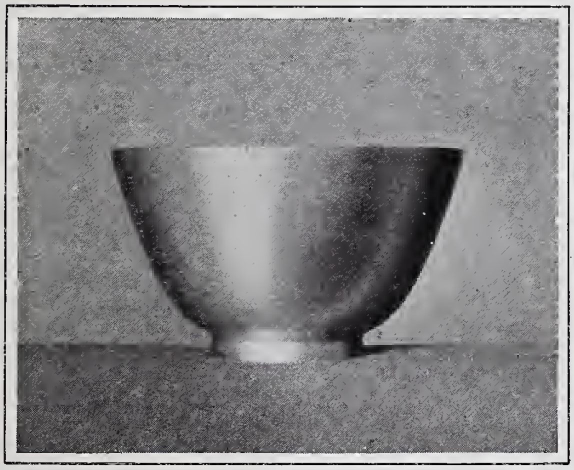

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NEW YORK
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A LETTER

Dear Mr. Hambidge:

My colleagues of the National Society of Art Masters and myself are greatly indebted to you for your clear and illuminating lecture on "The Dynamic Symmetry in Greek Design."

I feel personally that a clear and correct understanding of this vital system will have a most profound influence on the art of Britain and Europe during the next decade, and I shall lose no time in using the "Diagonal" methods in my own school here. The logical reasoning of the whole scheme compels conviction immediately the origin and application are adequately realized. . . .

I am convinced that the rapid spread of the system of Dynamic Symmetry will lay the foundation of our artistic salvation.

With best wishes for all success,

Cordially yours,

Walter G. Raffe, A.R.C.A., F.R.S.A.

Chairman, London Federation of the
National Society of Art Masters,
Great Britain.

DYNAMIC SYMMETRY: THE GREEK VASE

By Jay Hambidge

TO BE PUBLISHED SHORTLY

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THE DIAGONAL

THE DYNAMIC AND THE STATIC AREA

FROM the material which has been presented in the issues of THE DIAGONAL up to the present, the reader has probably recognized that the symmetry conception is, in reality, a system of measurement. This is indeed the case; and the reason that a system of measurement is essential in design lies in the fact that the designer cannot prepare a plan unless it is measurable. This measurableness is not only necessary for the designer himself but, if it were possible to make a plan of any design without measurableness, workmen would be utterly unable to carry it out. Carpenter, stone mason, decorator, workers in precious metals, all must use a measuring system of some sort which will permit the determination of definite details, no matter how these details may be arranged. At different times in history different measuring systems have been used by artists and craftsmen. The most obvious system is that based upon a unit length divided into multiple parts, such as the cubit, the foot or the meter. This is the system we use in all the arts and crafts; it results, however, automatically, in the lowest type of static symmetry. One of the reasons for this lies in the fact that the relation of the parts to the whole in a design laid out with an evenly divided unit of length, such for example as the foot, automatically produces ratios of line which are too easily grasped by the mind and ratios of area which are too complex to be easily understood. At different periods in history artists and designers have recognized that the length unit system might be discarded and a system based upon measurable units of area substituted. Curiously, we find that whenever a system based upon measurable units of area has been used that period marks one of the great art epochs, and in every case the area measurable scheme, whatever it was, may be shown to be closely connected with nature. The possible schemes are few and extremely simple. During the great Gothic period of art the methods employed to achieve this were based upon the regular two-dimensional figures, the equilateral triangle and the square. We know this is true because of the quantity of evidence, bearing directly upon art and craft practice during this period, that has survived. In the South Kensington Museum in London, there is an architect's plan and elevation of a fifteenth-century German Gothic tabernacle. The elevation of this tabernacle measures 4950 mm. in height and 675 mm. in width. The lesser measurement divided into the greater produces the ratio 7.333 plus or seven and one-third. This is clearly static symmetry. The interesting point, however, is that the scheme of measurement is one of commensurable area, the static ratio of seven and one-third being merely incidental to the area measuring method. The ground plan shows that this scheme was based upon the square and was developed in a characteristic Gothic manner. In Fig. 1 this fact is apparent by inspection. The lines outside of the central construction indicate that the overall arrangement was an octagon. The inner or central construction is an adjustment of squares on 45 degree angles; but these squares all bear a definite measurable area relationship one to the other. This design is unique in that it is one of the very few surviving architectural plans which show clearly the designer's method of procedure. There is nothing unusual in the fact that an octagon and a series of squares appear in the scheme. These figures often appear in architectural plans today. But there is this difference; Gothic architects took extreme care to have the simple figures they used measure up in terms of themselves; the modern architect ignores this relationship, uses a measurable length unit, such as a foot divided

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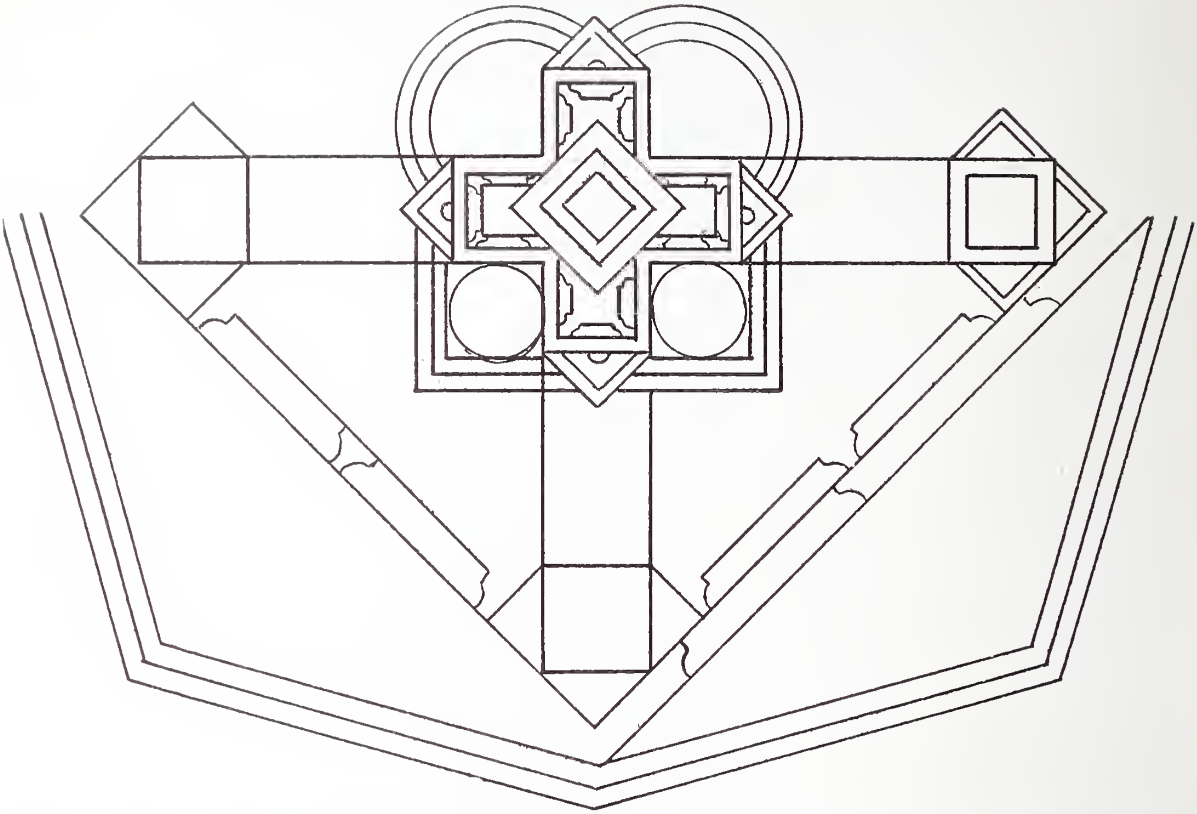


Fig. 1. Fifteenth Century Architect's Plan for a Tabernacle. Original in South Kensington Museum, London.

evenly, and fixes the sizes of the figures by it. By this proceeding he utterly destroys proportional relationship and introduces incoherence into his design. The plan of this Gothic tabernacle, as it exists today, proves conclusively that this was not the Middle Age procedure. The plan was drawn on a heavy, well-made paper with a hard pencil, or stylus of some sort, by construction and desired lines afterwards inked. The construction lines were afterwards rubbed out. The pressure on the point was so firm that the eraser could not remove the deep indentation in the paper. The result is that the plan stands with these construction lines in place, and they are plainly visible, especially when the drawing is tipped toward the light. These lines show that this plan is an example of the use of static symmetry of the best type and that the coherence of arrangement depends upon static measurable area.

Other examples showing general design practice during the Middle Ages are furnished by sketches from the note books of Leonardo da Vinci and Wilars de Honnecourt, a thirteenth-century Gothic architect of France and designer of the cathedral at Cambrai, one of the celebrated structures destroyed during the revolution. This architect was well known in his day, as is evidenced by a commission given to him to visit far Hungary and there build a cathedral. While making the journey to Hungary, he made notes of various celebrated edifices he passed on the way. These notes are accompanied by drawings, but, unlike the sketches made by traveling architects in modern times, who embellish their note books with perspective sketches of "bits" that captivate their fancy, the old French builder confined himself to defining the symmetry or theme scheme upon which the buildings were planned. Wyatt Papworth, the English architect, several years ago found in the archives at Milan the recorded proceedings of a quarrel which had taken place during the building of the celebrated cathedral of that city. While the structure was in course of erection it seems to have become necessary to engage a new master mason. One was obtained from Germany, but the Italian workmen evidently did not like his method of proceeding as they complained and charges were brought against him that he was substituting the German method of co-ordinating the work by square instead of following the local procedure by triangle. He was dismissed and a French master mason engaged. The Italians, however, seem to have had no better success with the new master. After two years he was charged with adhering to the method of the square. He was dismissed, fined and compelled to tear down his work of two years. The incidents serve to show that mediaeval builders regarded seriously their methods of obtaining measurable areas in order to maintain coherence in their designs. It would be difficult to imagine a like situation today. The foot, inch and meter have entirely supplanted the ancient methods of area measuring by construction.

The Gothic builders' conception of area and volume measurement by construction is in one way analogous to the idea the Egyptians and classic Greeks had; the difference between the two lies in the means used. The Gothic builders employed the regular two dimensional figures to solve their problems; the Egyptians and Greeks the incommensurable lines obtainable from diagonals. The Gothic process produced static symmetry exactly like that of the crystal. The older builders' designs resulted in dynamic symmetry such as we find in the shell and in leaf distribution in plants. In the very early days of Egypt and Greece a simple static scheme was in vogue by which the elements of design were defined and related by a line measuring unit divided into even multiples such as one and a half, one-third, one-fourth, one-fifth, etc. This arrangement resulted in a division of areas into squares and simple multiples. This plan evidently did not entirely satisfy the early Greek artists because we find them resorting to many devices to obtain a higher degree of subtlety of form than that directly obtainable from the line unit. In many designs there is found a cunning use of diagonals to obtain what might be called hidden or secret squares. An example of such a device is furnished by a large amphora in the Metropolitan Museum of New York City.

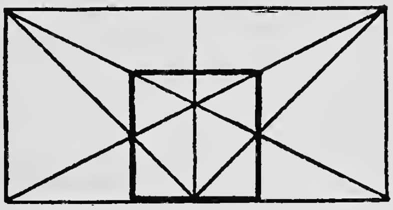


Fig. 2

Fig. 2 is an area composed of two squares and the diagonals to this shape and diagonals to the two composing units. Intersections of these diagonals are utilized to define a small secret square in the center. After many experiments analogous to the above, and, apparently, much dissatisfaction with the limitations imposed by the use of the measurable line unit scheme, some genius hit upon the astonishing idea that certain lines, incommensurable in themselves, that is, one could not be divided into the other, nevertheless produced a type of area delightfully measurable as such, and designs correlated by this new plan were unmistakably superior to those produced by the older method. The first step in this new direction was made by using the side and the diagonal of a square. From this was created that curiously fascinating family of root-two shapes, which constitute about fifteen per cent of Greek design. Later, apparently, some other gifted designer noticed that a diagonal to two squares, in relation to a side of one of the generating squares, produced a much superior area measurable plan making device. This was the full revelation. The diagonal to two squares is called root-five by the mathematicians because it is the side of a square equal in area to five times the area of one of the generating squares. The diagonal to two squares is the base of the most effective scheme so far discovered for correlating the elements of design. From it is obtained the whirling square rectangle and all of the interesting compound shapes which constitute some eighty per cent of classic design bases.

THE FALLACY OF THE "SO-CALLED GOLDEN COMPASS"

AN extraordinarily large number of the best British artists are today using a German proportioning instrument called the "Golden Compass." This device is simply a metal arrangement constructed on the principle of the pantograph, differing only in that it is set to one proportion, this being the Greek extreme and mean ratio. The base of the apparatus is shown in Fig. 1. It is difficult to trace the cause of the popularity of this device especially as only a casual inspection is necessary to show how utterly misleading and pernicious its employment may prove.

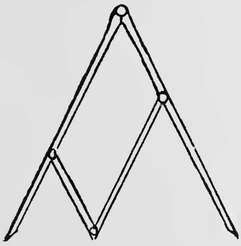


Fig. 1

If the writer had not heard eminent artists explain how they used this compass and maintain that it was a solvent for some of their most difficult problems of composition, he would not have believed that men, apparently rational and capable in all other respects, could be so credulous. One well-known sea painter seems to depend upon it entirely, declaring that he would not think of carrying out a composition unless the essential points of interest in his picture were first placed by this little machine of brass.

The method of using the instrument is in reality no method. The artist simply opens it to the width or length of his picture and permits the middle leg to fix approximately the point where he places the most important action of his composition. If ever anything were devised by man for the purpose of negativing brain exercise and introducing pure mechanism into art this compass easily takes the prize. British artists don't stand isolated, however, as report says that artists on the continent in like numbers are leaning on the frail support. Indeed, the handy aid to thought has made its appearance in America. Artists attending the writer's lectures have frequently had it in their possession.

Because of the facts related it seems necessary that an explanation be made of the effect of the use of this machine. The term "golden section" was coined in Germany early in the past century to describe the ratio the Greeks are supposed to have discovered and named "extreme and mean ratio." Of course the Greeks in all probability did not make the discovery. It was simply part of the loot acquired by an expedition made to spoil the Egyptians some time during the sixth century B. C. This peculiar ratio has engaged the attention of man practically ever since classic Greek days. The Italians called it the "divine section" (see Appendix notes to the "Greek Vase" by the writer), the Germans the "golden section" and the English the "medial section." (See Leslie, the English mathematician.) Pacioli, the Italian mathematician and friend of Leonardo da Vinci, wrote a mystical book about it. Kepler, the astronomer, called it the "divine section." Zeising, a German author, thought this engaging ratio rested at the base of all earthly form. The fact is that no one since Greek days has understood the exact nature of the subject. "Extreme and mean ratio" accurately describes mathematically the condition created by the cutting of a line in this proportion, as is shown in the Eleventh Proposition of the Second Book of Euclid. There it is treated as a line cut in such manner that a square described on the greater segment is equal in area to the rectangle made by the whole line and the lesser segment. To accomplish this, Euclid shows that the first step is the determination of a diagonal to two squares, Fig. 2, *a* and *b*.

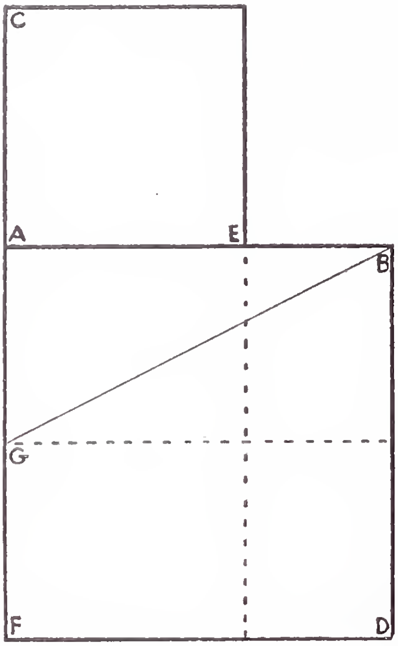


Fig. 2*a*

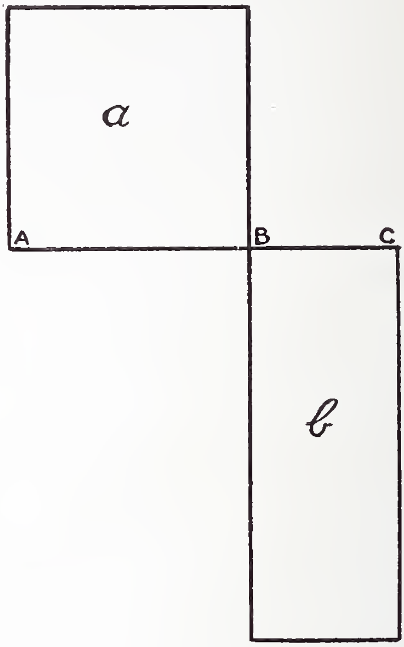


Fig. 2*b*

Euclid uses this construction in the Fourth Book to comprehend a pentagon in a circle. As the Thirteenth Proposition of the Sixth Book the matter appears in another form as an illustration of the Greek conception of the "application of areas." The general idea of this proposition is "to apply to a square an area equal to the square and that shall exceed by a square." Fig. 3 exhibits this.

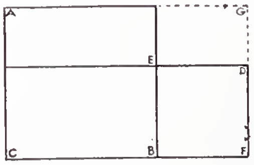


Fig. 3

AB is the given square and CD is the applied area equal to AB and exceeding by the square BD. In Greek phraseology, this square BD would be hyperbolic. The Greeks were much nearer the truth of the matter when they made their proofs by these "applications of areas." Moderns determine the ratio, or cut a line in this proportion by fixing the diagonal to two squares and subtracting unity as in Fig. 4.

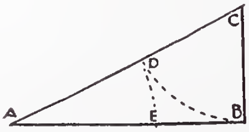


Fig. 4

AB is the given line and at right angle to B the line BC is drawn equal in length to one-half AB. C and A are connected and CD made equal to CB. AE is made equal to AD. This cuts the line AB in the desired ratio at E. Most modern text books on geometry use this construction, but in so doing lose all the value of the area idea, which the Greeks were so careful to maintain. The proportion appears in the Greek classification of irrationals in the Tenth Book of the Elements. The first six propositions of the Thirteenth Book are devoted to it and in this book Euclid proves the following:

(1) That diagonals to a pentagon cut each other in extreme and mean ratio, Fig. 5.

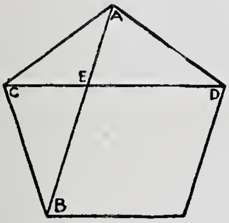


Fig. 5*a*

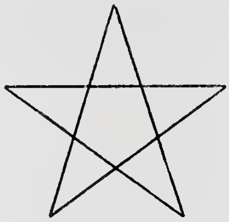


Fig. 5*b*

ACBD is a regular pentagon and AB, CD are diagonals. They cut each other at E. The line AB is cut in extreme and mean ratio at E, as is also CD. This suggests a possible reason for the adoption by the early Pythagoreans of the re-entrant pentagram as a mystic symbol.

(2) The side of a hexagon and the side of a decagon inscribed in the same circle, added, produce a line which is cut in extreme and mean ratio at the point of juncture of the two composing lines.

(3) In a rectangle (which has by the writer been given the name of "the rectangle of the whirling squares" because of a certain peculiar property unknown in modern times) wherein this relationship of extreme and mean ratio exists between end and side, if the side is used as a radius to describe a circle, this line is the side of a hexagon, the end is the side of a decagon and a diagonal to the rectangle is a side of a pentagon described in the same circle.

Referring to Fig. 3 the reader will recognize the area AF as that of a whirling square rectangle; CD is a 2.618 rectangle, its reciprocal being .382 and AE is a similar figure to CD. If the line CB equals 1 then BF equals .618. CF is cut in extreme and mean ratio at B, and BG is a reciprocal area to AF.

The above rough sketch of what is known of this peculiarly fascinating ratio will indicate to the reader that it is a very engaging subject and, no doubt, destined to play a large part in future design practice. The revival of interest in the matter is largely due to the modern discovery of the presence of the ratio in the architecture of the plant. (See November, December and January DIAGONALS.) Absorbing as "extreme and mean ratio" is, however, its application to problems of design is quite useless if it is used merely as a line. Its value depends upon its power as a co-ordinating factor when employed as a combining element in certain areas. As a rectangle, the end of which is unity and side 1.6180, it is seldom found in Greek design. In fact, it does not seem to be a basic ratio; it appears to be a derived proportion from the square root of five. (See lessons on The Elements of Symmetry.)

Artists' attempts to apply the "golden compass" to define the composition plans of their pictures show that it is invariably used as a line quite irrespective of whatever area condition may exist. Before a group of prominent artists and craftsmen in London recently, a well-known English painter maintained that the ratio could be applied successfully to all the photographs of celebrated pictures published in his book on drawing. It was pointed out by critics that nothing but the original picture would settle this point. This artist was assuming that a photograph does not lie. It has been the writer's experience that it never tells the truth and a simple experiment would settle the matter. An object may be measured accurately and then photographed. It would be quite impossible to determine accurately the proportions of the object from the printed image. This incident is mentioned because it is characteristic of the loose methods of artists in determining measurable relationship.

If the ratio 1.6180 is to be of any value in design the breadth as well as the height of a canvas must be considered and when this is done it will be found that only one rectangle is possible, that is, a shape the end of which is unity and the side 1.6180. In other words, the rectangle of the whirling squares. If anything greater or less than unity is used as an end measurement then the proportion 1.6180 disappears; the rectangle becomes something else and the new shape supplies its own proportion, whatever the end and side relationship may be. Suppose, for example, we take some ordinary canvas shape, say 16 by 24. The lesser divided into the greater produces the ratio 1.5 or one and a half, a purely static affair. But suppose we divide the side, that is, 24, into extreme and mean ratio. This may be done at once by multiplying 24 by the reciprocal of 1.6180, that is, by .6180. The line 24 is now divided into two sections, one measuring 1.4832 and the other .9168 parts. When we divide 16 into either of these figures or either into 16 an extraordinarily complicated system of proportions results and, whatever power 1.6180 possessed in connection with unity, it is utterly lost in the new system. This is the sort of condition resulting from an unintelligent use of an instrument such as the "golden compass." But suppose we use a 1.6180 line in connection with another line of the dynamic series, say 1.309, the lesser divided into the greater produces the ratio 1.2360 or two .618 rectangles standing side by side. This new shape produces its own characteristic subdivision scheme and the ratio 1.618 becomes a minor factor. Suppose our lines are 1.4472 by 1.6180, the resulting ratio would be 1.118, an area composed of two root-five rectangles. Even with these dynamic rectangles, the "golden compass" would be a joke. The only value an instrument of this type could have, would be as an adjustable device much like a pantograph with some of the dynamic series of ratios indicated on the movable legs.

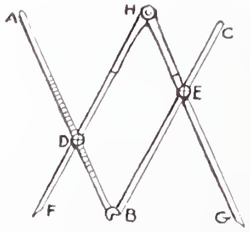
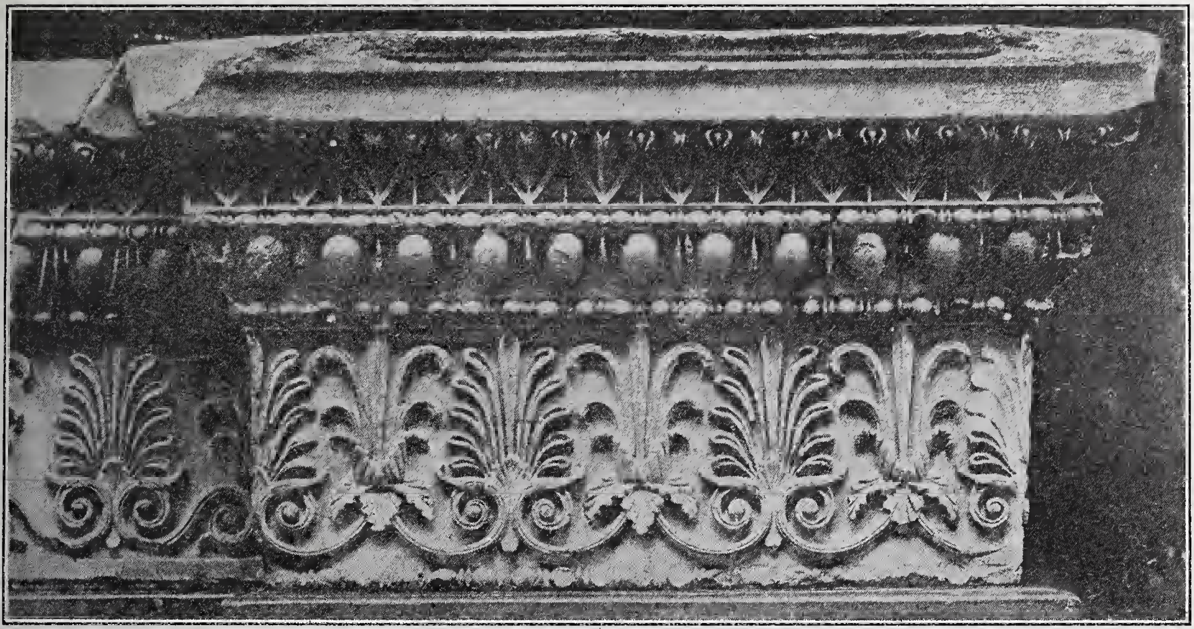


Fig. 6

If we take two legs such as HF, HG, Fig. 6, and to them attach two other legs, such as AB, BC, and fasten them by adjustable pins at D and E we may arrange the points F B G in any ratio we may desire. We could determine a dynamic rectangle by such a device but the matter would end there. All such instruments are useless because a simple metric scale and right angle will do everything the machine can do and infinitely more. An instrument such as the one described divides a line into extreme and mean ratio and makes no provision for dividing an area. The line means nothing to design, the area means everything.



Pilaster Capital from the Erechtheum, British Museum

THE ERECHTHEUM FRIEZE, Second Article

THE photograph of the pilaster capital of the Erechtheum shows clearly the character of the pattern decoration of the East face with the crowded condition of the anthemia units, the irregular arrangement of the tongue-and-groove and egg-and-dart units above and one unit of pattern immediately to the left. The width, from center to center, of the East face units of the capital, is 1.191; for the unit immediately to the left it is 1.4472. On the short North face of the capital it is .9045. Adjoining this on the North return it is 1.4472. For the three slabs further to the left it is 1.309. (See previous article.)

For the general proportional subdivisions of all the pattern units we may examine one unit from slab number 3 (see drawing, Fig. 1*b*). The most striking feature of the arrangement is that the point E is in the center of the rectangle AB. This rectangle is exactly one-tenth of the slab face. The ratio is 1.764. AD is a square and DF a 1.309 or .764 area. BG, FM are squares and their diagonals cross on the line JK. The areas BM, FG are each composed of two whirling square rectangles. OG, IM are root-five shapes. The point R explains the relation of the line OI. This line does not appear on the marble but it defines the full height of the pattern; OA measures .2. FN is the face of the lower course (see cross section in the April DIAGONAL). On the capital this course is cut into egg-and-reel moulding. ST is a deep-cut section to receive a piece of marble. Either the Greek stonecutter made an error or the marble was imperfect at this point. The argument would seem to be in favor of a workman's error. On this slab a piece of a tongue-and-groove unit was similarly treated.

The planning process for this decoration seems to have been as follows: the face of a slab probably was divided into a series of dynamic rectangles; on the face for the direct elevation of the pattern units and on the end for the different depths shown by the cross section. An unfinished pattern decoration on a cornice fragment in the British Museum, of a much later date, shows that the stone mason carefully outlined the pattern unit with a chisel. Afterwards the stone was cut away to a required depth and the pattern left in high relief. The process on the Erechtheum must have been similar. On all four slabs drills of different sizes were used freely. It is possible that the architect supplied the mason with a stencil. When the proportional heights of the different elements of a unit are established, the varying proportions of other units and their interrelationship may be shown by the measurements and ratios.

For the widest pattern units on the section of the East front immediately adjoining the capital, we have the following measurements and ratios:

Height 2.3090
Width 1.4472, ratio 1.5955

This ratio is connected directly with the measurement of the unit width of the East face of the capital; that is, 1.191. The fraction .5955 of the ratio, if multiplied by 2, equals 1.191. This fraction .5955 subtracted from unity leaves .4045; the width measurement of the unit of the North face of the capital is .9045. This latter fraction multiplied by 2 equals 1.8090.

North capital face units:

Height 2.3090
Width .9045, ratio 2.5528

The fraction .5528 is the reciprocal of 1.8090. Reference to the drawing of this unit (4*c*) shows AB a square and BC an area composed of a square plus a 1.809 figure.

East capital face units:

Height 2.3090
Width 1.191, ratio 1.9387

The fraction of this ratio, .9387, is the reciprocal of 1.0652, an area composed of root-five and a whirling square rectangle, .4472 plus .618. The area of each pattern unit of this face, therefore, would be composed of a square plus a whirling square and root-five rectangle. The ratio 1.0652 is connected with the measurement 2.0652.

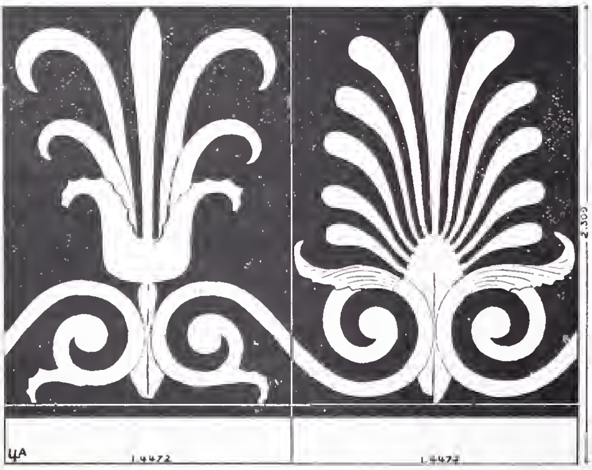


Fig. 1*a*—Pattern Unit on Slab 4
Measurement: Height, .2309; Center to center, width, .14472; Ratio, 1.5955.

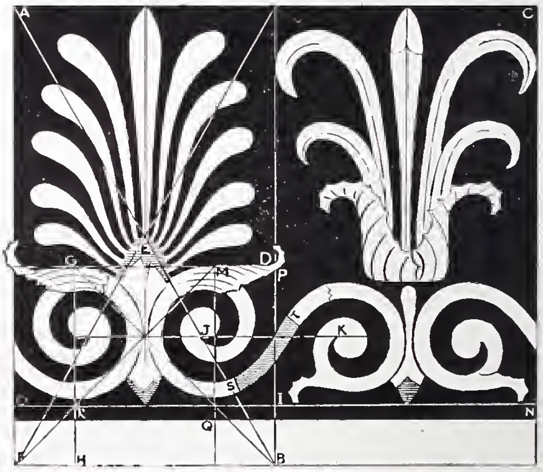


Fig. 1*b*—Pattern Unit for Slabs 1, 2 and 3
Measurement: Height, .2309; Center to center, Width, .1309; Ratio, 1.764.

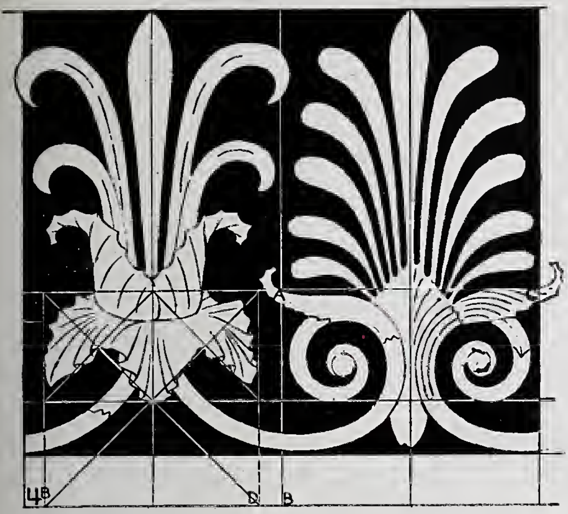


Fig. 2*a*
Pattern Unit of the East Face of the Capital
Measurement: Height, .2309; Center to center, Width, .1191; Ratio, 1.9387.

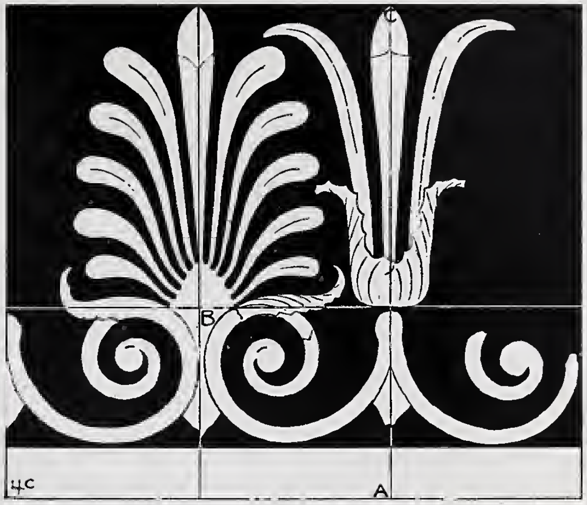


Fig. 2*b*
Pattern Unit, North Face of Capital
Measurement: Height, .2309; Center to center, Width, .09045; Ratio, 2.5528

It was noted in the previous article that the subpattern units, the tongue and groove and egg and dart, registered perfectly with the anthemia pattern on slab 3; on the others they did not register. On slab 3 they are even in number; on the others they are odd.

[To be continued.]

DYNAMIC SYMMETRY OF MAN FOR ADVANCED STUDENTS

PREVIOUS articles upon this subject have carried us, in a general way, through most of the proportional details of one skeleton. There remains, however, for this example, the matter of hands and feet. Dynamic analysis of the human hand shows a wonderful engineering construction. A hand from the skeleton being discussed, in its bare proportional relationship of detail, is an illustration of this. The total length of this hand from middle finger tip to the top of the wrist is 2.118 dm. The wrist itself measures .3541; the bones of the hand and finger respectively show .6545, .5, .382 and .2275 d. When the first three measurements are examined the ratio 1.309 is developed; *i.e.*, .6545 divided by .5 or .5 divided by .382. The measurement .2275 does not subscribe to the ratio. The reason for this probably appears in Fig. 2. If we arrange the bone lengths, as in the drawing, which simply represents the structural base of the fist, we find a perfect right-angled adjustment and a diagonal to the whole, that is, a diagonal to a 1.309 rectangle, .6545 divided by .5, cuts the diagonal of the reciprocal, that is, a 1.309 rectangle, .5 divided by .382, at right angles at the point A. The first bone of this finger is too short by the distance from B to C. The shortness of this bone may be due to the fact that if it had the right ratio length, the hand, when covered with flesh, could not be closed tightly without injury to the palm. The flesh pads on the end of the finger and the palm just make the necessary adjustment so that

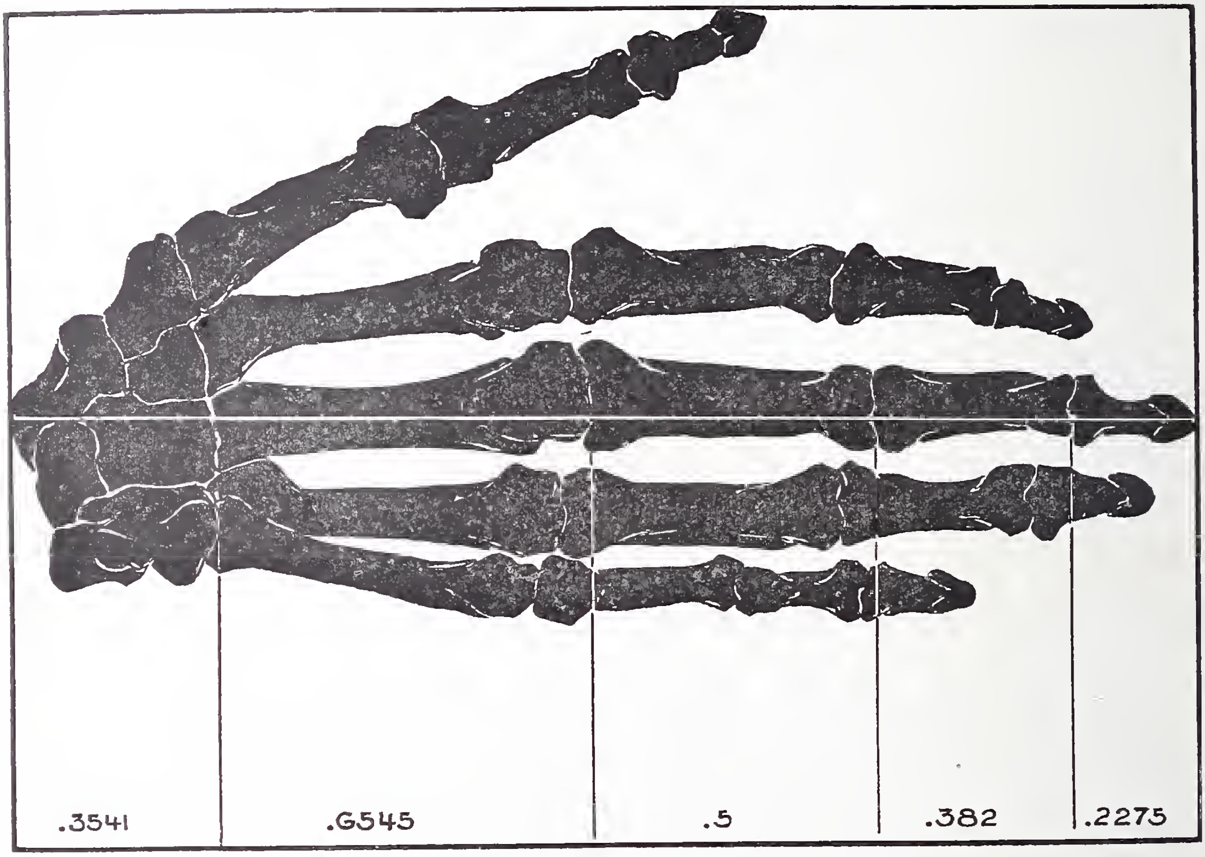


Fig. 1. The Ratio of the Bones of the Middle Finger, Ratio 1.309

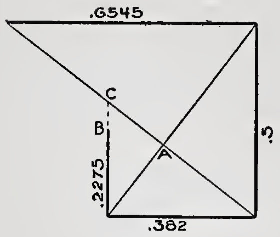


Fig. 2

when the hand is closed we find we have the strongest possible structure arrangement with minimum of material. We have here, really, the basic principle of bridge building. In fact, as engineers have frequently pointed out to the writer, the entire dynamic idea of reciprocals, wherein diagonals cross each other at right angles, is, in its basic form, an engineering conception with structural binding secured by simplicity of means and maximum strength obtained by minimum of material.

The structural base of the foot is that of the arch adapted to a combination of movements. In the drawing, Fig. 3, one set of proportions only is shown and for this purpose it was not necessary to make a true elevation where the toes would be in place, one immediately behind the other. The true height of this foot, to the top of the ankle bone, is .764, the total length 2.618± d., practically identical with the forearm of this example. The ratio is 3.427. It will prove interesting practice for the student to work out ratios for the measurements .927, 1.309, 2, 2.309 and 2.4045 in relation to the height of .764. The series of ratios 1.2135, 1.713, 2.618 and 3.1472 are connected with the symmetry of this entire skeleton.

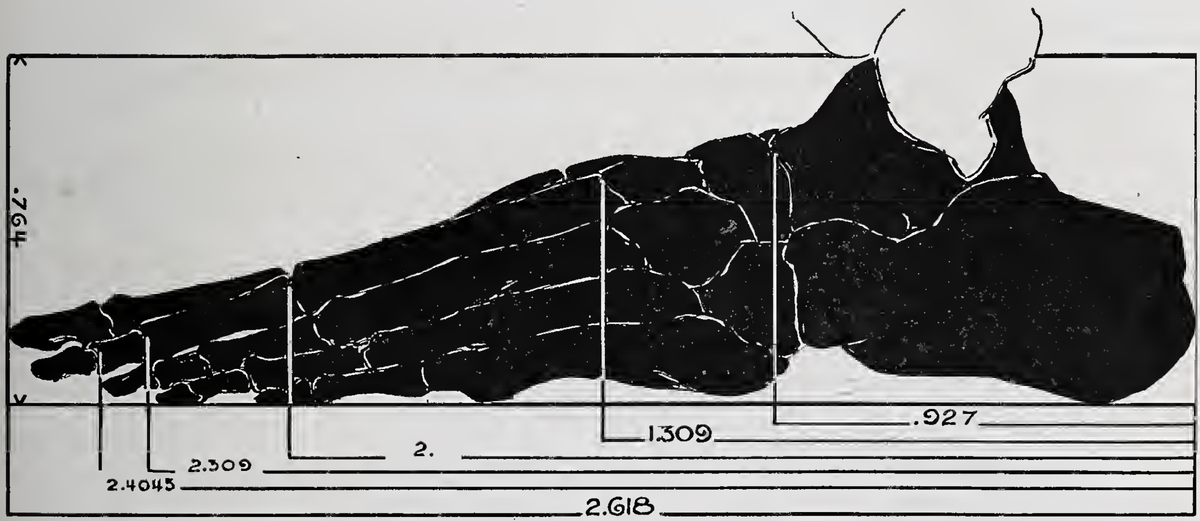


Fig. 3

It will not be necessary for the present to go further into other proportions of this foot as, for construction purposes, the foot of the live model is better. The ground is now prepared for the consideration of other skeletons. This, however, may often be done by simply stating the measurements and ratios—the student should be able to work out for himself the resulting forms.

Before taking up other aspects of human symmetry, it may be well to have a brief description of measuring units and something more about the so-called Polykleiton Canon.

Robert Fletcher, who compiled the bibliography of man measurement statistics for the Provost-Marshal General's Bureau at Washington, found 115 published schemes of proportion, distributed among the different nations thus: Italy 33, Germany 22, Belgium and Holland 13, Spain 5, France 25, England 16, United States 1. This list is up to 1883. Since then many other works have appeared.

In a lecture delivered at Washington about the time he compiled his bibliography, among other things bearing upon the subject, Fletcher said:

"The cubit, or forearm, measured from the point of the elbow to the tip of the middle finger, was the standard measure of Egypt, Assyria, Babylonia and Judea. It is the most ancient measure known, being used long before the foot was employed. In exploring a temple at Karnak, an ancient Egyptian rule was discovered which a workman had carelessly left in an aperture of the masonry which had been afterward built up. It was a two-cubit rule which thus came to light 3000 years after it had been laid aside, and it is now in the British Museum. It corresponds precisely to the width of the entrance to the Great Pyramid of Ghizeh, which was built centuries before the temple at Karnak, proving the great antiquity of the measure. In some parts of Arabia, even now, the natives measure their cloth by their forearm and throw in a hand's breadth over as an allowance for variations.* The ell was originally the same measure as the cubit, being derived from the ulna, the Latin name of a bone of the forearm. The foot was first employed as a measure in Greece, and according to tradition, the foot of Hercules was the standard. This is probably a corruption of a story told by Aulus Gellius, the authority for which he ascribes to Plutarch, and as it is the first recorded instance of scientific anthropometry, I must repeat it. In instituting the Olympian games, Hercules measured out the stadium with his foot, making it 600 feet long. Other stadia in Greece, measured by ordinary men, proved shorter than the original, so Pythagoras, by simple rule of three, obtained the size of the foot of Hercules, and, being deeply versed in Egyptian lore, he applied their canon of proportion and from the foot deduced the height and other dimensions of the hero. Hence the proverb, *ex pede Herculem.*

"Other bodily measures are the digit, or finger's breadth; the palm; the hand's breadth, which is still in use as horseman's measure, being fixed at four inches; the span; the pace, in the Latin *passus,* and from a thousand paces, *mille passuum,* we get our word mile. From the Saxon *gyrdan,* to girdle, signifying the girth of the body, we get our English *yard.* Also from the Saxon *fadhm,* to embrace, being the length of both arms from finger-tip to finger-tip, we get *fathom,* a measure now established as six feet. The old French inch is the *pouce,* being the breadth of the thumb. In pharmacy we still use the terms a handful and a pinch, or, in decorous Latin, *manipulus* and *pugillus.* In domestic pursuits the middle finger, from knuckle to point, is popularly supposed to represent an eighth of a yard, and the *nail,* measuring from the end of the thumb nail to the second joint, half that quantity.

"Vitruvius, whose work on architecture was written but a few years before the birth of Christ, gives a scale of human proportion which, he says, was used by all the great sculptors and painters of antiquity, and this scale has been generally believed to be the canon in question. Nevertheless some misgivings arose from the fact that two of the dimensions given were evidently erroneous, when a fortunate discovery of recent date dispelled all doubt. In the library of the University of Venice are preserved some drawings and manuscripts of the great master, Leonardo da Vinci, and about thirty years ago there was discovered among them a translation into Italian of this famous passage of Vitruvius.

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*The cubit measurement must be spontaneous. When the early white settlers in America wished to purchase or sell supplies, they found that they had to deal with Indian money or wampum, a sort of bead made of clam shells and strung on thong. One Indian treasury was on Long Island and it is recorded, that, when doing business with the Dutch settlers on Manhattan, the natives selected men who had short forearms to measure the wampum.—Ed.

It is in the handwriting of Leonardo and illustrated with a drawing by him. Upon the paper is written in Greek characters 'The Canon of Polykleitus' and, most important of all, the two erroneous dimensions have given place to a corrected and accurate form. The drawing illustrates that part of the canon which describes the relations of height to the outstretched arms. I must again remind you that Leonardo da Vinci was not only one of the foremost artists of the Renaissance, but he was a sculptor, an architect and a geometrician. He was an engineer and constructed canals, sewers and fortresses. His knowledge of anatomy, perspective, physics and chemistry, was far in advance of his day, and with these scholarly tastes he was also a man of wealth. It is extremely probable that he had in his possession a manuscript copy of Vitruvius's work in which the error found in the *editio princeps,* and perpetuated by it, did not exist.

> "On the field of the drawing Leonardo had written in characters copied from an ancient Alexandrian manuscript the 14th verse of the 139th psalm: 'I will praise thee, for I am fearfully and wonderfully made'; at the top, in the same character, is written, 'The Canon of Polykleitus.' "

Leonardo's translation: "The standards according to which all measurements are wont to be made, are likewise deduced from the members of the body; such as the digit, the palm, the foot and the cubit, all of which are subdivided by the perfect number which the Greeks called Teleios.

"Nature in the composition of the human frame has so ordained that the face, from the chin to the highest point of the forehead where the head begins, is a tenth part of the whole stature; the same proportion obtains in the hand, measured from the wrist to the extremity of the middle finger; the head, from the chin to the top of the scalp, is an eighth. From the top of the chest to the highest point of the forehead is a seventh. From the nipples to the top of the scalp is a fourth of the whole stature. If the length of the face, from the chin to the roots of the hair, be divided into three equal parts, the first division determines the place of the nostrils; the second the point where the eyebrows meet. (The ear likewise is a third of the length of the face.) The foot is a seventh part of the height of the entire frame; the cubit and the chest, the width across the shoulders, are each a fourth.

"If a man should lie on his back with his arms and legs extended, the circumference of the circle which may be described about him with the umbilicus for a center, would touch the extremities of his hands and feet.

"The same obtains if we apply a square to the human figure; for, like the contiguous sides, the height from the feet to the top of the head is found to be the same as the distance from the extremity of one hand to the other, when the arms are extended."

Professor Fletcher adds:

"But although we have this reconstruction of the Canon of Polykleitus, it is not evident what part of the body he selected as his modulus.

"Alberti, known as the Florentine Vitruvius, celebrated as artist, sculptor, architect and geometrician, published early in the fifteenth century his scheme of a perfect human figure, obtained from the measurements of living models and classic sculpture. He took for his modulus one sixth of the entire height and this he divided into ten degrees and each degree into ten minutes. His proportions have been found to correspond very closely to the results obtained in our day. His employment of a decimal system is also worthy of note.

"Vitruvius, architect, records in his work on architecture, that the measures of man are by nature distributed in this wise, that is: that four fingers make a palm, and four palms make a foot; six palms make a cubit; four cubits make a man; and four cubits make one step; and twenty-four palms make a man; and these measures constitute the human edifice.

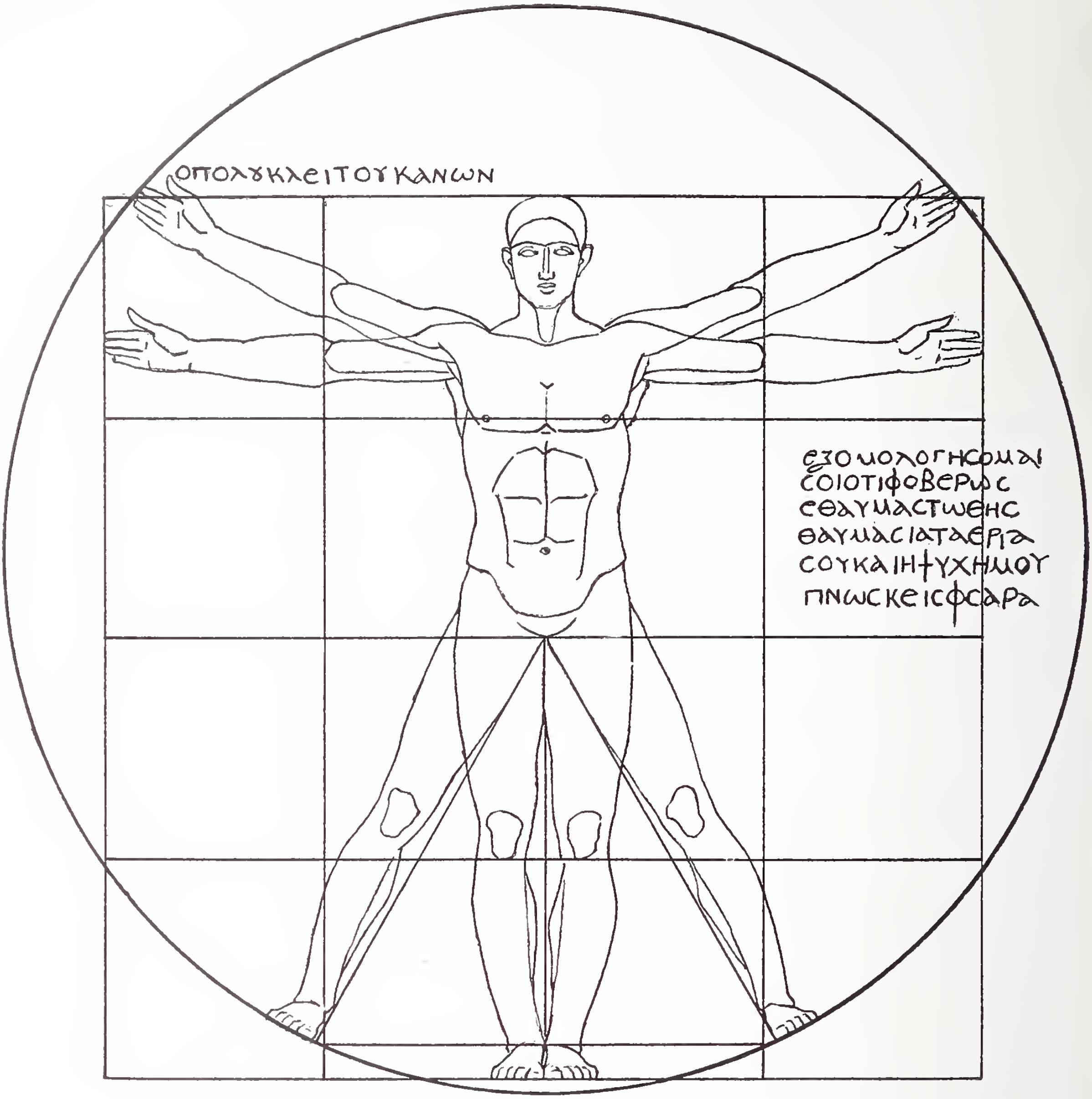

Fig. 4. Leonardo's Conception of the Vitruvian Interpretation of the Greek Canon. Published by Joseph Bonomi, 1873.

"If thou expandest so much thy legs that thou lose one fourteenth of thy height, and expand and raise thy arms that with the longest finger thou shouldst touch the line at the top of thy head, know that the center of thy expanded members is the navel, and the space that is found between the legs is an equilateral triangle.

"From the roots of the hair to the outline of the chin is the tenth part of the height of the man.

"From the outline of the chin to the top of the head is the eighth part of the height of the man.

"From the top of the breast to the top of the head is the sixth part of the height of the the man.

"From the top of the breast to the roots of the hair is the seventh part of the height of a man.

"From the nipples to the top of the head is the fourth part of the height of the man.

"The greatest width of the shoulders contains in itself the fourth part of the man.

"From the elbow to the end of the middle finger is the fourth part of the man.

"From the elbow to the end of the shoulder is the eighth part of the man.

"The entire hand is the tenth part of the man.

"The foot is the seventh part of the height of the man.

"The horizontal line in the center of the diagram marks the middle of the figure.

"From below the knee to the horizontal line in the middle of the diagram is the fourth part of the man (and from below the knee to the ground is likewise a fourth).

"The parts that are found between the chin and the nose, and the roots of the hair and that of the eye-brows, each space by itself is equal to the ear, and is the third of the face."—"Proportions of the Human Figure, etc.," Joseph Bononi, 1873, 4th ed.

[To be continued.]

THE ELEMENTS OF DYNAMIC SYMMETRY

Much of the work upon which the Elements of Dynamic Symmetry is based was done during the year 1919 under the Sachs Research Fellowship of Harvard University. This fellowship was founded by Samuel Sachs of New York City and, under the terms of the foundation, is assigned to scholars of proved ability, whether students, instructors or others, for the general purpose of advancing the Fine Arts. This acknowledgment is but a faint expression of the writer's appreciation of the benefits enjoyed from a fellowship of a type too rare in American Universities. JAY HAMBIDGE.

THE addition of unity or some multiple of unity, such as a square or two squares, to the areas of dynamic symmetry, is frequently found in Greek design. These additions result in a transformation of the original shape in such a manner that the new area may be subdivided to produce schemes differing entirely from the subdivisions of the original area. In the last lesson we considered a 1.191 rectangle and a rather obvious method of subdivision. If we add unity or a square to the 1.191 area the resulting ratio is 2.191. The student should remember that often many methods of subdividing the same dynamic rectangle are possible and that each method produces its own peculiar arrangement of form. Also, two rectangles of the series may be very close to the same ratio. Regarded as simple rectangles it may be impossible to tell, without measurement, which is which; yet when each is subdivided in its own terms, confusion is impossible for the reason that the subdivided arrangements are totally unlike.

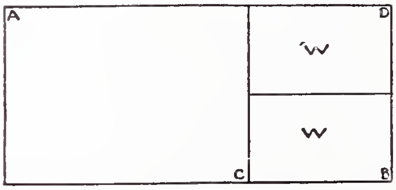

Fig. 1

AB, Fig. 1, is a 2.191 rectangle. If we apply to this area a 1.382 shape the excess area is composed of two whirling squares or the reciprocal area .809. AC is a 1.382 rectangle and CD the .809 form.

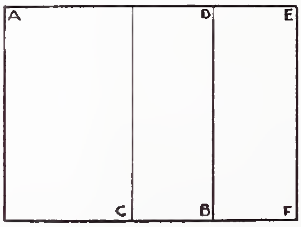

Fig. 2

A 1.382 rectangle is constructed geometrically by first applying to a square a whirling square rectangle, as AC in AB, Fig. 2. .618 subtracted from unity or a square leaves a .382 area, CD. The area BE is equal to CD and AF is a 1.382 rectangle.

If a 1.382 rectangle be divided by 2, as in Fig. 3, the result is two .691 reciprocal ratios or two squares and two root-five rectangles.

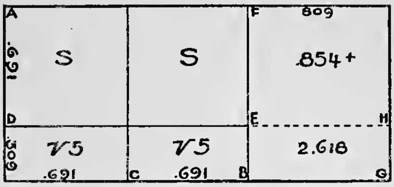

Fig. 3

AB is a 1.382 area; AC, CF two .691 areas, AE two squares and DB two root-five rectangles. AG is a 2.191 area and FG an .809 shape. AD measures .691 because it is equal to the length CB. If DE is continued to H the area BH is defined. This area will measure .309 by .809 and the resulting ratio will be 2.618. FG is an .809 area and FH measures .691 by .809, the ratio being either 1.17082 or .8541, depending on whether we divide .8090 by .691 minus, or the reverse. (See some of the ratios found in the first human skeleton analyzed in THE DIAGONAL.) We may obtain this result in another way. The area FG may be regarded as an .809 or a 1.236 rectangle, each being composed of two whirling square rectangles. If we subtract the area EG from FG we must use the reciprocal of 2.618, *i.e.,* .382. When .382 is subtracted from 1.236 we have left .854.

If from a 2.191 rectangle we apply a 1.309 area the excess is .882, Fig. 4.

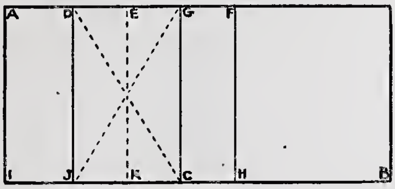

Fig. 4

A 1.309 rectangle is made by construction as follows: AC is a square and CD is an applied whirling square rectangle, the end of which measures .618, *i.e.,* the line DG. .309 is equal to one-half .618, or the area CE. The shape CF is equal to CE and AH is a 1.309 rectangle. FB is an .882 area. This latter area is equal to .5 plus .382 or two squares plus a 2.618 rectangle. JA is a .382 area because it is the difference between .618 and 1, or the square AC. The area JE is .309 and .309 plus .382 equals .691, or a square plus a root-five shape. The area KF is equal to .309 plus .309 or .618. The two reciprocal ratios, .691 and .618, added equal 1.309. If to the whirling square rectangle AB of Fig. 5, the square AC is applied, DC measures .618 and CB .382. The line FC produced to G divides the .882 area DE into the two figures BG and GD. The area GD measures .618 by .882 and produces the ratio 1.427 plus. BG measures .382 by .882 and the ratio is 2.309. If the line FC be continued to H, the area HI, .382 by .691, is a 1.809 rectangle, and JF, .618 by .691, has the ratio 1.118, or two root-five rectangles, one being over the other.

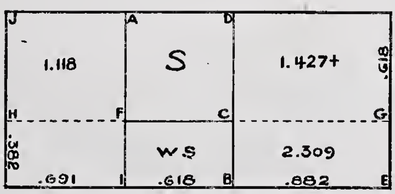

Fig. 5

[To be continued.]

THE BEGINNING OF DYNAMIC SYMMETRY IN ANCIENT GREECE

EXAMPLES of the designs of the craftsmen of ancient Greece, which have survived, represent the products of different men and different localities. Rarely do we find a sufficient number of examples of one man's work to be able to determine, to any degree of exactness, his general planning procedure. Fortunately, however, we have several examples from the hand of an early black-figured pottery designer which show, on analysis, a definite step from static to dynamic symmetry. This man's name was Tleson and he signed his work.

In the Museum of Fine Arts in Boston there is one of these Tleson vases, a handsome kylix. Dr. Caskey of the Museum measured it and found it was constructed on the dynamic theme of root-two. (See Fig. 1.) The overall rectangle was composed of three root-two rectangles. The ratio was 2.1213 or .7071 multiplied by three. In this case the sides of the root-two rectangles are represented by unity and the ends by the root-two reciprocal .7071. The width of the foot equaled the end of one of these root-two areas. Dr. Caskey's analysis shows how one of these root-two forms is subdivided to determine the minor proportions of the vase. Every element of the design is simply expressible in terms of the containing rectangle.

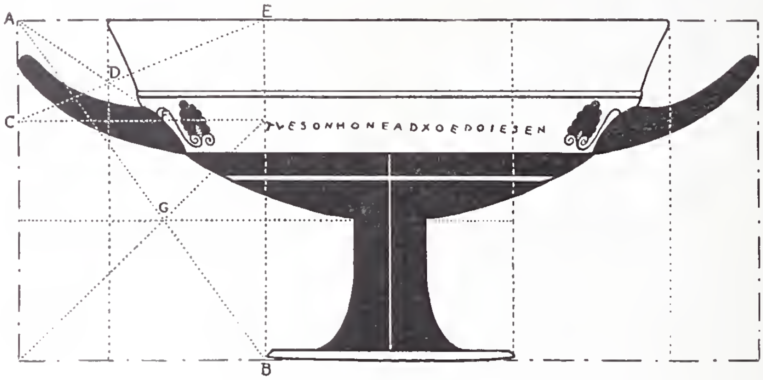

Fig. 1

In the British Museum recently the writer found three other kylikes signed by Tleson. One of these vases is clearly static throughout. The other two are root-two themes. The containing rectangle of the static example is composed of two squares. The area of the bowl projection, that is, the vase without the handles, is a square and a half. (See Fig. 2.)

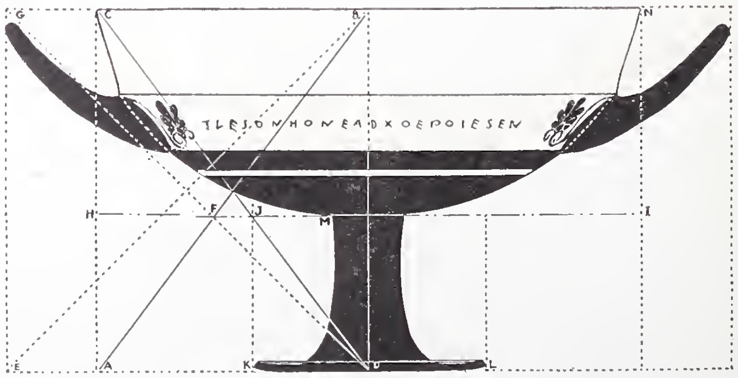

Fig. 2

There is much ingenuity and skill shown by the designer in adjusting the minor proportions to those of the bowl and overall shape. The method of adjustment exhibited here is characteristic of all static work by Greek designers. In the analysis of the vase plan, Fig. 2, EB is one of the major squares and AB is half of the square and a half area of the vase without its handles. Diagonals of these two areas, as GD and AB, cross at F to establish the line HI. This fixes the height of the bowl. The line HI crosses the diagonal CD at J and JK fixes the width of the foot. The point M is placed by a diagonal to the square and a half area, as AN. This is an excellent example of Greek static symmetry.

In Fig. 3, evidently, we have a design example which shows this artist trying to break the bonds of a tradition or a method of personal practice. The overall rectangle, as in Fig. 2, is composed of two squares. But here the resemblance ceases. We now have a subdivision of the major squares, which can in no way be accounted for by a logical subdivision in even multiples. To determine the relation of the composing elements of this design we must abandon the sides of the major squares and their multiple subdivision and turn for light to their diagonals. When we do this the analysis of the design is simple. Undoubtedly the process was, first, the construction of two squares, as MJ. One quarter of a diagonal to one of these squares is used to fix the line QD. This fraction of a diagonal determines the length of the line GD, which is a side of one of the two major squares HG. GR is a major square and GJ a minor. Diagonals of these squares intersect to fix the points N O P and S. F is a center of a major square. KL and M are apparent. A line from T through K to B fixes the width of the foot; a line from B to A places the point U.

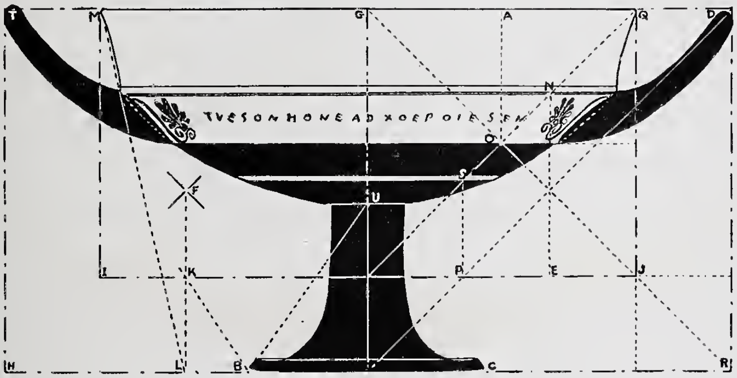

Fig. 3

The overall ratio of the kylix of Fig. 4 is the same as that for part of the Greek bronze

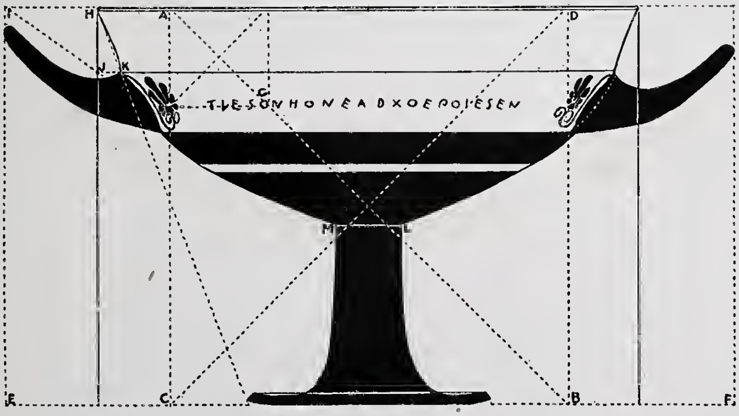

Fig. 4

mirror in the November Diagonal, 1919; that is, 1.8284 or 1.4142 plus .4142. The area is fixed by the diagonals of the square AB. The line AG is one-quarter of AB and AH is one-half of AG. BA is equal to BE and CD to CF. The width of the foot is one-third the overall width. The point J is fixed by a line from B to I. A diagonal to the whole places K, and M and L also lie on diagonals to the whole. The projection of the bowl without foot or handles is an area of two and one-half squares. The height of the bowl in relation to the overall width is three and one-third.

These examples of the early Greek designer Tleson show unmistakably that in him we have an artist who was struggling incessantly to improve the quality of the form of his creations. Beginning evidently in an older static manner wherein he employed squares and even multiples of them to secure area measurableness, he seems to have, in the middle of his career, possibly, awakened to a realization of the fact that a diagonal to a square, while incommensurable with a side as a length, nevertheless produced an area possessing delightful qualities of measurableness. The result of work by craftsmen of this fiber probably enabled Greek scientists, long afterwards, to make the extraordinary generalization that lines incommensurable in length were not necessarily irrational as they might be commensurable in square.

THE ROOT RECTANGLES

A SQUARE on the end of a root-three rectangle applied to a side of that shape changes its form and becomes three squares.

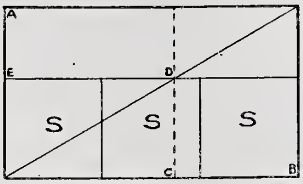

Fig. 6

AB is a root-three rectangle and AC is an applied square on an end. A side of this square cuts a diagonal of the whole at the point D. The area BE is equal to the square AC and is composed of three squares, see Fig. 6.

A diagonal of a root-three rectangle is equal to the side of a root-four rectangle on the same unit base.

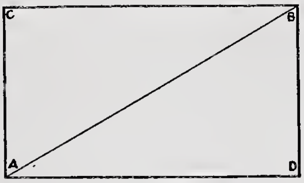

Fig. 7

CD, Fig. 7, is a root-three rectangle and AB, a diagonal, is equal to the side of a root-four rectangle if the line AC is the end of such a shape.

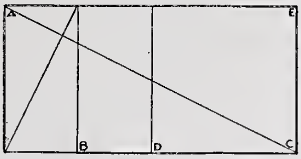

Fig. 8

The square root of four is two, consequently a root-four rectangle is composed of two squares. A reciprocal of a root-four rectangle is equal to one-fourth the area of that shape. AB, Fig. 8, is the reciprocal of the area AC, *i.e.,* it is the reciprocal of two squares and is itself composed of two squares. The excess area of a root-four rectangle, after a reciprocal has been defined, is composed of a square and a half or the figure BE. It will be noticed that a root-four rectangle may be treated either dynamically or statically. It is found in Greek design used both ways. When used dynamically it is subdivided either by root-five or the whirling square rectangle.

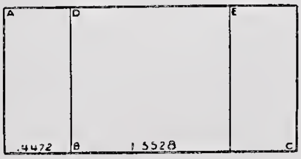

Fig. 9

If the reciprocal of a root-five rectangle, *i.e.*, .4472, be applied to a root-four rectangle the excess area is represented by the difference between .4472 and 2, or 1.5528. AB, Fig. 9, is a root-five rectangle reciprocal. CD is a 1.5528 rectangle. The fraction .5528 is the reciprocal of a 1.809 rectangle, consequently the area CD is equal to a square plus a 1.809 shape. The area EC is a root-five rectangle and BE is equal to .5528 multiplied by two or 1.1056, *i.e.*, 2 minus .8944 or .4472 multiplied by 2.

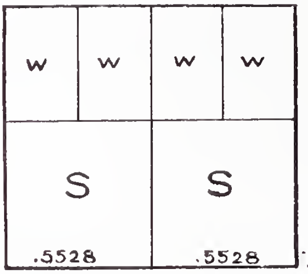

Fig. 10

If the 1.1056 area is divided in the middle and the two 1.809 rectangles defined, the entire area is divided into two squares and four whirling square rectangles, Fig. 10. The reciprocal of two squares is .5. Four whirling square rectangles, or .618 multiplied by 4, *i.e.*, 2.472, is .4045. This fraction plus .5 equals .9045, and this again is the reciprocal of 1.1056.

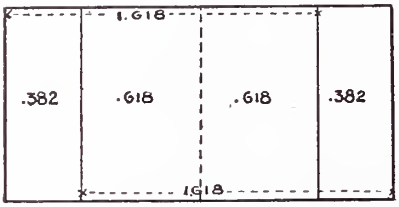

Fig. 11

If a whirling square rectangle, *i.e.*, 1.618, be applied to a root-four rectangle the excess area is .382. If a 1.618 rectangle be applied from the other end of the shape the two whirling square forms overlap and the area of the overlap is composed of two whirling square rectangles, *i.e.*, two .618 shapes or the rectangle 1.236, Fig. 11. If the ratio 1.236 is subtracted from 2 the excess is .764 or .382 multiplied by 2. The fraction .764 is the reciprocal of 1.309.

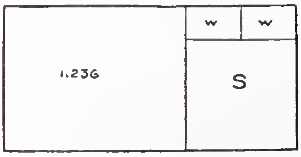

Fig. 12

Consequently the area of a root-four rectangle may be considered as composed of two whirling square rectangles plus a 1.309 area, *i.e.*, a square plus two whirling square forms, Fig. 12.

If a square on an end be applied to a side of a root-four rectangle the area of the square changes its shape and becomes four squares, Fig. 13.

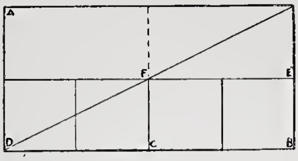

Fig. 13

AC is a square applied to the area of the root-four rectangle AB. A side of the applied square cuts the diagonal of the whole at the point F. The area DE is equal to the square AC, and is composed of four squares.

A root-four rectangle may be regarded as unity multiplied by 2, and any dynamic arrangement applied to one square is equally appropriate for two squares.

[To be continued.]

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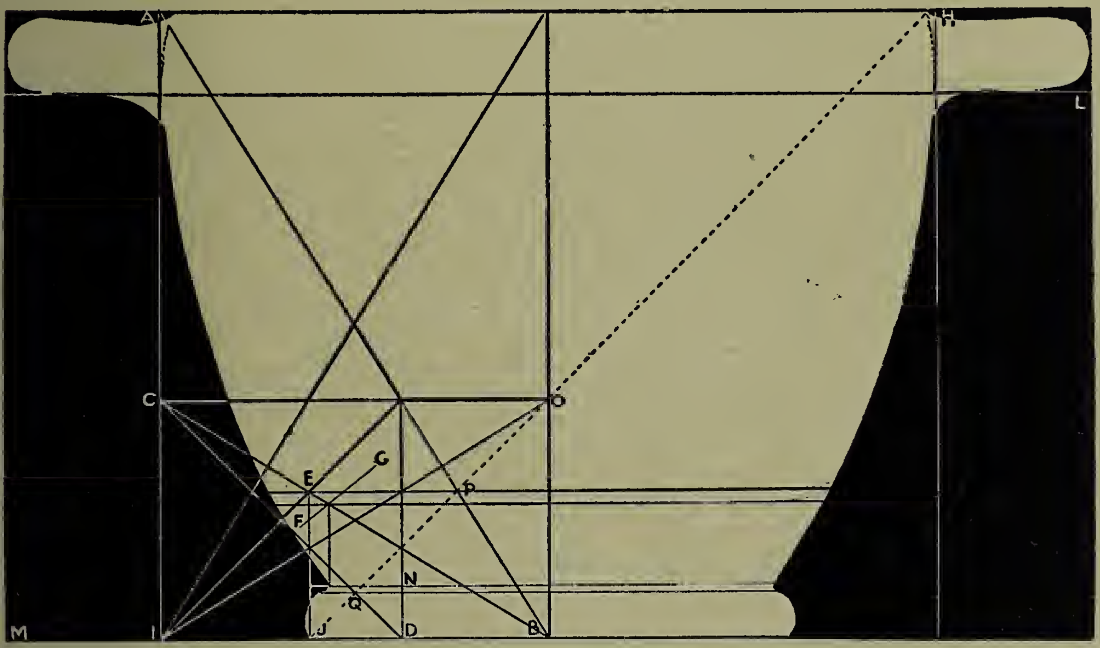

See article on "An Important Skyphos" in this number of THE DIAGONAL

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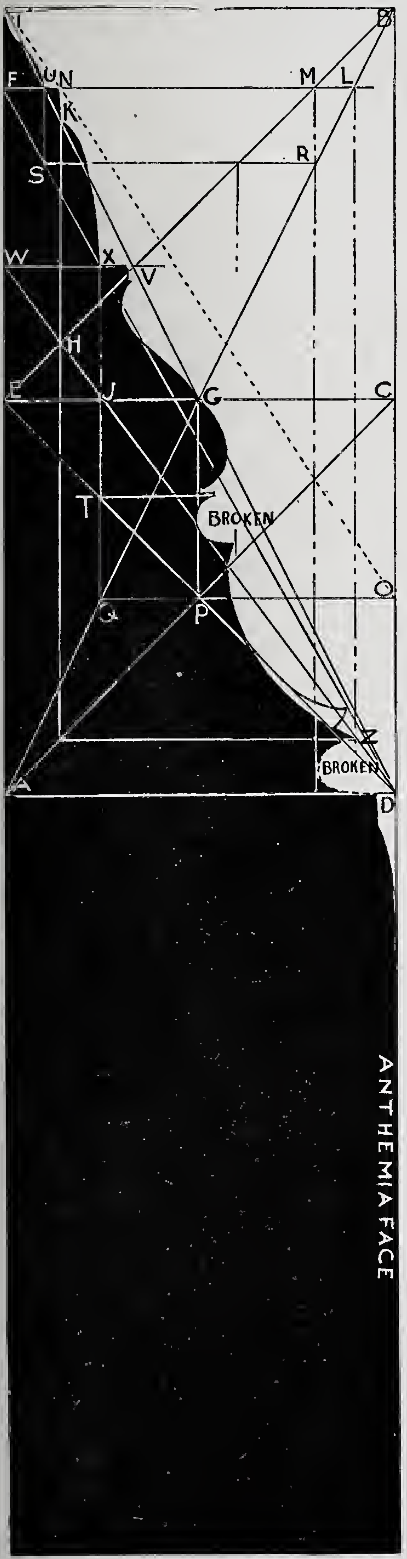

Fig. 1

THE ERECHTHEUM FRIEZE; CROSS-SECTION AND MOULDING PROPORTIONS

A CROSS-SECTION of the Erechtheum frieze in the British Museum gives us for the overhang of the moulding the measured length .1236. The height being .4854 we have for the ratio of the rectangle 3.927, *i.e.*, .1236 into .4854. From the bottom of the bead-and-reel moulding, as shown by the line AD, Fig. 1, to the top of the frieze we have the area AB, which is composed of two squares or a root-four rectangle, and the center of this area is G. It will be noticed that the curve theme of the moulding follows the line ID, one of the diagonals of the two squares. A diagonal to two squares is a root-five line, *i.e.*, the proportion of DA to DI, is as one is to the square root of five. EB, ED are squares. The arrangement of the curve and straight line elements of this moulding, in relation to the containing rectangle of two squares, should be studied carefully as there are symmetry ideas in it characteristic of Greek mouldings generally. The best way to do this is to draw two squares carefully, using a metric scale. The scale will assist the student to understand proportions which may not be comprehended from a construction.

In Fig. 2*a*, we have the diagonals to a root-four rectangle and diagonals to the two composing squares. P is the center of a square. Draw the line PQ parallel to the line AD. The point V is the intersection of a diagonal to two squares with a diagonal to one square. Draw VW. In Fig. 2*b*, draw the diagonal WD. This line cuts a diagonal of a square at H. Through the point H draw the line KY and at Y, at right angles to KY, draw the line YZ. In Fig. 2*c*, P is the center of a square. Draw the line PO parallel to AD. Draw a diagonal to a square and a half, as the line OI. In Fig. 2*d*, continue the line YK from K to N and draw FL, parallel to IB. At right angles to FL draw LZ. In Fig. 2*e*, draw the diagonal FD; it cuts VW at X. Draw XQ. From M draw MR and at right angles RS and again SU. These detailed references to the sub-figures of Fig. 2 will enable us to understand the proportional explanatory lines of Fig. 1. It should be remembered that this explanation is not

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given as the actual Greek method of procedure; these proportions may have been obtained in other ways by the dynamic scheme, but it does make clear the proportional relationship as it exists in the moulding. The bead-and-reel moulding is badly broken and therefore cannot be accurately determined. There are some unlettered points and lines which are closely connected with important proportions but these may be worked out by

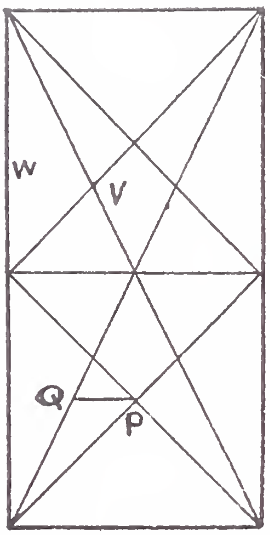


Fig. 2*a*

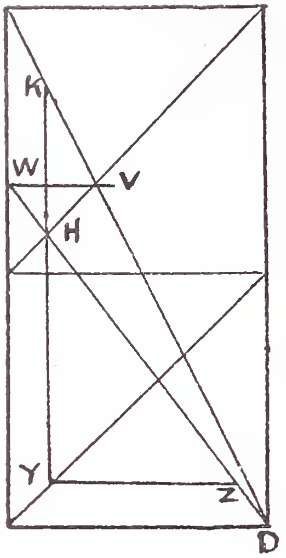


Fig. 2*b*

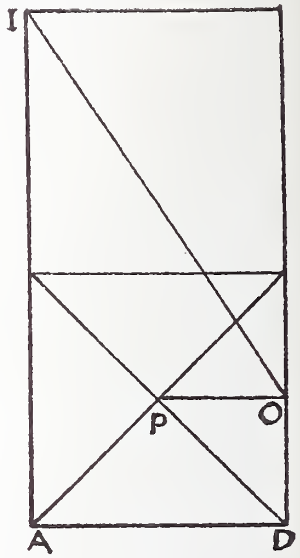


Fig. 2*c*

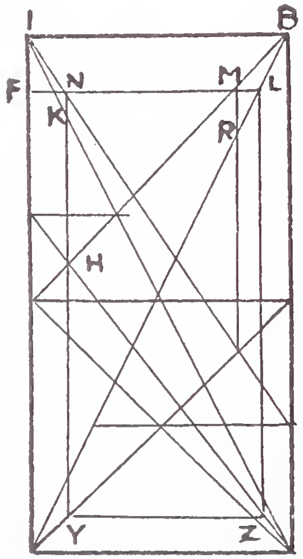


Fig. 2*d*

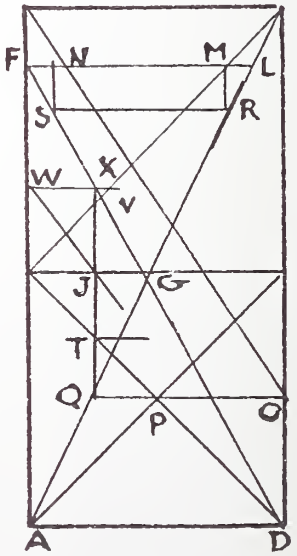


Fig. 2*e*

the student. In closing these notes on the frieze fragment it may be pointed out that the subject is still open and final corrections cannot be made until Athens is visited. The measurements in English feet and inches given in the museum catalogue are inaccurate. In the writer's opinion the Erechtheum is a Greek building exhibiting characteristics of design decay. The old tradition seems to be still in force but there are so many elements of overwork and over-refinement in details of the design that we are compelled to conclude that when this building was completed the backbone of the great age was weakening.

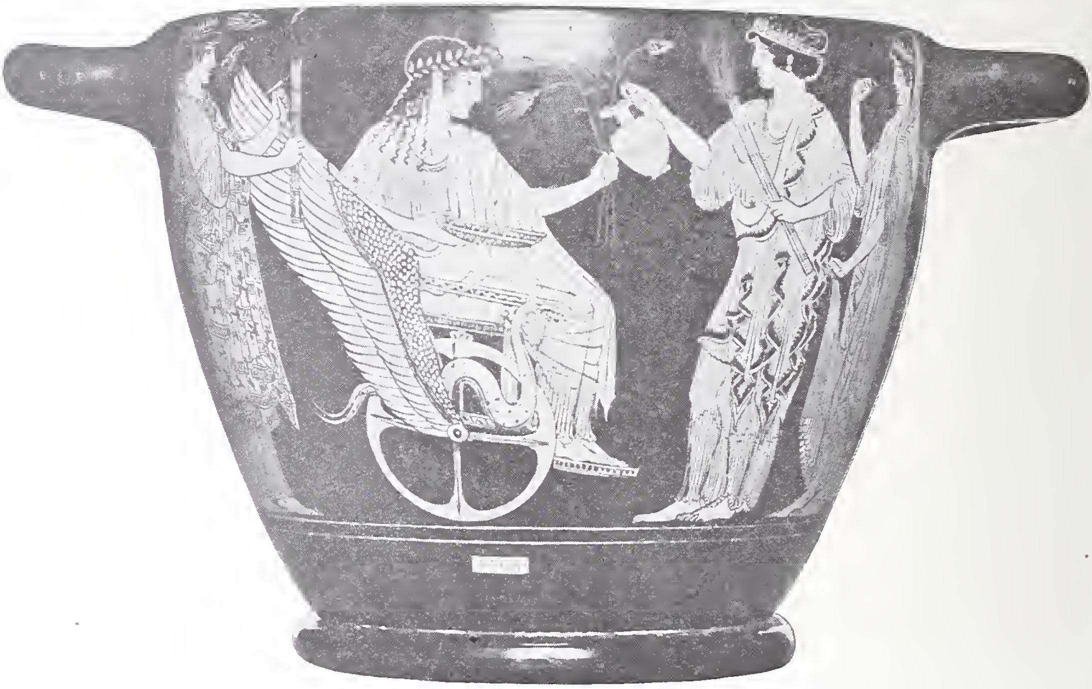


Fig. 1*a*

AN IMPORTANT SKYPHOS IN THE BRITISH MUSEUM. (Signed by HIERON)

A LARGE and important Skyphos or Greek drinking vessel, signed by Hieron, and kept in an individual case in the British Museum, supplies us with a bowl proportion identical with that of many other vases of this class. This proportion is 1.236, or two whirling square rectangles, 618×2. Dr. Caskey of the Boston Museum of Fine Arts, has found many skyphoi with this bowl proportion, as also has the writer at Boston, Yale and New York. From the many examples so far brought to light it would seem that this is a proportion which the Greek designers may have regarded as a norm for vessels of this type.

A peculiarity of this example is that the overall ratio, *i.e.,* the height with the width, including the handles, appears to be a root-three rectangle. At the Boston Museum of Fine Arts there is another important skyphos which has this same ratio of overall dimension and bowl proportion of 1.236. So far in the dynamic investigation these are the only two examples found in any Greek design where this peculiar relationship is manifest. It is not clear, however, that the overall ratio of root-three, in either example, is the right one. There is a whirling square combination which is very close to it but inasmuch as there is no detail between the bowl and the extremities of the handles the area cannot be calculated.

The Measurements

Height, .2120 m. Width, including the handles, .3665 m. Ratio, root-three. Width, without the handles, .2630 and .2620 m. .2120 multiplied by 1.236 equals .2620 m.

Width of foot, .1665 m. (Compare with .3665.)

Distance of the base of the composition band from the top of the vase, .165, *i.e.,* the width of the foot with this band distance from the top of the vase equals a square.

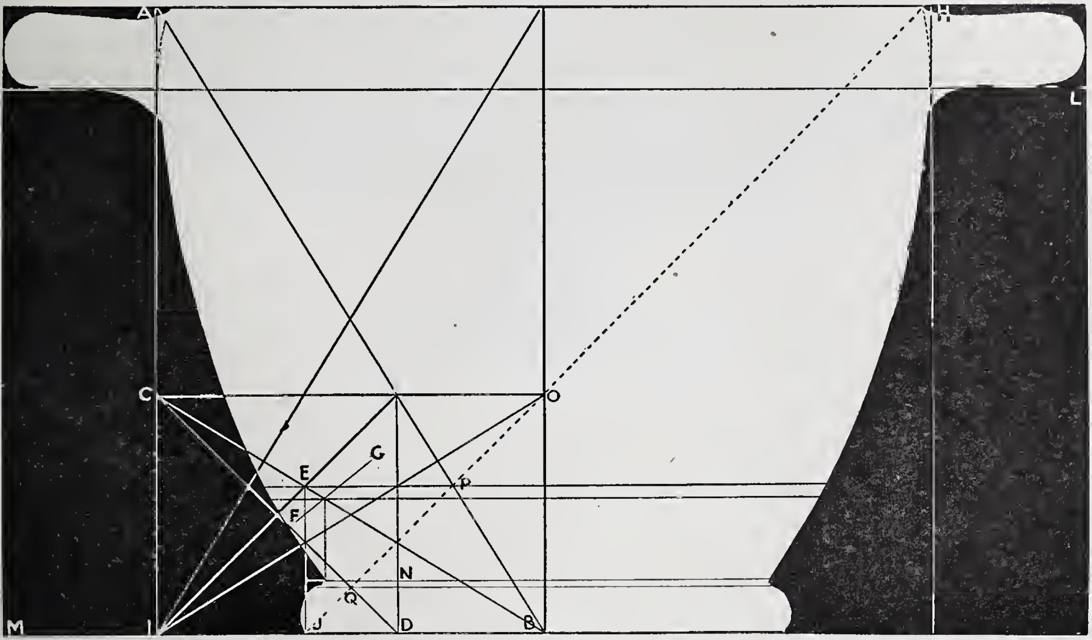


Fig. 1*b*

Explanation of the Areas

AB, Fig. 1*b*, is a .618 rectangle, as is also BH.
CB is the reciprocal of AB.
CD is a square in CB and F is its center.
CB is a reciprocal diagonal cutting a diagonal of the square CD at E.
FG is a diagonal to the entire bowl proportion, *i.e.*, the area HL.
The area ML is composed of two squares.
JO is a square, therefore JH is a square.
The points P and Q are apparent.

The width of the foot at its base, and this base line was probably the important factor in this relationship, is equal to the distance of the overlap of two squares applied to the ends of a 1.236 rectangle. This distance equals 1 minus .236 or .764, *i.e.*, the width of the foot at its base with the height of the vase supplies a 1.309 rectangle, .764 being the reciprocal of 1.309.

A further subtlety of proportion exists in the relation of the two lines at the base of the picture composition with the width of the foot. The top line distance from the top of the vase is equal to the width of the foot at its base. As we have seen, the bottom line distance is equal to the full width of the foot. If we should produce the top line EP until it should touch the line AI on one side and the line defining the width of the bowl on the other, the area thus inclosed would equal the area of a whirling square rectangle. The distance of this top line from the top of the vase measures .1620 m.; the width of the bowl measures .2620 m. Whatever unit of measure was used in this design it was astonishingly close to the modern meter, as the lengths throughout are but a little greater than the decimal subdivision of the meter-unit: .1618 divided into .2618 m. equals the ratio 1.618. If we assume that the width of the bowl is .2618 m. and the ratio 1.236, then the height should be .2118 m. This is the sort of thing which appears constantly in Greek design analysis.

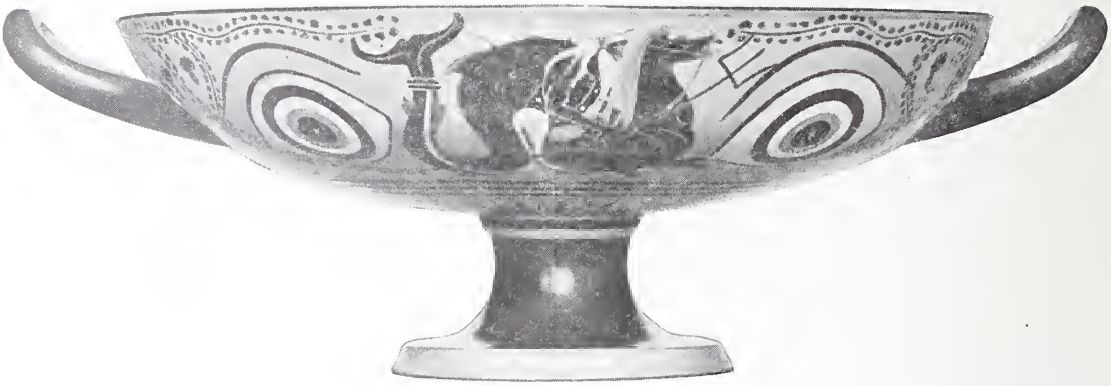


Fig. 1*a*

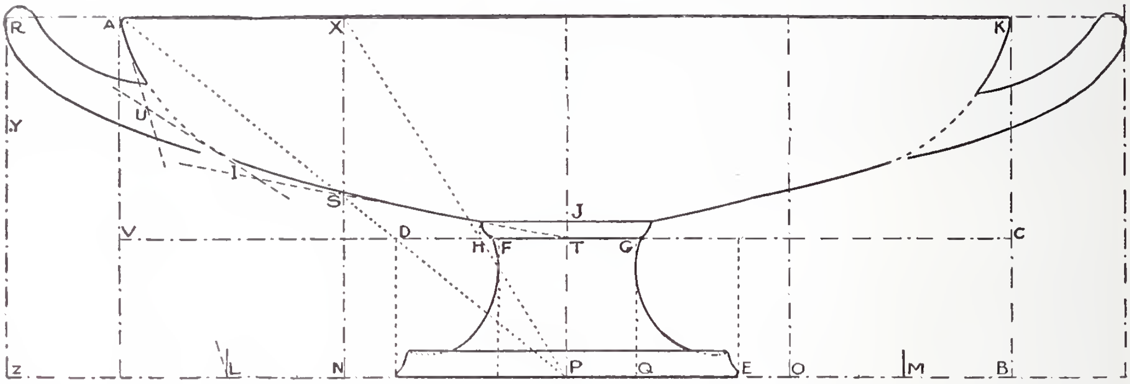


Fig. 1*b*

THE FIRST VASE OF THE BRITISH MUSEUM COLLECTION TO BE EXAMINED DYNAMICALLY

THE first vase of the collection of Greek pottery in the British Museum, a large black-figured eye-kylix of the best type, to be analyzed for its symmetry, showed an overall proportion of 3.118 and a bowl ratio of 2.472. The overall area is composed of two squares plus two root-five rectangles, 2 plus 1.118. The 2.472 area is equal to four whirling square rectangles, .618×4. The bowl area, without the foot, is composed of four squares. The foot, including the stem, occupies the area of four whirling square rectangles, *i.e.*, the foot-stem area echoes the area of the projection of the vase without its handles.

Measurements

Height, .1220, .1220, .1215, .1200, .1220, .1220, .1200, .1230, .1235, .1240 m.

Width, including the handles, .3800 m.

Width, without the handles, .3020 m.

The ratio 3.118 multiplied by .1220 m. equals .3803 m. Error three-tenths of a millimeter.

The ratio 2.472 multiplied by .1220 m. equals .3017 m. Error three-tenths of a millimeter.

Explanation of the Areas

AB is the area composed of the four whirling square rectangles. AC is the area composed of the four major squares of these rectangles.

AP is a diagonal to half of the 2.472 area. This line cuts the line CV at D. Because the line AP is a diagonal to two whirling square rectangles, the area DP is composed of two whirling square rectangles; but the area DP is half the foot-stem area, consequently the full foot-stem area is composed of four whirling square rectangles.

XP is a diagonal to a whirling square rectangle and it cuts the line CV at H. This point defines the width of the ring at the base of the bowl.

FQ is a square within the area DE.

The points L and M are found by the centers of the two whirling square rectangles AN and KO.

The point J is the intersection of lines drawn from L to K and from A to M.

The thickness of the foot will be fixed by a line from R to P cutting a diagonal of the square FQ. If TS is produced to Y, the area AY is a square in the area AZ.

The Curve Tangents

Draw lines from S to T, from P to R, and from A to L. A free-hand curve touching the tangents TS, SI, IU, and UA will be the curve of the bowl.

THE DYNAMIC SYMMETRY OF THE HUMAN FIGURE FOR ADVANCED STUDENTS. LESSON V.

IN previous numbers of The Diagonal we have outlined the general dynamic proportions of a human figure and shown by drawings the nature of many of its modulating areas. The character of these areas shows clearly that a human figure canon, such as that explained by Vitruvius as the Greek canon for man's proportional structure, is entirely inadequate to account for the subtle interrelationships of area and volume which the actual symmetry of the figure discloses. Evidently the classic tradition, by the time it reached the Romans, was but a husk of what it must have been during the days of the great Greek effort. But an analysis of one skeleton will not enable us to understand fully the significance of the area method of measurement. We must have many such examples, if possible at least a hundred, before we can comprehend the extraordinary power of the method to unfold the modulating forms of the human structure. The publication of a hundred human skeleton drawings, however, would be impracticable and unnecessary. Careful study of one skeleton, supplemented from time to time by normal and exceptional examples, will enable the student to use a table of measurements, apply them to the proper rectangles and work out the resulting themes. Also, this plan is advisable for another reason. The human figure themes will probably be used more often for general design purposes than for figure construction. When the dynamic plan is illustrated by the live model, however, we must exhibit the themes for the different examples by drawings.

An illustration of the character of a human shape theme obtained from a specific example is furnished by a skeleton recently analyzed. The base of the area theme of this skeleton is that part of a whirling square rectangle which exceeds a root-five rectangle applied to the end of the shape. This area is CB of Fig. 1.

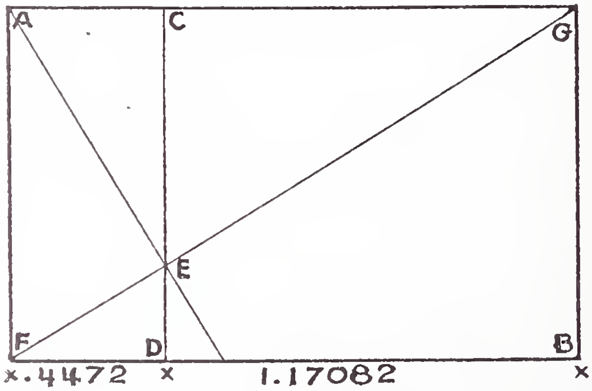


Fig. 1

AB is a whirling square rectangle and AD is a root-five rectangle applied to an end. A root-five rectangle is applied to an end of a whirling square rectangle by drawing a diagonal, as FG, and a diagonal to a reciprocal. (See lessons on Dynamic Symmetry.) These diagonals cut each other at right angles at E. The line CD, drawn through E parallel to AF, defines the root-five area AD. For accuracy and to make the analysis clear it will be necessary to use the fourth decimal of the ratio for the area CB. This may be obtained from a table of square roots. The square root of five will be the necessary number. The line DB is equal to FB minus FD. The line FD is the end of a reciprocal of a root-five rectangle. This is one-fifth of the square root of five. To the sixth decimal this is 2.236068. Dividing this by five we obtain .4472136 for the line FD. The line FB, to the fifth decimal, is 1.618034. FD subtracted from FB gives 1.170820 for DB. This is also the ratio for the area CB because AF equals unity.

The reciprocal of 1.17082 is .8541. If we multiply this reciprocal by two we obtain 1.7082, the reciprocal of which is .58541. The fraction .7082 divided by two equals .3541, and multiplied by two equals 1.4164; this again multiplied by two equals 2.8328. The reciprocal .8541 divided by two equals .42705, and this again by two is .21352, etc.

These simple operations and the figures resulting therefrom are given here because of their connection with the skeleton under examination, as will be explained later.

Using the metric scale we obtain the following measurements and ratios:

Height, 1.7082 m.

Width of shoulders, .3541 m. Resulting ratio, 4.8240.

Height of the torso from the base of the pelvis to the top of the head 8790

Width of the shoulders 3541

Ratio, 2.48240.

(The difference between .7082 and .8790 is .17082.)

It will be noticed that the fractional part of this ratio is equal to the overall ratio with the decimal moved one point to the left.

Distance from the ground to the bottom of the pelvis 82917

Shoulder width 3541

Ratio, 2.34164.

(This ratio is equal to 1.17082 multiplied by two.)

The width of the pelvis, front elevation 2559

Height 21854

Ratio, 1.1708.

Width of the pelvis, side elevation 21854

Height 14164

Ratio, 1.5427.

(The fraction .5427 is equal to the great span ratio of this example, *i.e.,* 1.08541, divided by 2.)

Side Elevation of the Figure

Full height 1.7082

Thorax or rib-basket, front to back 24120

Ratio, .7082.

(The thorax section has a slightly rectangular form.)

Hip Width and Full Height Ratio

Full height 1.7082

Hip width 2918

Ratio, 5.854.

(.2918 is the difference between .7082 and unity.)

Total Figures for the Arm Spread

Upper arm	.342705	
Lower arm	.25326	
Hand and wrist	.18944	
	.78541	total length of the arm.
Multiplied by 2	2	
	1.57082	total length of the two arms.

Arm socket to socket, .28328 m.; this added to 1.57082 equals 1.8541 for the total arm spread; this divided by the height, 1.7082, furnishes an overall ratio of 1.0854. (.28328 equals .07082 multiplied by 4.)

Ratio of the ribs, front elevation, height .3306

width .27082, ratio 1.07082.

The Hand

Total length of the hand, .18944 m.
It is divided as follows:

Wrist	.03618
Hand bones, through the middle finger	.0618 .0427 .0295 .01926
	.18944

Ratio for the four bones of the hand, 1.4472.

The composition of the overall ratio:

1.08541, among others, is composed of the following:

.3618 multiplied by 3
.6382 plus .4472
.7236 plus .3618
.8090 plus .2764

1.08541, minus .5 equals .58541 or the reciprocal of 1.7082.

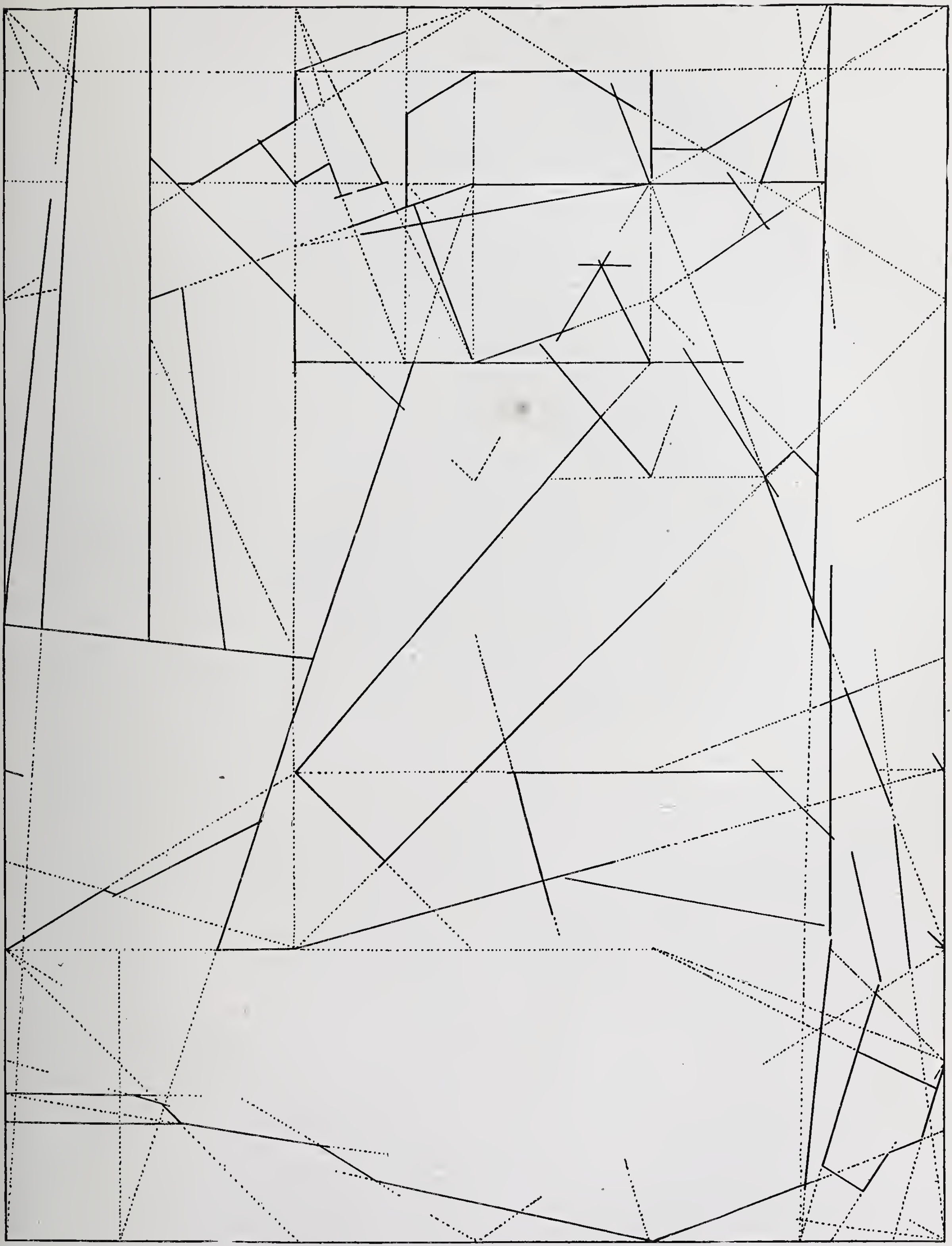
Composition of the 4.824 ratio:

This is composed of 4 squares plus the fraction .8240.

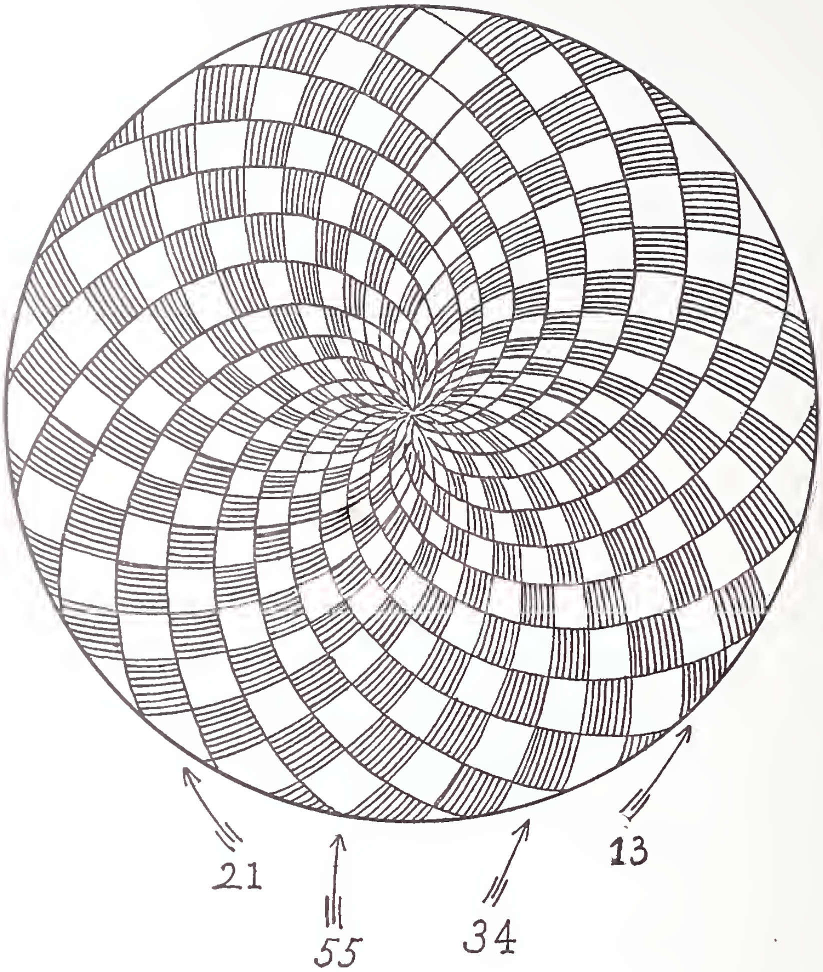
.8240 is the reciprocal of 1.2135. This ratio multiplied by 2 equals 2.42705 and .42705 multiplied by 2 equals .8541.

In the succeeding lesson these ratios will be explained at greater length in relation to a geometric diagram of the figure showing the position of each of the composing elements.

[To be continued.]



Summer 1919. Scheme in 1.309 for out-door picture of girl and dog. Wm. Sergeant Kendall.
(Arrangement in Yellow. Canvas 37.3 x 28.5 inches).



April 17, 1920.

My dear Hambidge:

Your mention of the radiating rows of seeds in the sunflower led me to count the similar rows of disk-florets in a considerable number of last summer's daisies—enough to learn that the normal number is 21 against the clock, and 34 with it. When the flower first opens, the center is a flat disc in which the rows show very plainly and may be easily counted. A little later when the flower is fully developed the center grows up assuming a hemispherical form, and then occurs a phenomenon which may be familiar to others but which is new to me, and which will interest you if it has not already come to your notice.

In a good specimen the 21 and 34 rows, if one looks directly down upon them, are still perfectly evident, and in addition if one looks at the flower from the side another series of rows, nearly vertical but slanting from left to right, can be plainly seen. This series numbers 55. Making this arrangement in diagrammatic form, I find that not only can these three series of rows be followed on the flat drawing, but that a series of 13 rows which is not evident in the plant becomes at once apparent. In the common daisy, therefore, we find 13, 21, 34 and 55, all in extreme and mean ratio so nearly as whole numbers can express it, and all blood relatives to the diagonal of two squares.

Sincerely yours,

Albert A. Southwick.

THE ELEMENTS OF DYNAMIC SYMMETRY

Much of the work upon which the Elements of Dynamic Symmetry is based was done during the year 1919 under the Sachs Research Fellowship of Harvard University. This fellowship was founded by Samuel Sachs of New York City and, under the terms of the foundation, is assigned to scholars of proved ability, whether students, instructors or others, for the general purpose of advancing the Fine Arts. This acknowledgment is but a faint expression of the writer's appreciation of the benefits enjoyed from a fellowship of a type too rare in American Universities. JAY HAMBIDGE.

AFTER becoming more or less familiar with the simple bases of dynamic symmetry and, possibly, using them synthetically in design, occasions will frequently arise wherein the student will find it necessary to use an analytical method to determine the nature of unknown areas. There is a vast difference between synthesis and analysis. In the former, given a reasonable equipment for a journey, we advance freely in the direction of our goal. In the latter the objective is unknown and the finding of a solution to a puzzle or the picking up of a cold trail are matters which may tax ingenuity to its utmost. It is in analysis that the great value of simple arithmetic is proven—without its assistance the task of unraveling a design plan would be practically impossible. The function of arithmetic in this connection is twofold; it straightens out entanglements and furnishes positive proof of correctness of results.

Mathematicians have frequently asked the writer why he doesn't use algebraic symbols. The answer is simple; symmetry is immediately connected with design and design depends to a great extent on similarity of figure; *i.e.,* figures of the same shape but not necessarily of the same size; or figures of different shapes but belonging to a specific sequence of theme. If we used algebraic symbols for the units of dynamic symmetry we should have too great a confusion of values within a specific design example. Moreover, the average designer or the average layman is not sufficiently familiar with the processes of either algebra or geometry and to use mathematical formulae, would result in placing design knowledge beyond the reach of those most interested in the subject.

When we are confronted with the problem of determining the nature of unknown areas we may have recourse to several methods of analysis. If we have used a dynamic base for the development of a design we may be assured, if our procedure has been consistent, that the subdivisions of that area are dynamic whether or not we are able to define their composition. If the unknown area is not recognizable as being composed of two or more familiar areas we may determine its reciprocal, or, if the ratio is less than unity, find the ratio of which it is the reciprocal: having fixed the reciprocal we may examine the excess or defect area: the logical position of a reciprocal is as an area, similar to the whole, applied to the end of a rectangle: we may turn the reciprocal sidewise and consider it as a subdivision of a square: we may apply it to the side of an overall area exactly as we should apply a square, and then analyze the excess area: we may apply a square or squares, or other known areas, and again examine the excess or defect.

Suppose we select some rectangle with which we are familiar and use a process of subdivision with which we are unfamiliar. Let the area be that of a 1.3090 shape. Fig. 1 is such an area.

It is composed of the square AB and the excess area BC. Suppose we draw the diagonal CD. It cuts the side of the square HB at G. The line EF is drawn through G. The major area is now subdivided into the minor areas AG, GC, GD and BF. This process of subdividing an area is one of the oldest known. It is practically the base of the second book of Euclid and, by authorities, is generally conceded to be an Egyptian method of fig-

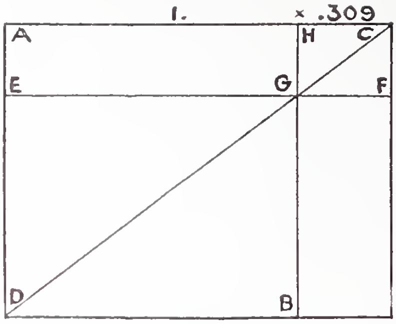


Fig. 1

ure dissection. The right angled shaped figure BC, GA, is known as a gnomen or carpenter's square. It is that figure which, added to a figure, increases the size of the latter but maintains its original shape. The figure DG is similar to the figure DC. The area AG is equal to the area BF, and these two shapes are known as complements. The area GC is similar to the area GD. The simple geometric proof that AG is equal in area to BF is shown in Fig. 2.

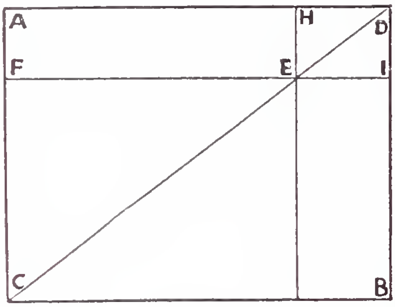


Fig. 2

AB is a rectangle composed of the two right angled triangles ACD and BCD. They are equal in area because each represents half of the rectangle AB. CE and ED are similar figures to the whole; because they are composed of similar and equal triangles they may be cancelled. The area AE must equal the area EB because ACD is similar and equal to BCD. Some have called this the theorem of the gnomen. The word gnomen also means "that by which something is known." Later, in classic Greek days, the name gnomen was given to the upright shadow caster of a sun-dial. We may continue the analysis.

We have decided that Fig. 1 is a 1.3090 rectangle and AB is an applied square on the end. BC is the excess area. The line EG fixes the area EB, a similar figure to the whole, within the square AB. But the area EB is the reciprocal of the area of the whole. (Reference to an earlier lesson wherein the different methods for determining reciprocals are explained will make this point clear.) If the area EB is revolved until it is at right angles to its present position and the line GB coincides with the line AH the diagonal DG will cut the diagonal DC at right angles. The numerical value of the end of a reciprocal to a 1.3090 rectangle is .764, minus. If AD equals unity ED equals .764 and AE equals 1 minus .764, or .236. (See the Root Rectangles in this issue.) The area AF measures .236 by 1.3090 and produces the ratio 5.546—for a more exact determination of this ratio we divide 1.309017 by .23606 and obtain 5.5452; for convenience we fix the ratio as 5.545, *i.e.*, five squares plus the fraction .545. We obtain the extension of the fraction of a ratio by extending the fraction of the square root of 5. To the fifth decimal this 2.23606. This gives us .618034 for the reciprocal of a whirling square rectangle, and by dividing this by two we obtain 1.309017 for the 1.3090 rectangle. The area BC measures .3090 by 1 and fixes the ratio 3.236 or 1.618 multiplied by 2, *i.e.,* two whirling square rectangles. The area AG, .236 by 1, is a 4.236 area.

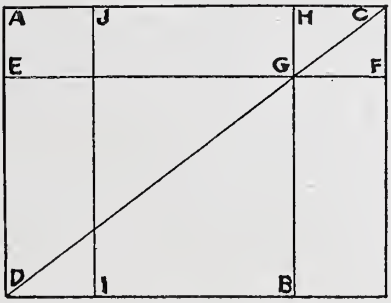


Fig. 3

In Fig. 3, CD is a 1.3090 rectangle; AB, CI are squares applied on either end. They overlap to the extent of the area HI. The line DB equals 1 and DI .3090; consequently the line IB equals .691: HB equals 1. The area HI, therefore, is composed of a square plus a root-five rectangle,—.691 is the reciprocal of 1.4472. The line GB equals .764 and BI .691; the area GI therefore is a 1.1056 rectangle. This ratio will be readily recognized by its reciprocal, .9045. (See the first human skeleton analyzed in THE DIAGONAL.) The area JG, .236 by .691, is a 2.927 rectangle—.927 equals .3090 plus .6180. The area FB, .3090 by .764, is a 2.472 rectangle or a shape composed of four whirling square areas,—.618 multiplied by 4 equals 2.472. See an eye-kylix from the British Museum in this issue of THE DIAGONAL.

[To be continued.]

AN APPRECIATION OF DYNAMIC SYMMETRY BY AN ENGLISH CRITIC—(From the London *Spectator*, Feb. 21, 1920)

ART, like nature herself seems jealous of her secrets. That she should have succeeded in hiding the true system on which she based the most beautiful works the world has known for over 2300 years seems incredible in these days of radium and wireless telegraphy, but such apparently is the case, and it is only the patient labor and ceaseless research of Mr. Jay Hambidge which have at last wrested from the treasures of her storehouse the secret to which they owe their existence.

Had we in our schools an Ictinus, a Callicrates, a Phidias or a Scopas, who knows but that some day we too might raise monuments of beauty and adorn them with sculpture which might compare not so very unfavorably in their symmetry and design with those of the Periklean age? Be this as it may, we can but conclude by the theories expounded by Mr. Hambidge that, if the rediscovered system could but thoroughly permeate our arts and crafts, beauty of balance and rhythm would be the result; beauty, living and vibrating as that of the human form, and such as has not been known in its entirety since the chisel of the masters left us the Demeter of Cnidos, and the *so-called* Fates of the Parthenon.

Going back over the ages the student wonders why Roman art with all its marvels of construction, its ever enduring concrete, its vaults and domes, its use of the "Five Orders" and its overladen wealth of decoration, leaves one sighing and looking over one's shoulder, back to the land that gave virtue to the gods.

Why is the Roman work so different, why is it so *dead* compared with the Greek? Yet we are told that like the builders of the Parthenon the Romans worked on a system based on the human figure; and herein lies the discovery made by the American artist.

It appears that in the time of Vitruvius the tradition still obtained that classical Greek art was designed in conformity with the principles of symmetry derived from the human figure. This, as a fact, has proved to be correct, but the statement bears with it a possibility of misinterpretation, and to this misinterpretation is due the failure, not only of Roman art, but of the art of all succeeding ages, to attain to the heights of the classical Greek prototypes.

The Romans interpreted the tradition as the measurement *in length* of the human form, the symmetry of which, it appears, is in reality incommensurable in length, to be true to nature, and this is the rock upon which the Roman architects and sculptors alike, all foundered. Mr. Hambidge has proved by the deep study of statistics that the human figure cannot be expressed (in length) in even multiples of any one member—such as the head, hand, foot, arm, or leg; for these members are incommensurable one with the other: but—what he has found is that in accord with the known principles of Euclid, that which is incommensurable in length is not necessarily irrational *but may be measurable in area.* This then is the secret.

In his thesis he proceeds to explain the difference between static and dynamic symmetry, the one, the measurement in length by the repetition of units; the other, the measurement in area and in particular the use of the "root-five" rectangle. *Static symmetry* is obtained, as we see, by measurement in length by any given unit or equal part of a unit. "As used by the Copts, Byzantines, Saracens, Mahommetans and the Gothic Renaissance designers it was based upon the pattern properties of the regular two dimensional figures, such as the square, and the equilateral triangle." The Archaic Greek sculptors practiced "static symmetry" and apparently had no knowledge of the secret of Egyptian dynamic symmetry.

The Tenea Apollo, a sixth century B. C. example, although closely resembling the Egyptian prototypes of dynamic symmetry is, in itself, "static" and has a symmetry theme of three and two-thirds, that is to say, the projection of the statue can be contained in a rectangle the area of which consists of three squares and two-thirds, and every member of the body can be expressed in terms of this shape.

It is clear that the archaic sculptors of Greece ignored the secret of Egyptian symmetry, and even later when it became known, work on the same lines of ignorance and with the identical symmetry theme as the Tenea Apollo, persisted, in a lesser degree, contemporaneously with the more enlightened dynamic system, throughout the classic period. The artists producing this inferior work, Mr. Hambidge suggests, were possibly not members of the craft guilds.

Curiously enough with the decline of Athens and the decadence of art, the "static" system became more and more usual until finally, in the first century B. C., the "dynamic" had entirely disappeared. Little wonder therefore at the misinterpretation of the system by the Romans, since the connecting link, as handed down to them was not of the true, but of the archaic type.

The dynamic "is a symmetry suggestive of life and movement. Its great value to design lies in its power of transition or movement from one form to another in the system."

It is derived from nature herself as shown in the human form, the skeleton, the distribution of leaves in plants, the sunflower disk, the pineapple and pine-cone, the cockle shell, and other natural products.

The ancient Egyptians were the first to practice and use dynamic symmetry in their art, and after them the Greeks for a brief, but incomparable period.

Its application to art apparently evolved from the Egyptian surveyors or "rope stretchers" who "corded" the temples and whose work gradually developed into a generally recognized science. The "rope stretchers" were regarded by the Greeks as expert geometers.

From certain wall sculptures in Egyptian temples and tombs it is suggested that there was some sort of ritualistic significance attached to the "rules of the cord" as it was called by the Hindus who later, in the fifth and eighth centuries B. C., also possessed a slight knowledge of the science, which formed part of a sacrificial altar ritual.

But we have no examples left of Hindu art in which its presence is discernible.

A curious connection has been discovered between the "root-five" rectangle in which terms the spiral of the shell may be expressed when reduced from curves to straight lines, and the ratio produced by the summation series of numbers which are to be found in the distribution of leaves in plant growth and such as is found in the disk of the sunflower. This ratio is 1.6180 and the area of a root-five rectangle is said to be equal to a 1.6180 rectangle plus its reciprocal. This 1.6180 rectangle is known as the "golden measure."

In art, especially that which is destined for reproduction, the system usually employed is by means of the diagonal of the rectangle, since any shape drawn on the area of the rectangle having the same diagonal in common, is bound to be in proportion with the whole, *but the value of the diagonal of the reciprocal,* the diagonal which is at right angles to the diagonal of the whole, and which determines the shape of the reciprocal, has been ignored, we are told, by all artists since the days of the classical Greeks, and it is the use of this diagonal which is the key to dynamic symmetry.

If a 1.6180 rectangle be drawn and its reciprocal determined and the diagonal of the reciprocal be again cut at right angles, we again get the same result, and so on to infinity.

It seems as if here we have the revelation of a great secret, the point of escape as it were, essential to all works of genius. Is one traveling too far into the realms of phantasy in reading into Mr. Hambidge's theory a connection with the scheme of the universe? One would like to be able to do so and to see in the shell, the plant, the human form, yea, even in its skeleton, together with the work of the classical Greeks, when all are reduced to a common language, the language of the "rope-stretchers" of Egypt, the stamp, the mark of infinity upon them!

If stones can speak, the group of the East Pediment has whispered me the answer; Demeter, I know, would acquiesce.

Claire Gaudet.

THE RECTANGLES

A DIAGONAL to a root-four rectangle becomes the side of a root-five rectangle on the same unit base.

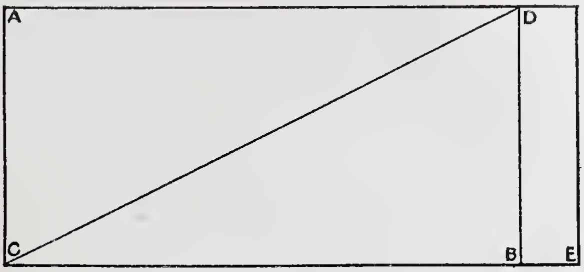


Fig. 1

AB, Fig. 1, is a root-four rectangle and CD is a diagonal. CE is made equal to CD. AE is a root-five rectangle. The area DE represents the difference between a root-four and a root-five rectangle, or .236. The square root of four is 2, and of five is 2.2360. The ratio .236 represents a dynamic shape. Because it is less than unity it is the reciprocal of some ratio greater than unity. When .236 is divided into unity the ratio 4.236 is disclosed. This ratio or its corresponding area may be subdivided in many ways. It may be regarded as root-five, *i.e.,* 2.236, plus 2, or two 1.6180 shapes, *i.e.,* 3.236, plus 1; as three squares plus two whirling square rectangles, *i.e.,* 3 plus 1.236; as 1.809 plus 2.427, etc. The .236 area occupies an important position when whirling square rectangles are comprehended within a square. When whirling square rectangles are described on the four sides of a square they all overlap to the extent of a .236 rectangle. See Fig. 2.

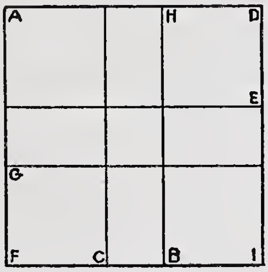


Fig. 2

AB, CD, EF, GD are four whirling square rectangles described on the four sides of a square. They overlap to the extent of the areas GE and CH. These areas are .236 areas. If we subtract .236 from unity the remainder is .764, the reciprocal of 1.309, or a square plus two whirling square rectangles. The areas BD and AC, added, side to side, equal a 1.309 shape. So also do the areas AE and GI. If we divide .764 by 2 the result is .382 or the reciprocal of a 2.618 rectangle. AC, BD, AE and GI are these shapes.

When the square on the end of a root-five rectangle is applied to that area the excess area is composed of two whirling square rectangles. See Fig. 3.

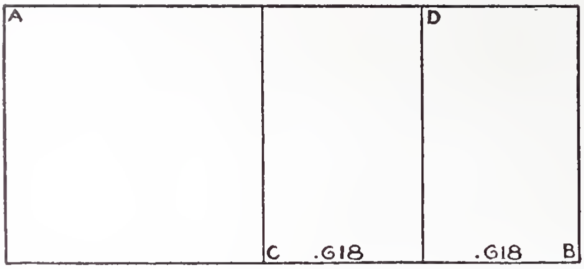


Fig. 3

AB is a root-five rectangle, AC is a square and CD, DB each a .6180 or whirling square rectangle. The square root of five is 2.2360. With 1 subtracted the remainder is 1.2360: this divided by 2 equals .6180.

A square on the end of a root-five rectangle applied to a side of that shape changes its form and becomes equal to five squares, Fig. 4.

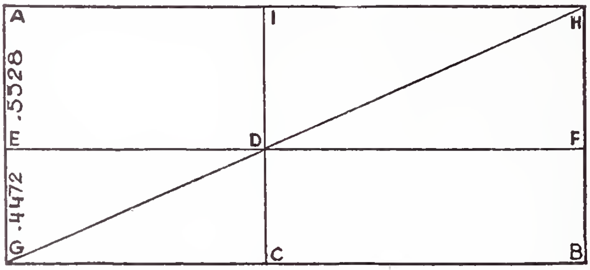


Fig. 4

AB is a root-five rectangle, AC is a square applied on the end. A side of this square is CI and it cuts a diagonal, as GH, at D. The line EF is drawn through D parallel to GB. The area EB is equal to the area AC, and EB is composed of five squares.

If we take the area of the root-five rectangle as shown in Fig. 4 with the five squares defined by EB and divide the entire area into its reciprocals, *i.e.*, into five equal parts as shown in Fig. 5, it will be noticed that each reciprocal stands with a square applied on its end.

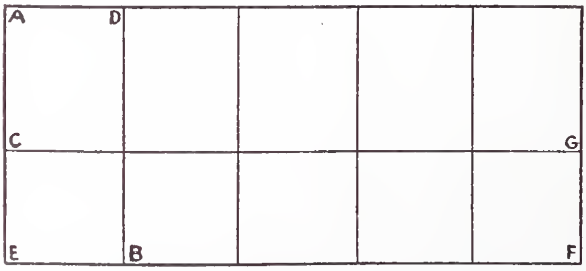


Fig. 5

For each reciprocal of the root-five area, as AB, we have the square CB and the excess area CD. Because a reciprocal of a rectangle is a similar shape to the whole AB is a root-five rectangle and the line EB is one-fifth of the line EF or .4472 (2.2360 divided by 5 equals .4472). The line CE equals .4472. The line AE equals 1; AC therefore is equal to

1 minus CE or 1 minus .4472, *i.e.,* AC equals .5528. EF equals 2.2360 and .5528 divided into this ratio equals 4.045. If we divide the ratio 4.045 by five we obtain the reciprocal ratio .8090. This fraction is the reciprocal of 1.236, or .6180 multiplied by 2, *i.e.,* the area CD is equal to two whirling square rectangles and the area AG is composed of ten such shapes.

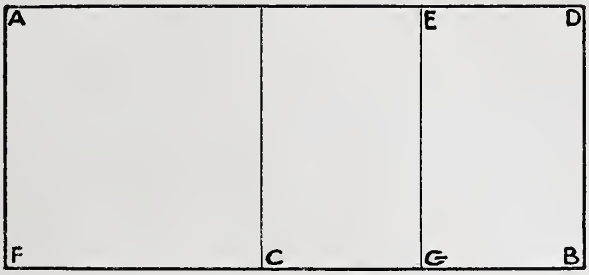


Fig. 6

AB, Fig. 6, is a root-five rectangle and AC is an applied square on the end. The area CD is composed of the two whirling square rectangles CE and EB. The line FB equals 2.2360; FC equals 1; CB equals 2.2360 minus 1, or 1.2360. CG and GB equal .6180.

[To be continued.]

STATEMENT OF THE OWNERSHIP, MANAGEMENT, CIRCULATION; ETC. REQUIRED BY THE ACT OF CONGRESS OF AUGUST 24, 1912,

Of The Diagonal, published monthly, at New Haven, Conn., for April 1, 1920, State of Connecticut, County of New Haven, ss.

Before me, a Notary Public, in and for the State and County aforesaid, personally appeared G. G. Wyant, Secretary, who, having been duly sworn according to law, deposes and says that he is the publisher of The Diagonal, and that the following is, to the best of his knowledge and belief, a true statement of the ownership, management (and if a daily paper, the circulation), etc., of the aforesaid publication for the date shown in the above caption, required by the Act of August 24, 1912, embodied in section 443, Postal Laws and Regulations, printed on the reverse of this form, to wit:

1. That the names and addresses of the publisher, editor, managing editor, and business managers are:

Publisher, Yale University Press, 143 Elm St., New Haven, Conn.; Editor, Jay Hambidge, Hotel Netherlands, New York City; Managing Editor, None; Business Managers, None.

2. That the owners are: (Give names and addresses of individual owners, or, if a corporation, give its name and the names and addresses of stockholders owning or holding 1 per cent or more of the total amount of stock.) Yale University Press, New Haven, Conn.; George Parmly Day, New Haven, Conn.

3. That the known bondholders, mortgagees, and other security holders owning or holding 1 per cent or more of total amount of bonds, mortgages, or other securities are: (If there are none, so state.) None.

4. That the two paragraphs next above, giving the names of the owners, stockholders, and security holders, if any, contain not only the list of stockholders and security holders as they appear upon the books of the company but also, in cases where the stockholder or security holder appears upon the books of the company as trustee or in any other fiduciary relation, the name of the person or corporation for whom such trustee is acting, is given; also that the said two paragraphs contain statements embracing affiant's full knowledge and belief as to the circumstances and conditions under which stockholders and security holders who do not appear upon the books of the company as trustees, hold stock and securities in a capacity other than that of a bona fide owner; and this affiant has no reason to believe that any other person, association, or corporation has any interest direct or indirect in the said stock, bonds, or other securities than as so stated by him.

5. That the average number of copies of each issue of this publication sold or distributed, through the mails or otherwise, to paid subscribers during the six months preceding the date shown above is (This information is required from daily publications only.)

G. G. WYANT, Secretary.

Sworn to and subscribed before me this 8th day of April, 1920.

L. PHELPS SOULE.
(My commission expires February 1, 1925.)

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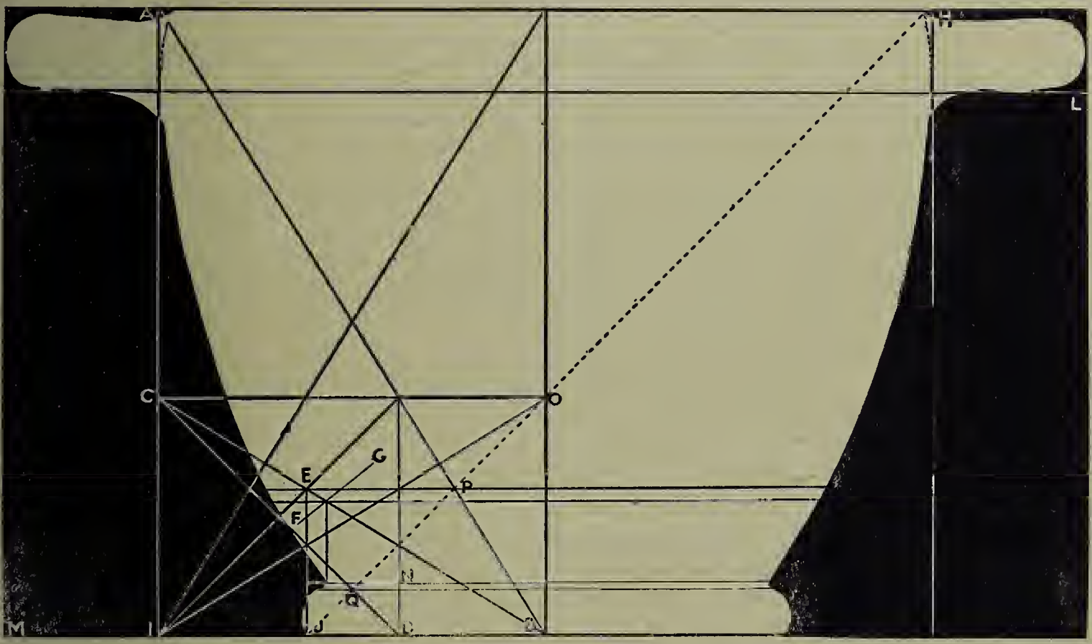
WITHDRAWN

THE DIAGONAL

AN ILLUSTRATED MONTHLY MAGAZINE DEVOTED TO THE EXPLANATION OF THE REDISCOVERED PRINCIPLES OF GREEK DESIGN, THEIR APPEARANCE IN NATURE AND THEIR APPLICATION TO THE NEEDS OF MODERN ART

EDITED BY JAY HAMBIDGE



See article on "An Important Skyphos" in this number of The Diagonal

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47 5th Avenue
New York

May 30, 1920.

My Dear Hambidge:

Your book has arrived. It is well worth the waiting and I congratulate you on the lucidity of both text and plates. It is a message to the art world which I hope and believe will be carried far and wide. It establishes beyond any possibility of successful controversy that dynamic symmetry exists and that the Greeks used it. It further establishes, which must be a source of well-earned satisfaction to yourself, that Hambidge did it. To those of us who are learning to use this incomparable aid to elegance and precision, it is an added but welcome obligation.

Sincerely yours,

ALBERT A. L. SOUTHWICK.

Copy of a letter from the head of the design department of Tiffany's.

THE DIAGONAL

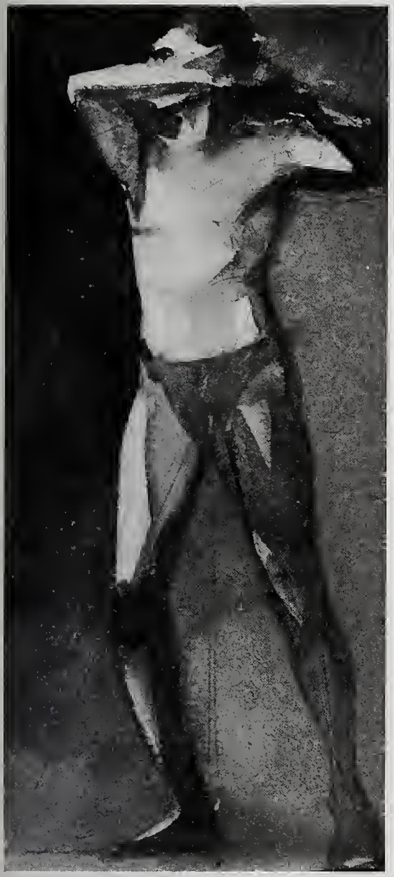

Fig. 1

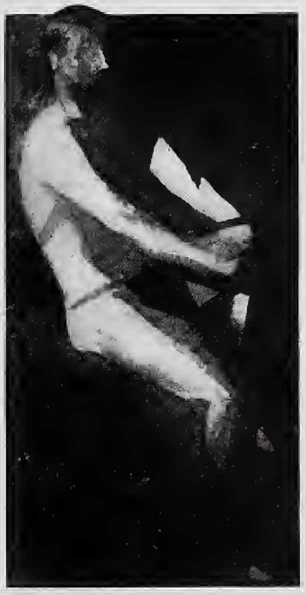

Fig. 2

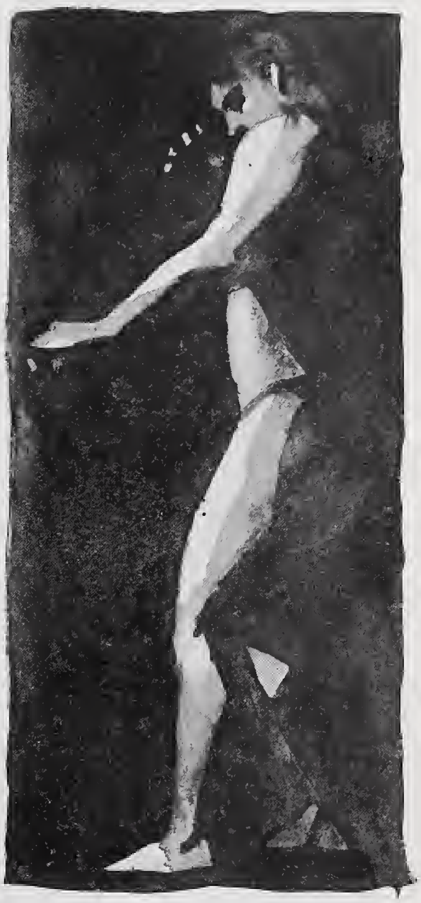

Fig. 3

DYNAMIC SYMMETRY AND LIFE CLASS WORK

UNDER the guidance of Mr. Howard Giles, the American painter, dynamic symmetry was tried out in the life and illustration classes of the New York School of Fine and Applied Art during its past term. Mr. Giles was a member of the group of painters, designers, sculptors and architects who listened to a lengthy course of lectures on this symmetry in New York two years ago.

Taking a large class of students, the majority of whom were unfamiliar with drawing from the figure, Mr. Giles organized and started them to work on entirely new and unacademic lines. The results of this experiment with unskilled figure draughtsmen are somewhat startling as the layout plans accompanying this article testify.

We are all familiar with the usual academic method of teaching drawing from the human figure. The model and the throne, the conventional posing and meaningless actions,

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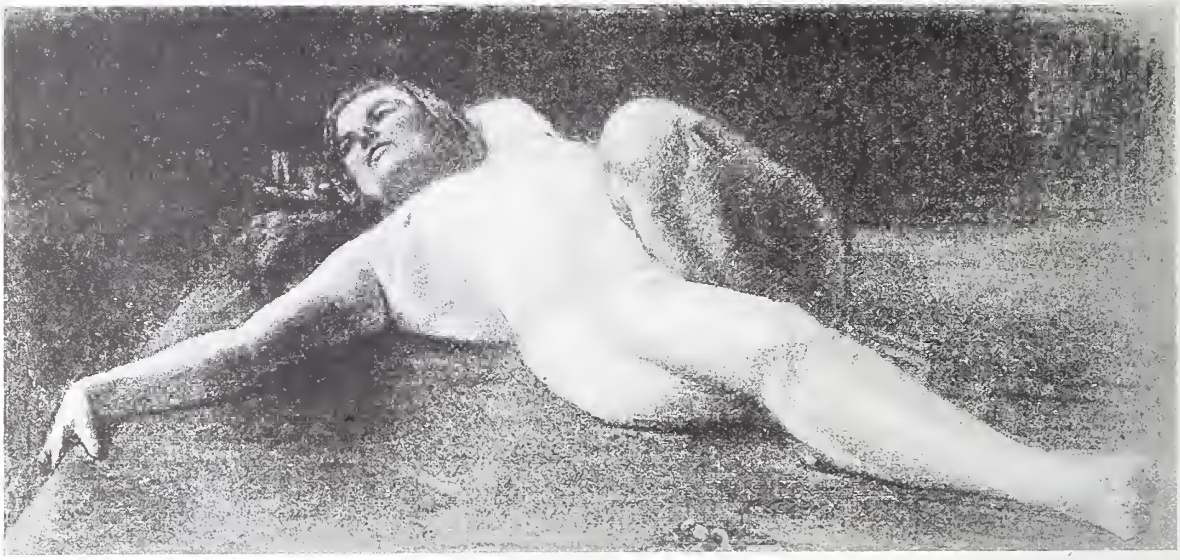


Fig. 4. Scheme in $\sqrt{5}$ rectangle. Diagonals made by leg and arm are diagonals of the overlapping whirling square rectangles. The body, including the left arm and left leg, is an arrangement within the $\sqrt{5}$ square.

the smooth surfaces produced by crayon sauce and stomp, the uniform charcoal paper size, the lack of construction and photographic rendering of surfaces; all characteristic conventions of modern school figure study which produce the rubber stamp quality which

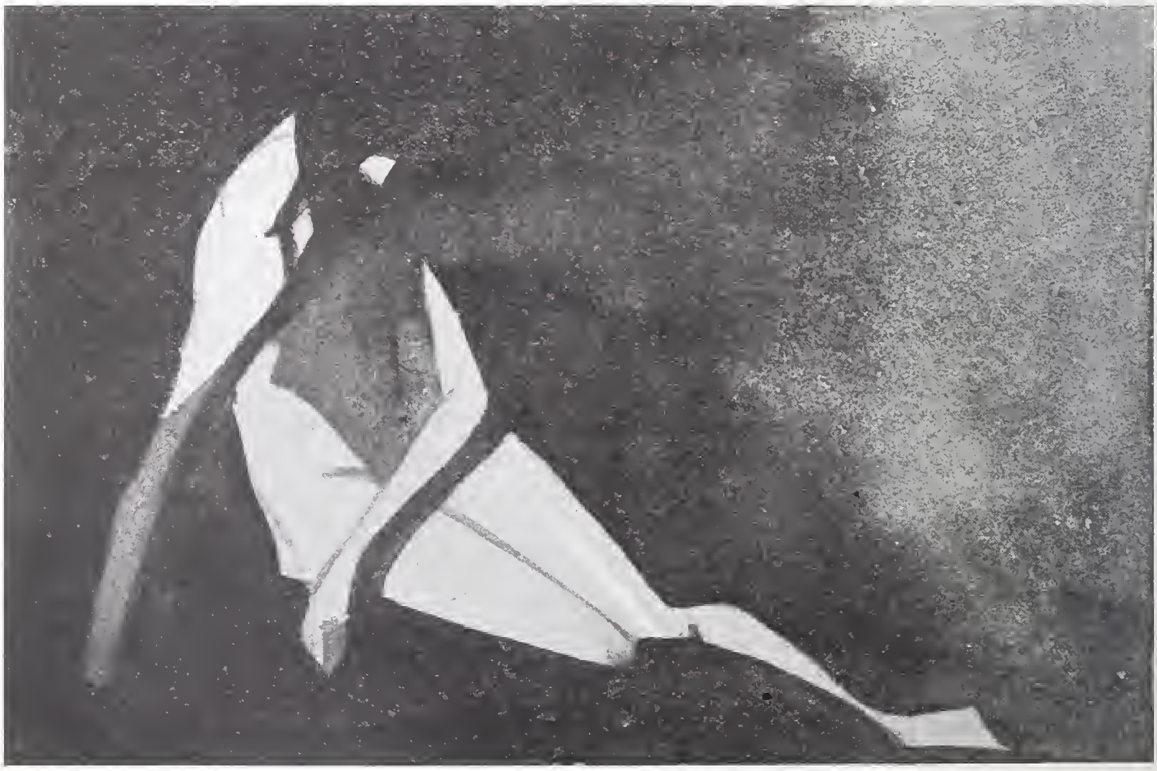


Fig. 5. A suggestive beginning.

has come to be accepted as the hall-mark of classroom work. Mr. Giles has banished all this and has begun the blazing of a new trail directed toward the goal of real artistic achievement.

Mr. Giles began by first explaining to his class something of the nature of the pattern of the human figure itself and then, by a simple practical illustration, made clear the fact that slavish copying of the superficial aspects of the model would utterly defeat any attempt to put force and power into composition. The members of the class were first asked to make a drawing of a figure in action—actually a man striking a blow. This was done without the use of the model. Then a similar drawing was made by using the model. The two best drawings, one by each method, were then selected and pinned up. The members of the class were told to imagine that they had to submit to being struck by one of the two figures but that they were privileged to select the one supposed to do the striking. The class was unanimous in selecting the figure which was drawn from the model. The figure drawn from imagination seemed too competent. This object lesson made it clear to each student that the imaginative element in a picture was most important. After this the class was shown how to make three simple rectangles by construction,—the arithmetical part being neglected. These rectangles were used as bases for plan arrangements for compositions. The students were now ready for an inspection of the model, but instead of beginning their work in the old manner by plumb line, sight measurement and feeling their

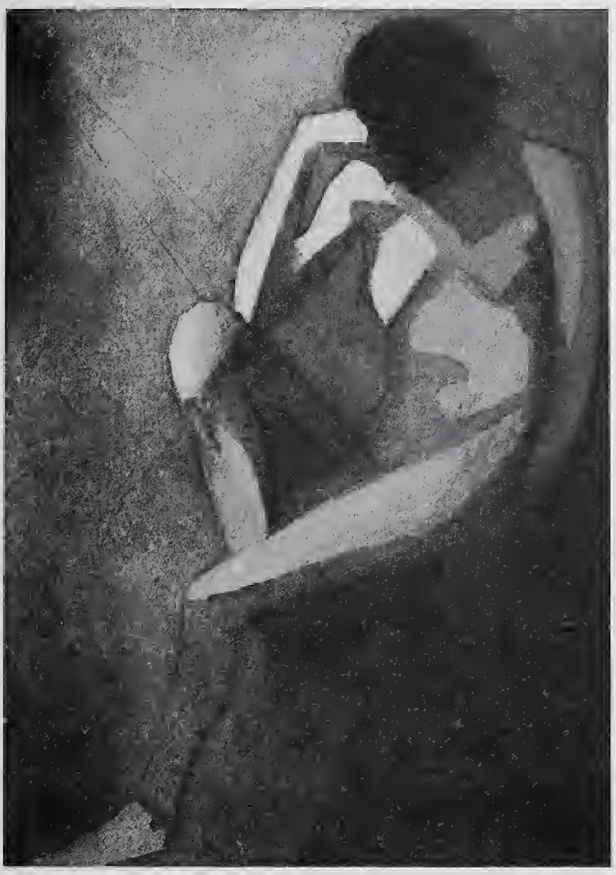


Fig. 6. Action suggested by dynamic sub-divisions.

way gingerly with the charcoal point on the white paper, each set about making a plan on a small scale, using for this purpose one of the simple constructions already learned. While this plan-making process was going on the model was neglected. These plans were made on a scale of about nine inches for the length and the medium used was, generally, a lamp-black wash, though some of the students preferred water-color. After the students had completed their plans they made more careful drawings on a larger scale, usually regular charcoal paper size.

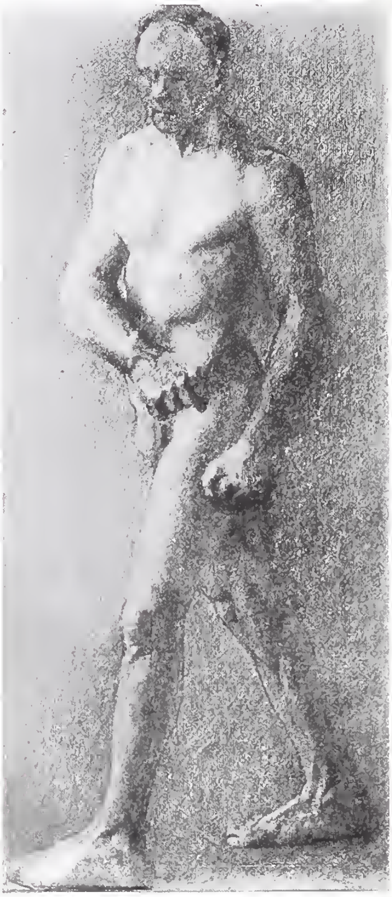


Fig. 7. Virile study by a 4 months' student.

The interesting point is that no attempt was made to use the model until the student had a definite idea of the compositional arrangement he wished to make. The preliminary plan furnished a clear and actual picture of the result he was striving for. Mr. Giles says the students were fascinated with the process, absorbed with their work and that the results were the opposite of mechanical. The plans themselves make this clear. The process seemed to encourage individuality. Whether using color or black and white the students appeared to be quite unconscious of the technical difficulties attending the employment of these mediums; they had clear conceptions of what they wanted and adopted naturally the simplest means to attain their purposes.

The symmetry of the human figure is dynamic, therefore the selecting of a dynamic rectangle for the purpose of correlating the units of a figure composition is most appro-

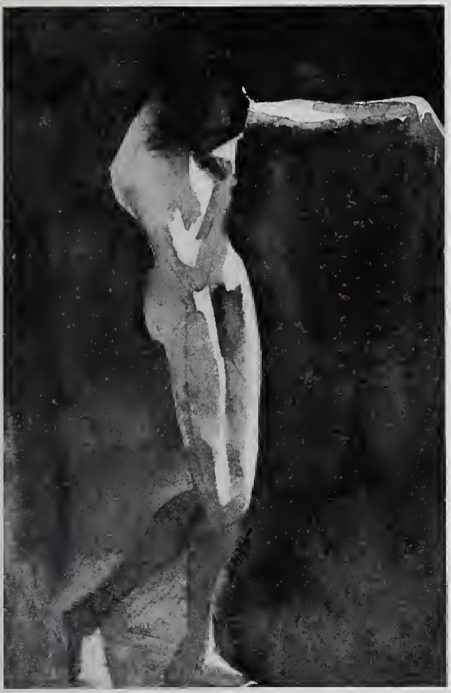


Fig. 8

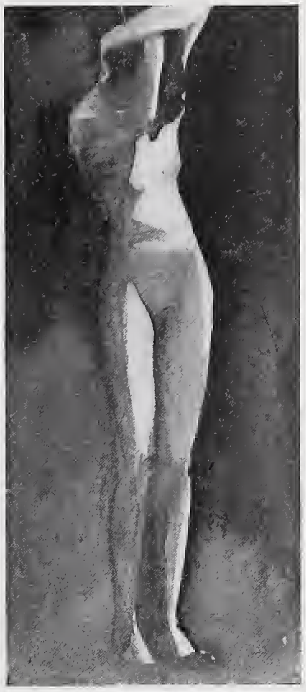


Fig. 9

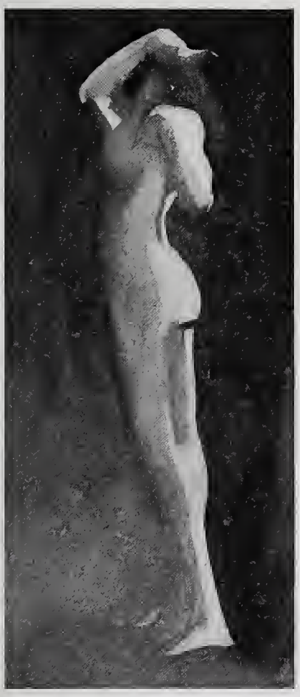


Fig. 10

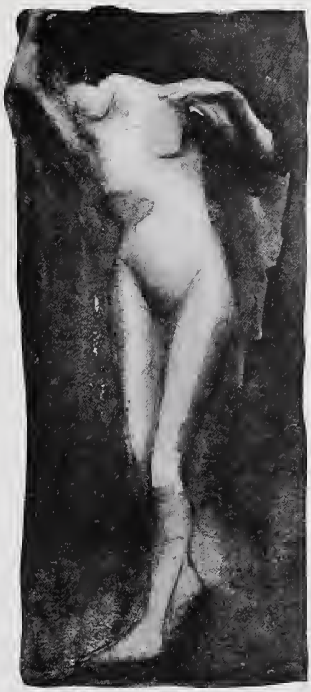


Fig. 11

Group of Plan—Sketches showing different student conceptions.

priate. It will be noticed that the actions in these sketches are most natural and logical. These are determined by the diagonals and proportional subdivisions of the rectangles themselves. These offer an infinite variety of movement suggestions which transcend invention. Dynamic symmetry also is an ideal enlarging and reducing scheme, being much superior to the old system of squaring. When a student has finished his preliminary sketch he may enlarge or reduce it to any proportion with the utmost ease. Mr. Giles says that practically no time is lost in this process.

One very interesting feature of these plan-sketches is that they do not look like the work of beginners. They have a professional, a painter-like quality which is found only in the work of like character done by artists after they have had a long studentship training. The reason for this lies in the fact that most of the difficulties which beset the beginner, and which are usually overcome only by long practice, are, by dynamic symmetry or the dynamic method of approach, solved more or less automatically. The obtaining of correct proportions in figure construction is probably the most embarrassing element which confronts the novice, but by using proper rectangles and understanding something of their modulating subdivisions much of this difficulty is solved at once and the student is able to advance his work on any scale with assurance. Feeling the reliability of this sure foundation instead of becoming more and more confused as he proceeds the student gains more and more confidence and self-possession. This not only enables him to express himself freely but in a large measure develops his individuality. The sketches themselves furnish the best evidence as to this. No two of them are alike; the individual equation in each is unmistakable. Moreover the confidence and feeling of power which the method engenders seems to assist materially in giving the student color control. Experienced artists know how difficult it is to express the nude human figure in water-color terms. Success in this not only demands great technical skill in the use of the medium but profound knowledge of color. Yet these beginners, apparently oblivious of these difficulties, use water-color with freedom, ease and success.

An interesting aspect of this experiment is that the students have been aroused mentally and have begun to think of many other things besides the technique of art expression. Dynamic symmetry compels thought. The art student or indeed the artist himself, as a rule, is not given to ponder much over matters which are not connected directly with his work. But the mere act of making the simple constructions necessary to start a composition in a dynamic way forces a certain degree of mental application. No one can go far in following out the ramifications of modulating dynamic form without being struck with the logic of the process. Like a brilliant demonstration in science the interweaving of the vital shapes of this symmetry disclose invariably a play of reason and inevitableness captivating in the extreme. Lincoln studied Euclid that he might apply reason to his law practice. Dynamic symmetry discloses veins of rationality of the purest character which are applicable in some degree to most human effort. Familiarity has had its effect. The students who have taken this step in a new and what seems a saner and healthier direction than that usually followed, have already given evidence of an awakened interest in phases of culture seemingly not connected with art training. Perhaps through art training we may revitalize, humanize, classic study. Perhaps this is the beginning of a new type of education which will seek to cultivate the mind by the study of beauty. At any rate when this school term closed these students, themselves taking the initiative, asked to have a course of lectures on logic.

(In the next number of THE DIAGONAL Mr. Giles will explain his methods in an illustrated article.)

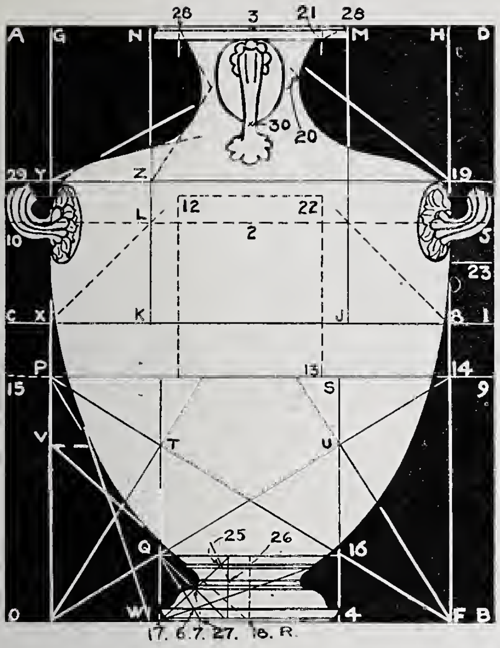


Fig. 1*a*

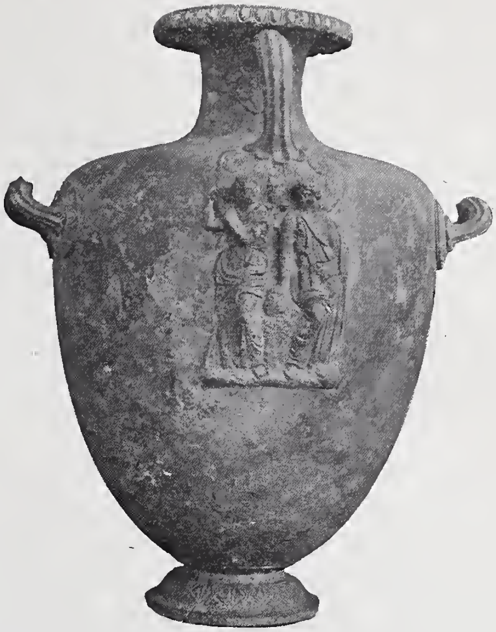


Fig. 1*b*

Bronze Hydria No. 312, in the British Museum.

A BRONZE HYDRIA IN THE BRITISH MUSEUM

BRONZE HYDRIA No. 312, in the British Museum, gives us an extraordinarily interesting combination of proportion. The overall ratio is 1.236 and the vase without the handles is 1.528. The catalogue description of the example reads:

"Round the lip egg-moulding; round the foot, Lesbian *cymatium*. Fluted handles, those at the sides ending in rosettes with lotos flowers between the points, while the handle at the back ends above in a shield-shaped plate with foliated patterns and rosettes in relief. Below is a large plate with two rosettes and leaves above and a relief of Dionysos and Ariadne. Dionysos to the front, beardless, with long curls, wreath, fawnskin over right shoulder, himation over lower limbs and round the left arm, and shoes; his right hand rests on a rock; he turns to left (?) towards Ariadne, who looks round at him. She wears an ivy wreath, bracelets, long girt chiton with *apoptygme* himation and sandals. Her hands are raised over her right shoulder to fasten up her hair behind; it is parted and rolled in front. Behind her is a *thyrsos*. Height of relief, 18½ inches; width 5¾ inches. From Chalke near Rhodes, 1885." (These measurements are approximate only.)

HOW TO DRAW THE PROPORTIONS AND CURVES OF THE HYDRIA. ACTUAL SIZE

Measure, with a metric scale, a 1.236 rectangle, .4672 m. high and .3780 m. wide. If we consider the width of this rectangle as unity the height will be 1.236. If the height is unity or 1, the width will be .809 or the reciprocal of 1.236. Divide 1.236 into 1 to obtain this, or .3780 m. into .4672 m.

Establish a reciprocal of a 1.236 rectangle by construction. This is done by drawing a diagonal to the overall shape and a line from one corner of the rectangle cutting this diagonal at right angles. (See Lessons on Dynamic Symmetry for methods.)

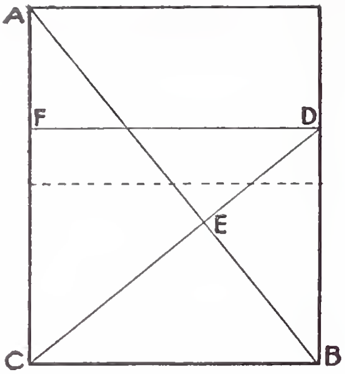


Fig. 2*a*

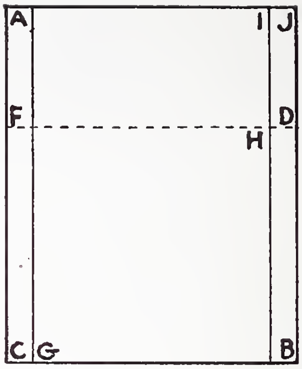


Fig. 2*b*

AB, Fig. 2*a,* is a 1.236 rectangle. CD is a reciprocal. The two lines cut each other at right angles at E.

If CB is 1, AC is 1.236 and FC is .809. The area CD is, like the area AB, composed of two whirling rectangles.

Within the reciprocal area CD, Fig. 2*b,* establish the square GH. The ratio of the sides of this square to the end of the containing rectangle, that is, the line CB of Fig. 2*a,* is .809 or the line CF.

Produce the sides of this square to the line AJ to establish the area GI. This area is the containing rectangle of the hydria minus the handles.

The end to side relationship of this rectangle is .809 to 1.236. Dividing the lesser into the greater we obtain the ratio 1.5278—1.528 for short; dividing the greater into the lesser gives us the reciprocal .6545. If the end of this new rectangle is 1, the side is 1.528; if the side is 1, the end is .6545.

The rectangle mentioned above is the area which gives us the detail proportions of the hydria.

So far we have: (1) a 1.236 rectangle, AB, Fig. 1*b*. This is composed of the two .618 rectangles CB and CD. (2) Within the 1.236 area we have established the 1.528 or .6545 area FG. This area is cut into halves by the line CI and we have the areas XH and XF. These two areas are 1.309 rectangles for this reason:

If HF is unity, the line HG equals .6545. This fraction divided into 1 or unity gives us 1.528. But H8 equals 8F; that is the line HF is bisected at the point 8; or if HF equals 1, H8 and 8F each equal .5 or half of unity.

Divide .5 into .6545 to obtain 1.309. Each of the areas HX and XF is a 1.309 rectangle. We should also obtain this result by multiplying .6545 by 2.

A 1.309 rectangle is composed of a square, as XM, plus a double whirling square rectangle, as HJ. The square XM is applied to the area XH and the area HJ is the excess.

Apply a square to the other end of the rectangle, as N8 and the excess area is XN, equal to HJ.

The area NJ or KM is composed of the square LM plus the root-five area LJ, or, the line KJ in relation to KN is .691 to 1. It is .691 because XG equals 1, or the side of a square, X8 equals 1.309; XK .309; J8 .309 and KJ .691.

But the fraction .691 divided into 1 equals 1.4472, *i.e.,* a square, 1, and .4472. This fraction is the reciprocal of 2.236 or the square root of five.

NM, a side of the square ML, is equal to the width of the lip.

EF is a square, see Fig. *2b,* and EH is a .528 rectangle. Dividing this fraction into 1 we find it is the reciprocal of 1.8944; that is, the area EH is composed of the square ML plus the two root-five rectangles EN and N11. The square is 1, and .8944 is equal to .4472 multiplied by 2.

Apply to the whirling square rectangle CB the area of two squares, as O9. The defining line of these two squares, *i.e.,* the line 9 to 15, cuts the sides of the .6545 rectangle at P and 14.

The area FP is a whirling square rectangle. Draw its diagonals and diagonals to its reciprocals. Draw lines through the eyes or intersections of the diagonals at T, U, Q, and 16.

The area S17 is a 1.382 rectangle. SQ is a square and Q4 is a .382 shape. This fraction divided into unity gives us 2.618, or the square Q18 plus the whirling square area 18 and 16, *i.e.,* 1 plus 1.618.

PROPORTIONS OF THE TOP

The width of the lip is the side of the square ML.

Draw a line from the middle of the lip to 5. The line 10 and 5 is the side of the reciprocal to the entire shape. The line 3 and 5 cuts HF at 19 to establish Y19 and fix the height of the handles. Draw a line from M to Y, from G to B and from H to O. These lines cut diagonals of the squares XM and N8 to fix the 1.309 area 12 and 13, which is the containing rectangle for the composition of Dionysos and Ariadne. The base of this area is fixed by the top of the whirling square rectangle PF. Produce the line 13 and 22 to 21 to fix the height of the lip.

A line from 3 to 23 is the diagonal of a square. It cuts a diagonal of the square XM at 20 to fix the width of the neck.

PROPORTIONS OF THE BASE

The details of the foot are fixed by the .382 rectangle Q4. The area Q18 is a square and 7 is its centre. Draw TR, a diagonal to two squares. This line fixes the points 25 and 26. The line Q27 is a diagonal to a whirling square shape in the square Q18. The line 17 and 16 is a diagonal to the entire .382 area. The point 6 is obvious.

THE CURVE AND ITS PROPORTIONS

The area VR is equal to half of the root-five rectangle VF. The point W is found by continuing the line NK. Draw WX. The line YM has been drawn. Draw the line Z3. The points 28 and 28 are equal to 12 and 22. Draw a line from 28 to F (two or three inches only are necessary). The point 30 is the intersection of diagonals of the two squares N8 and XM. Draw the line 29 and 30.

The lines RV, WX, XY, YM, 29 and 30, Z3 and 28F are all tangents to the curves of one side of the hydria. The curves are drawn free-hand against these tangents.

DYNAMIC SYMMETRY OF THE HUMAN FIGURE FOR ADVANCED STUDENTS

THE ratio 1.08541 is a compound form and has its origin in the .10782 part of a whirling square rectangle, as shown by the diagram in the April DIAGONAL. This example is interesting because we are able to trace the skeleton's proportions and practically all its details to a simple geometric base. If we scale this rectangle the nature of the 1.08541 area becomes apparent. It is equal to a square plus the area represented by the fraction as in Fig. 1.

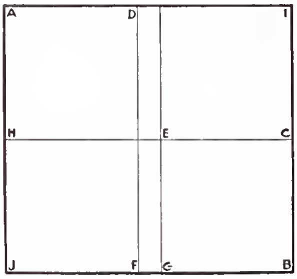


Fig. 1

AB is the overall rectangle. DG is the .08541 area.

AF and GI are each equal to two squares.

If we bisect a unity side as AJ and draw HC the area is divided into two parts, as AC, CJ.

HD is a square, as is also EI.

DE is equal to .08541 multiplied by 2, or .17082.

The area AC is equal to 1.08541 multiplied by 2, or 2.17082.

If we apply a whirling square rectangle to the area 2.17082, *i.e,* 1.618, the remainder is .5528 and this fraction is the reciprocal of 1.809.

For the area AC, therefore, we have the compound form of Fig. 2; a 1.618 rectangle plus a 1.809 rectangle.

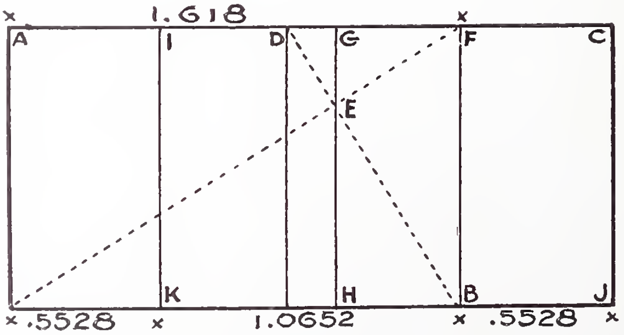


Fig. 2

AB is the whirling square rectangle and BC the .5528 or 1.809 area.

BD is a .618 shape and E is an eye.

DF equals .618 (measure this by scale).

GF equals .4472 and DG .17082 or .618 minus .4472.

But HC is a square.

If to the line HG we add another .618 area, as GK, then IJ becomes a 1.618 area and IB is a 1.0652 area. .5528 multiplied by 2 equals 1.1056 and this subtracted from 2.17082, or the area AJ, equals the area IB, *i.e.,* 1.0652.

1.0652 is composed of .618 plus .4472, *i.e.,* KG plus GB.

In Fig. 3 this 1.0652 area is more fully considered.

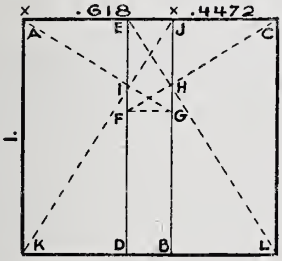


Fig. 3*a*

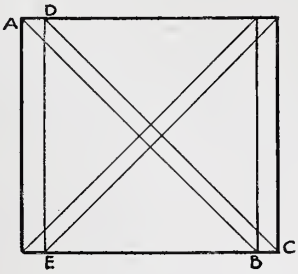


Fig. 3*b*

AB of *a* equals .618, BC .4472.

CD equals .618, AD .4472.

BE equals .17082.

AG, CF, JK, EL are diagonals and perpendiculars.

H and I are eyes. EJ is the width of the skull.

EB, *i.e.,* .17082, is the reciprocal of 5.8541, or is five squares plus the .8541 part of a square.

In Fig. 3*b* the 1.0652 area is exhausted by two applied squares, AB and CD, overlapping to the extent of DB or .9348.

AD equals .0652, as does also BC.

CE equals 1, and subtracting .0652 from this we have .9348 for BD.

These two overlapping squares seem to be important factors in the proportion of the shoulder width and thorax, as is apparent in Fig. 4.

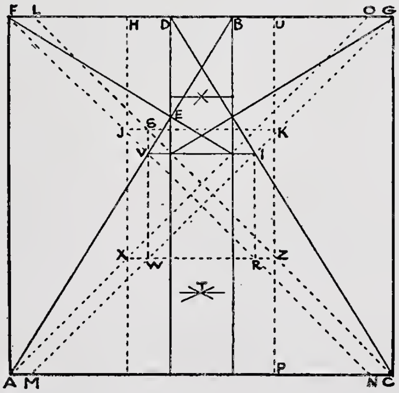


Fig. 4

AB, CD are the two overlapping .618 areas.

OA, MG, CL, NF are the diagonals of the two overlapping squares.

NF cuts AB at V.

Draw SW through V. It will be noticed that S and W lie on MG and CL.

Through W and R draw ZX; then XJ and ZK.

JK is the shoulder width.

WR is the thorax width, also the thorax base.

Draw lines from Z to M and from X to N. These lines cut each other at T, the top of the pelvis.

Complete the area HP.

This is a 2.412 area or half of the area formed by the shoulder width and full height of the figure. (See April Diagonal.) This ratio is 4.8240.

It will not be necessary to work out the many ramifications of form as it is disclosed by this skeleton example. It is but one of the countless themes of dynamic symmetry. Its principal interest lies in the fact of its curious consistency. The shape itself or any or all of its subdivisions may be used for any design purpose.

CONSTRUCTIONS FOR THE FRONT AND SIDE ELEVATIONS OF THE MOUTH

The student should remember that Dynamic Symmetry provides two methods of approach for the study of the human figure. By one we use the modulating forms of the symmetry to obtain general proportions and general lines of action or movement, the student using the principles very much like those of perspective as it is drawn free-hand. We know that an artist ignorant of perspective constantly violates its rules, but, being aware of its principles he seldom makes a mistake, often, in fact, if well grounded in the science, rendering the most difficult problems with ease and charm. For accurate perspective rendering the principles are rigidly followed. The former is the way the artist should use perspective, the latter the process the draughtsman and architect should employ. The former results in artistic charm, the latter in scientific precision. As is the case with perspective the artist cannot prevent himself from securing some symmetry balance in his work if he is aware that its principles exist.

It will be apparent to the student that we must first consider the precise symmetry of the figure. After symmetry precision is understood it may be applied with such degree of freedom as the artist may desire. Dynamic Symmetry is of great advantage to the artist as an enlarging or reducing scheme. Because of this we may take any part of the figure and draw it to a scale sufficiently large to enable us to inspect it in detail with the utmost ease once a construction is understood. Figs. 5*a* and 5*b* are front and side elevations of an actual mouth.

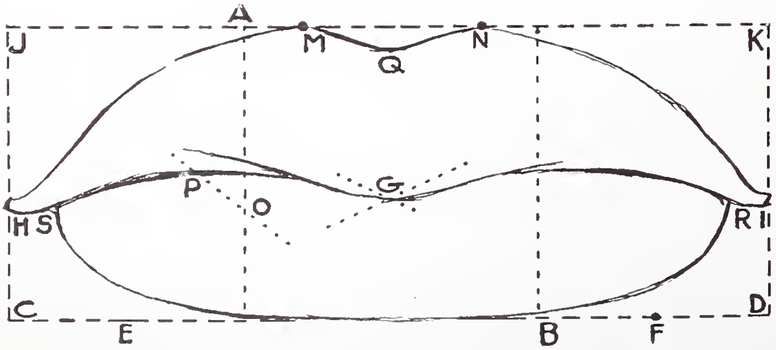


Fig. 5*a*

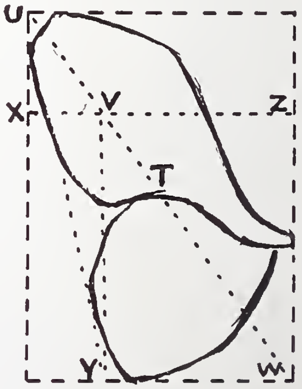


Fig. 5*b*

The rectangle of the front elevation is a 2.618 shape and AB is a square in the centre of this area. AC and BK are .809 areas; added they equal 1.618.

The point O is an intersection of a diagonal to half the overall area with a side of the centre square. This point fixes the line HI. This determines the droop of the upper lip.

Bisect the line BD at F and draw JF.

Draw a line from E to K. These two lines cut each other at G.

NK and JM are each equal to a side of the centre square.

MN, therefore, is the end of a whirling square rectangle in the centre of the square AB.

Draw lines from M to I and from N to H. They intersect at Q.

The actual length of the line JK is .0455 m.

SR measures .04045 m.

All curves may be drawn by tangents proportional to the overall shape.

Consideration of these is left for a future lesson.

The side elevation is a 1.382 rectangle.

WX is an applied square and a diagonal to the whole cuts XZ at V.

Draw VY perpendicular to XZ, and then a line from Y to U.

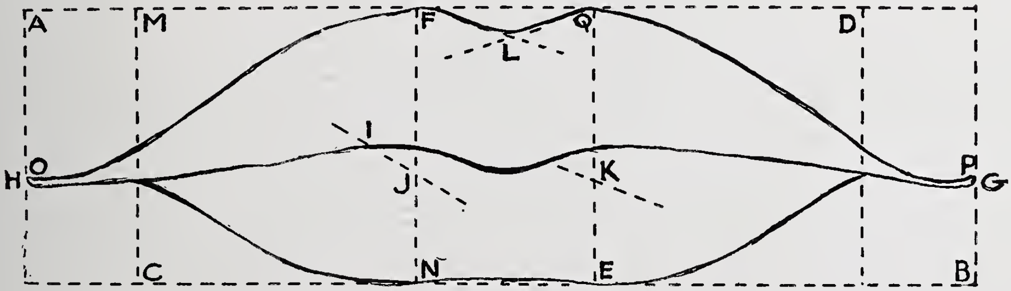


Fig. 6

Fig. 6 is a front elevation of another actual mouth of an entirely different character. Both examples are feminine.

The overall rectangle AB has the ratio 3.427.

CD is a 2.618 rectangle.

ME and DN are each whirling square rectangles overlapping to the extent of the whirling square rectangle FE.

Draw a line from E to M. It cuts FN at J. This fixes the line OP.

The point K is determined by a diagonal to the 2.618 area cutting a side of the square ED at K. This places the line HG.

Lines from Q to H and from F to G fix the point L.

The areas AC and DB represent the difference between 2.618 and 3.427, or .809. This fraction divided by 2 equals .4045, the ratio for the two small areas in excess of the 2.618 area.

[To be continued.]

THE ELEMENTS OF DYNAMIC SYMMETRY

Much of the work upon which the Elements of Dynamic Symmetry is based was done during the year 1919 under the Sachs Research Fellowship of Harvard University. This fellowship was founded by Samuel Sachs of New York City and, under the terms of the foundation, is assigned to scholars of proved ability, whether students, instructors or others, for the general purpose of advancing the Fine Arts. This acknowledgment is but a faint expression of the writer's appreciation of the benefits enjoyed from a fellowship of a type too rare in American Universities. Jay Hambidge.

FURTHER ANALYSES OF THE SQUARE

APPLY to a square a whirling square rectangle, Fig. 1.

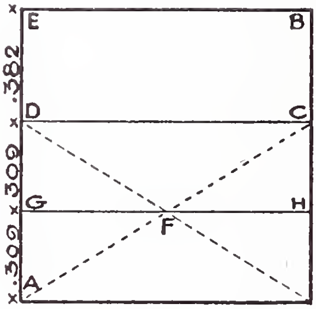


Fig. 1

AB is the square and AC the whirling square rectangle. CE is the excess area, F is the centre of the whirling square area.

Draw the line GH, parallel to the base of the square, and through the point F.

DE is equal to .382, GD to .309 and AG to .309.

GE is equal to .691, or .309 plus .382.

The area GB is a .691 or a 1.4472 rectangle within a square. The reciprocal of 1.4472 is .691.

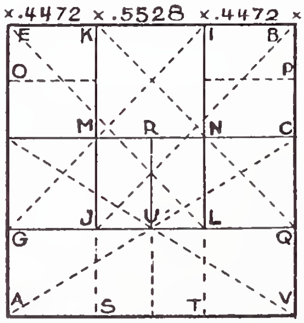


Fig. 2

In Fig. 2 GB is a .691 rectangle applied to the square AB. Apply a square on either end of this shape, as GI, BJ. They overlap to the extent of the area IJ.

If GE be considered as unity, EK is .4472, BI is .4472 and KI is .5528. EJ, BL are root-five rectangles and KL is a 1.809 rectangle. .5528 is the reciprocal of 1.809 and .809 is equal to 1.618 divided by 2. Determine the square in the 1.809 rectangle. It is IM. The area ML is composed of two whirling square rectangles. EM and NB are each double whirling square rectangles.

GN, QM are each root-five rectangles.

EN is a 1.809 area, as is also BM.

ES, BT are each double whirling square rectangles or each is equal to a root-five rectangle plus a square.

SL is equal to JN and MT or NS is a whirling square area.

SQ, GT are each root-five rectangles.

AM and NV are each composed of two squares.

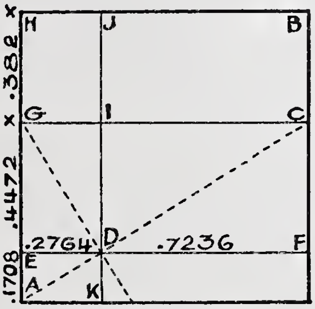


Fig. 3

Apply to a square a whirling square rectangle, as AC in AB, Fig. 3, and through an intersection of a diagonal of a reciprocal and a diagonal of a whole, as D, draw the line EF. The area EC will be a root-five rectangle. HG will be .382, GE .4472 and EA .17082. EG and GH added equal .8292 (more correctly .82918). .8292 is the reciprocal of 1.206.

ED measures .2764 and this multiplied by 3 equals .8292.

The area DH, therefore, is composed of three squares.

DF measures .7236; FB .8292.

DB is a 1.146 rectangle, or .382 multiplied by 3.

HI measures .2764 by .382. The lesser into the greater supplies a 1.382 rectangle.

AJ, .2764 by 1, is a 3.618 rectangle.

AF, .17082 by 1, is a 5.8541 shape.

KF, .17082 by .7236, is a 4.236 rectangle, or a root-five rectangle plus two squares. Its reciprocal is .236.

KB, .7236 by 1, is a 1.382 shape.

KC, .618 by .7236, is a 1.17082 shape and its reciprocal is .8541.

IB, .382 by .7236, is a 1.8944 area, or a square plus two root-five areas.

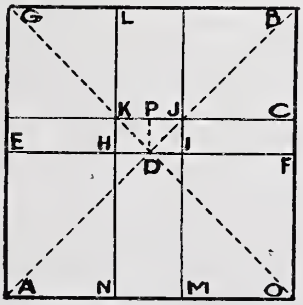


Fig. 4

Apply the whirling square rectangle AC to the square AB, Fig. 4. Find the centre of the square and draw the line EF, cutting the square into two parts. DB, DG are each squares.

In the whirling square rectangle AC the applied squares are OK and AJ. The area LM is a 4.236 area and its reciprocal is .236. NM measures .236.

EP, PF are each 4.236 areas. KD, JD are squares and EK, IC are each double whirling square rectangles.

GH and IB are each 1.309 areas, composed of the squares GK, JB and the double whirling square rectangles EK, JF.

GN, BM and CG are .382 or 2.618 areas. Other areas may be figured easily by the student. It is desirous that all the subdivisional areas be drawn to a fairly large scale and inspected carefully, as they all appear frequently in Greek design and in nature.

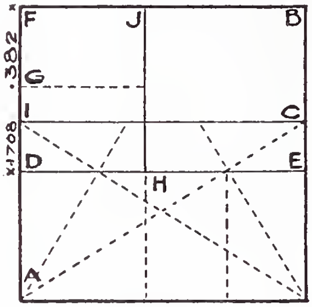


Fig. 5

To the square AB apply the whirling square rectangle AC and through the intersection of its diagonals with diagonals of its reciprocals draw the line DE.

FI measures .382 and DI .17082. Adding these we have .5528 for the line DF. As mentioned above, .5528 is the reciprocal of 1.809. This area is FE and EJ is its square. JD equals two whirling square areas, JG and GH. AE is a root-five rectangle.

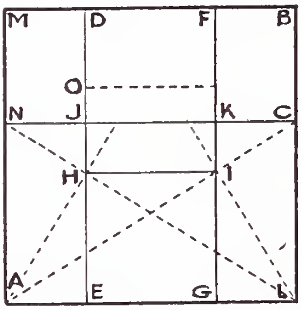


Fig. 6

In the square AB apply the whirling square rectangle AC and through the intersection of its diagonals with diagonals of its reciprocals, as HI, draw the lines DE and FG.

The area DG or EF is a root-five rectangle and GH is its square. HF is composed of the two whirling square areas IO and OF.

EG measures .4472 and AE and GL each .2764.

AD and FL are each 3.618 areas.

AJ, KL are root-five rectangles and MJ, BK are each 1.382 areas.

BJ and MK are each 1.8944 areas, *i.e.*, are composed of a square plus two root-five shapes.

MH, BI are each composed of two squares.

EC, GN are each 1.17082 areas.

[To be continued.]

Dear Mr. Hambidge:

I am sending you a little thing that occurred to me as a simple method of constructing a root-five and a whirling square rectangle in a square. Please tell me if it is of any use.

Faithfully yours,

Wm. Sergeant Kendall.

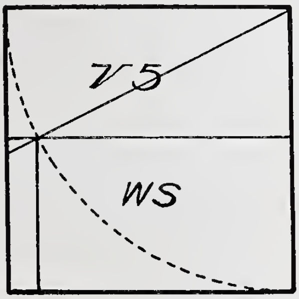


Fig. 1

THE construction is really a very simple one for the fixing of a whirling square rectangle within a square. There are also many other interesting arrangements in it. The general idea involved in the method will be clear if we reduce the construction to its base.

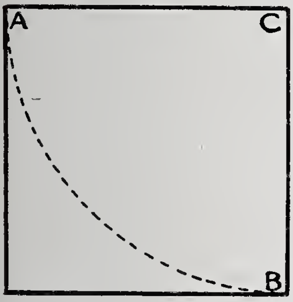


Fig. 2*a*

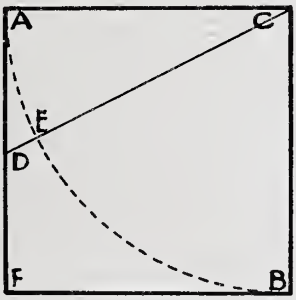


Fig. 2*b*

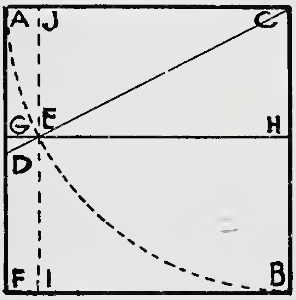


Fig. 2*c*

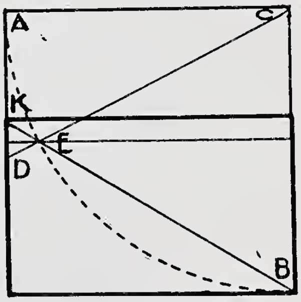


Fig. 2*d*

AB of Fig. 2*a* is a quadrant arc of a circle in a square and AC, a side of that square, is used as a radius. In Fig. 2*b* the side of the square AF is bisected at D and DC, a diagonal to two squares, is drawn. DC cuts the quadrant arc AB at E. In Fig. 2*c* the line GH is drawn through E to define the root-five rectangle GC or AH.

Through E draw IJ. EB is a whirling square rectangle and JH a root-four rectangle or two squares.

A whirling square rectangle plus two squares is equal to a root-five rectangle divided by 2, *i.e.,* 1.118. The square root of five is 2.236.

Two squares in this position are expressed numerically as .5. Adding .5 to .618 we get 1.118. JB, therefore, is composed of two root-five rectangles. .618 plus 1 equals 1.618. Subtracting .5 from this, or one-half of one, the result is 1.118; but 1.618 plus .618 equals 2.236, therefore .5 plus .618 equals 1.118.

If AH is a root-five rectangle, GB is a 1.809 rectangle. (If AF equals 1 then AG equals .4472 and GF is the difference between .4472 and 1, or .5528. This fraction is the reciprocal of 1.809.) But EB is a 1.618 area. The difference between 1.809 and 1.618 is .191, or the rectangle GI. The fraction .191 is the reciprocal of 5.236, or root-five, *i.e.,* 2.236 plus three squares.

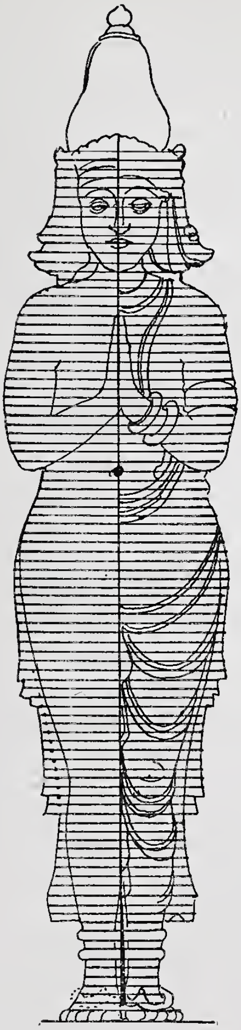
If AH is a root-five rectangle, JH is a root-four rectangle and the area AE is the difference between the two areas, *i.e.,* the difference between 2 and 2.236, or .236; but .236 is the reciprocal of 4.235, or root-five plus two squares.

GI equals 5.236 and AE 4.236. Added they equal 9.472.

If IB equals 1 or unity, BC equals 1.118. If BC be considered as unity, then BI equals the reciprocal of 1.118 or .8944 and FI is the difference between .8944 and 1, or .1056. This fraction is the reciprocal of 9.472.

In Fig. 2*d* the line BK is drawn through the point E. BK is a whirling square rectangle and KC is a .382 rectangle or the difference between .618 and 1.

This is indeed a simple method of constructing a whirling square rectangle within a square.



Tracing of a Hindu Figure by C. Purdon Clarke.

The proportions of this figure are taken from a photograph of one in Trimul Naik's Portico at Madura and are only approximately correct. Two arms holding a mace and a discus have been omitted. A centre line equal to the height of the required figure is divided into 96 parts. Then, starting from the base, with one part as the unit of measurement, offsets are dotted on either side through which lines are afterwards drawn to complete the figure. Ancient systems in use at Madura in 1882.—"Ars Quatuor Coronatorum," Plate 5.

A HINDU METHOD OF LAYING OUT A STATUE

IN a paper on "The Tracing Board in Modern Oriental and Medieval Operative Freemasonry," read before an English Masonic Lodge in 1893, C. Purdon Clarke describes a Hindu method of laying out a figure. The writer of this paper is well known in America because of his connection with the Metropolitan Museum of Art in New York City.

Transactions of the Lodge Quatuor Coronati.

"As a reliable account of the Architect Caste can be found in the well-known essay by Ram Raz', I shall condense their history into the statement that they, conjointly with the surveyors, carpenters and joiners, claim descent from Viswacarma, the Heavenly Architect, and to them belong some thirty-two, or, as some say, sixty-four books of the *Shastras* of which they, not the priests, are the custodians, and there is sufficient evidence to infer that in former times they were highly privileged and honoured. Their present position, however, is well expressed by Ram Raz' in the following extract from his preface: 'This class, perhaps, jealous of the Brahmans, whose sacerdotal authority they have always opposed with a spirit of independence, or, more naturally, apprehensive of competition in their trade, took particular care to conceal the sacred volumes, which have descended to them, from the rest of the people; but as they have on their own part been long denied the benefit of Sanscrit literature their treatises could be of little use to themselves; and the consequence has been, that while the practical part of the science continued to be followed up among them as a kind of inheritance from generation to generation, the theory became gradually lost to the whole nation, if not to the whole world. Even the few scattered fragments, that have escaped the hand which either jealousy or the fear of competition has raised to conceal or rather destroy the science, are now quite unavailable to themselves or to the priests, the former being compelled to refer to the latter for the interpretation of the superior dialect, and the latter to seek from the former the definition of technical terms, which neither one nor the other seem to have been able to explain or understand accurately.'

"Ram Raz' wrote the above in 1828, after some years' study of the subject, but, unfortunately for him, in provinces subjected to Mohammedan rule, where the Hindu architect had for generations been stultified by neglect and want of opportunity of practicing his craft. In 1881 I was more fortunate, and at the great temple at Madura found the 'Slhapati,' or architect, well able to read as well as apply the instructions of the *'Silpa-Shastra,'* relating to temple building and decoration. In my presence he worked out on the tracing board, by a curious system of centre stem and offsets, a design for a full-sized figure of one of their deities. The *'shastra,'* which was read aloud to him whilst he worked, by one of the others, seemed but a meaningless string of figures resembling a table of logarithms, but these, when marked down on offset lines on both sides of his centre stem, soon produced Vishnu with his flute in the usual characteristic attitude of standing on one leg. I was not permitted to take a copy of this figure, but noting that the whole height was divided into 96 parts and these parts used for units for horizontal measurements I applied this scale to an outline traced from a photograph of one of the figures in the temple at Madura with the result shown in Plate V."—"The Tracing Board in Modern and Medieval Operative Masonry," by C. Purdon Clarke. Reprinted from "Ars Quatuor Coronatorum," p. 102 and 103, Margate, MDCCCXCIII.

The reader will recognize that the Hindu scheme described above is simply an arbitrary method of repeating a design. It is quite apparent that Hindu figures, especially sculptured figures, are schematic. There is no evidence, however, that the Orientals understood dynamic symmetry as did the Greeks. At a very early period they employed some of the root rectangles in temple building, but their knowledge of these shapes was soon lost.

"Essay on the Architecture of the Hindus," by Ram Raz', native judge and magistrate at Bangalore. Published for the Royal Asiatic Society by J. W. Parker, West Strand, London, 1834.

The following extract from a letter by Miss Christine Herter, the American artist, is self-explanatory:

Dear Mr. Hambidge:

I have been working in two ways this winter,—carrying on a long slow portrait, a big thing on which I have not yet begun to paint, and at intervals I have been painting small rapid pictures. By small I mean life-size, of course, but within a comparatively small area. All of them I have painted on odd-sized canvases that happened to be about, with nothing but a few charcoal lines to start with and no *planned* composition. I find I work with a sense of judgment and freedom which I have never felt before, making plenty of mistakes but feeling an ability and confidence to right them which I am convinced comes from the study of the rectangles of Dynamic Symmetry. I had laid in a three-quarter figure on a 30 by 40 canvas and, after the whole thing was covered, I found the big division between light and dark in the background did not ring true (I had suspected it from the beginning). So, after thinking it over and deciding on what would be right, I rearranged the thing, and then, for fun, took my measuring stick and found I had exactly cut off the square from the bottom. This amused and pleased me not a little. I think the study of these rectangles is the most wonderful education for one's intuition that an artist can have.

Always sincerely yours,

Christine Herter.

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From

The Manchester Guardian

. . . . "Mr. Hambidge is no fool. He is worth reading. His key may not unlock such mysteries as Winged Victories of the prime, but he pokes about interestingly among minor cupboards. His first number of The Diagonal would be pleasant if only for his quotation of a fascinating passage by Professor A. H. Church on the mathematics of sunflowers. Everyone who has looked into the centre of a full-blown sunflower—the place where there are usually two or three preoccupied bees—must have noticed the extraordinary beauty and delicacy of the chased pattern made on a convex surface by the tiny facets of the hundreds of ovaries. Each facet is a rhomboid—Nature is no mere Cubist,—and they are so placed as to form such an exquisite design in intersecting curves that it is difficult to take the eye away from it. Professor Church has given Mr. Hambidge a lift by showing that in sunflowers there are found to be certain constant proportions between the longer and shorter of these curves—that, in fact, you can state a mathematical basis for this lovely design in a fuller sense than you do when you say how many points there are to a shamrock." . . . .

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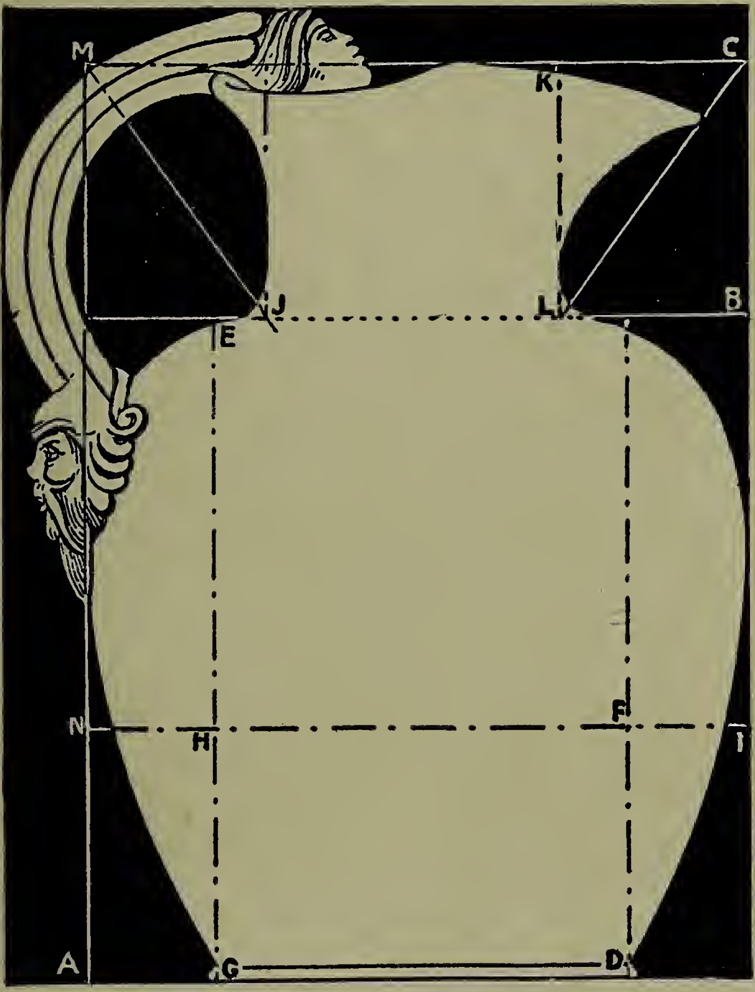

Bronze Oinochoe No. 2474 in the British Museum

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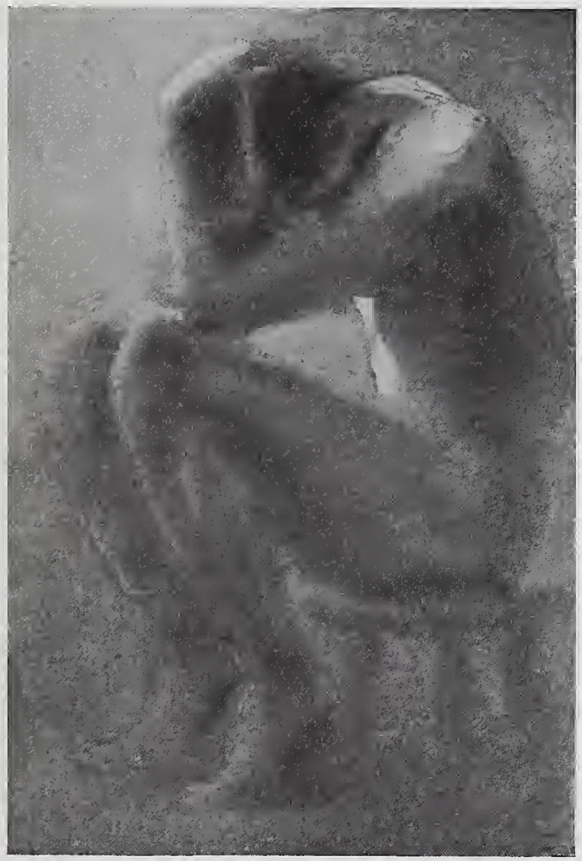

Fig. 1. Schematic movement in a Life Study.

DYNAMIC SYMMETRY IN THE CLASS ROOM

THE last few lines of Mr. Hambidge's article in the May DIAGONAL on the work of the students in the life and illustration classes in the New York School of Fine and Applied Art are self-explanatory: "—when this school term closed these students, themselves taking the initiative, asked to have a course of lectures on logic." In fact, the introduction of a prescribed, not elective, course on logic, coming as it did by request of the students, is inevitable; Dynamic Symmetry being in truth the graphic expression of logic.

The compositions reproduced in this article were done either entirely without the model or through reference prompted by an intelligent understanding of its uses.

Stress is constantly laid upon the power of understanding. We cannot suggest what we do not know. That the artist entirely without the knowledge of the laws of Dynamic Symmetry commits himself to an arbitrary shape when he chooses the size of his canvas

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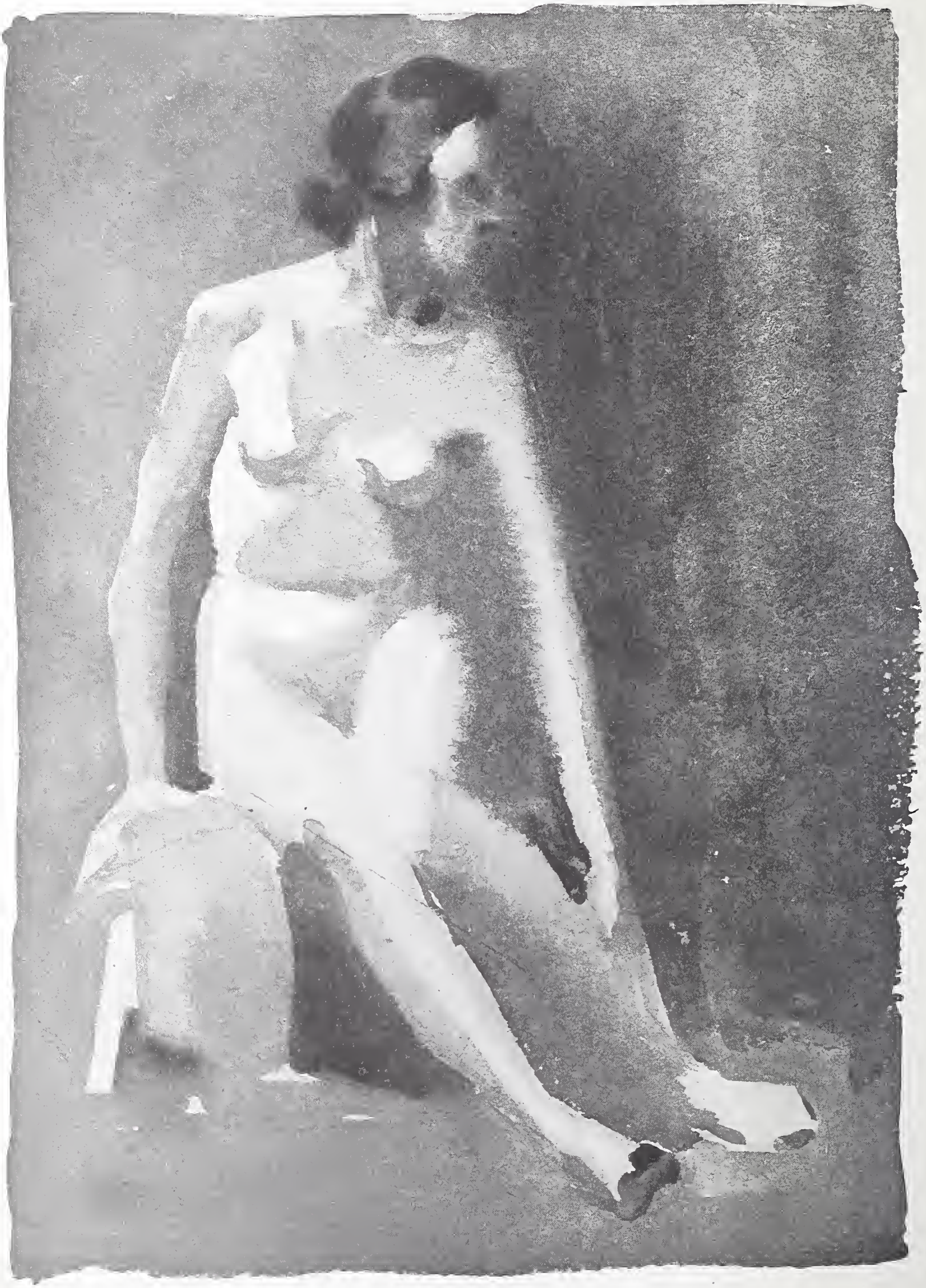


Fig. 2. Rapidly made study plan for character and movement.

or accepts a commission to decorate the pages of a magazine or the walls of a building, and even when he makes a spontaneous note on a piece of paper, becomes obvious; and his results, if he has a good set of intellectual habits, are worthy in proportion to his power to react to the fundamental truths of space understanding.

The students' questions are invited and through dialectics they soon convince themselves. In the matter of sports and games, the one who plays must first learn the rules, without which the game ceases to hold the interest of the participant or observer. The diagonal and resultant space arrangement (areas) of a root-five rectangle give the students their first insight to the order of the human figure and they react to a basic truth, *viz.,* the movement can never be greater than the diagonal of the containing shape. With the model posed from five to ten minutes, a few definite strokes are made and an impression of life given, seldom realized through dogged application alone. The result is pinned on the drawing board and by the simple process of projecting the diagonal they have a working space of similar shape, though greater in area, and the logical means of quickly transferring the theme without losing the spirit. Thus they constantly keep in front of them their own impression—in most cases one of spontaneous charm. In progressing from general to particular truths they find, as Amiel found, "the search for truths often defeats truth." The errors are understood and can be located in their own graphic record.

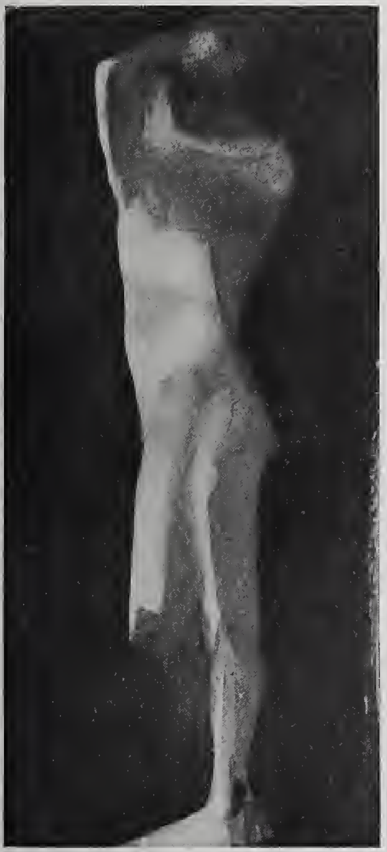


Fig. 3. Imagination.

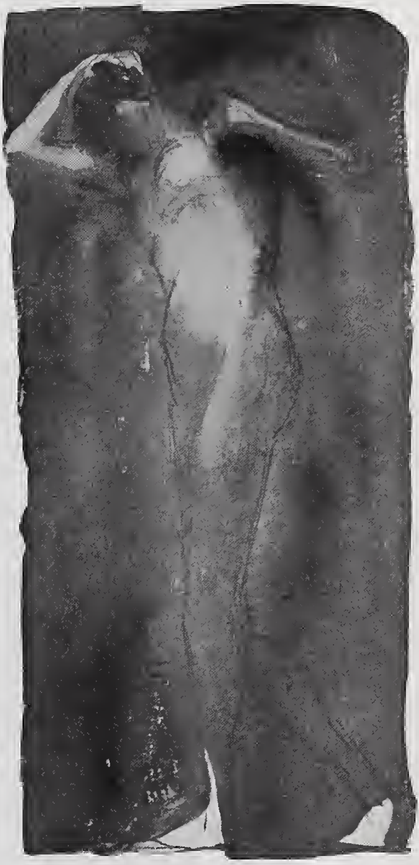


Fig. 4. Five minute memory study.

About the middle of the year the students of the illustration class were given the problem of drawing a chair; and told that the subject must be accessible, as it was safe to assume that each student lived in a room, also that in the room there would be chairs or at least a chair. The character alone of the chair was of importance, in that by close observation and graphic delineation they could tell the character to the spectator. When the work was done they were requested to take the next logical step by the simple process of turning the chair in their minds forty-five degrees and drawing it from memory. They could thus prove themselves to themselves. Various similar appeals to their reason were used, with the result that the old withering fault of charging to circumstances what they owed to themselves—"the model moved," "I was unable to find material," etc., was eliminated. As much as possible the use of the term "model" was discouraged and constant endeavor was made to make them realize that its relation to their work is comparable to that of the sextant to the mariner—something to make observations with; a base for compositional schemes. They learn to apply the rhythm of movement and its underlying structural truth; thus the picture grows in conformity with nature's plan and individual and personal interpretation become inevitable. The reader will gather from the foregoing that the method employed is in direct opposition to the atelier system, which we know is calculated to crush all personality except in the strongest spirit. In short Dynamic Symmetry teaches us *why*, not *how*.

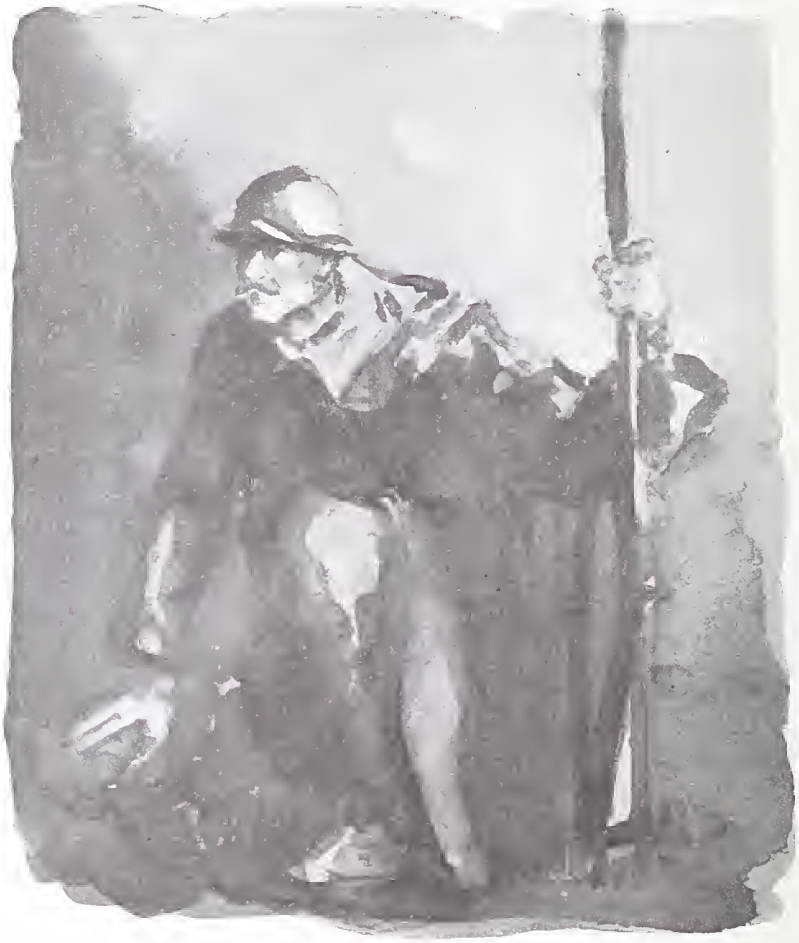


Fig. 5. Sketch for an illustration.

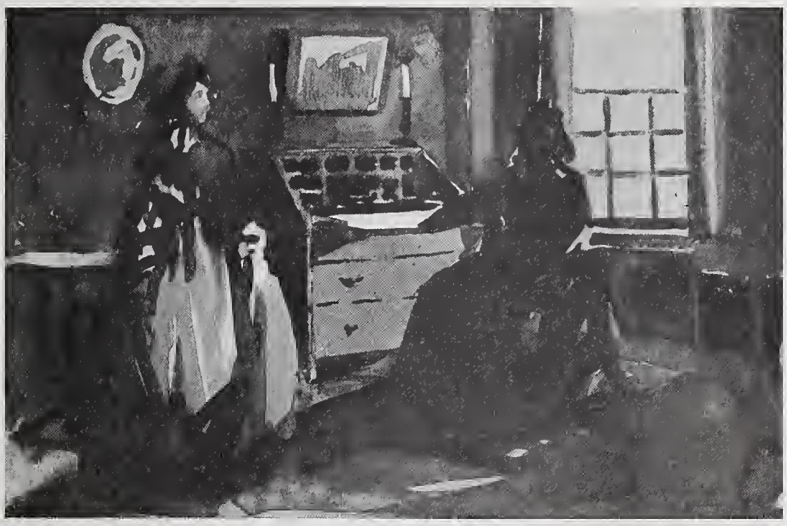


Fig. 6. Sketch for an illustration.

The accompanying reproductions of the students' work, which represent a few among many of equal merit, prove the wisdom of appealing first to their reasoning power. It has been a strong conviction with the writer that through this and only this process could beginners be taught to think as artists and realize joy in creation, without which no work reveals the individual characteristics of the artist. By paraphrasing a few lines of the introduction of Stuart Mill's admirable work, "System of Logic," the student has been made to understand that whereas an art may undoubtedly be brought to a certain not inconsiderable stage of advancement without any other logic than what all persons who are said to have a sound understanding acquire empirically in the course of their studies, there are limits, however, to what artists can do without principles of symmetry and to what

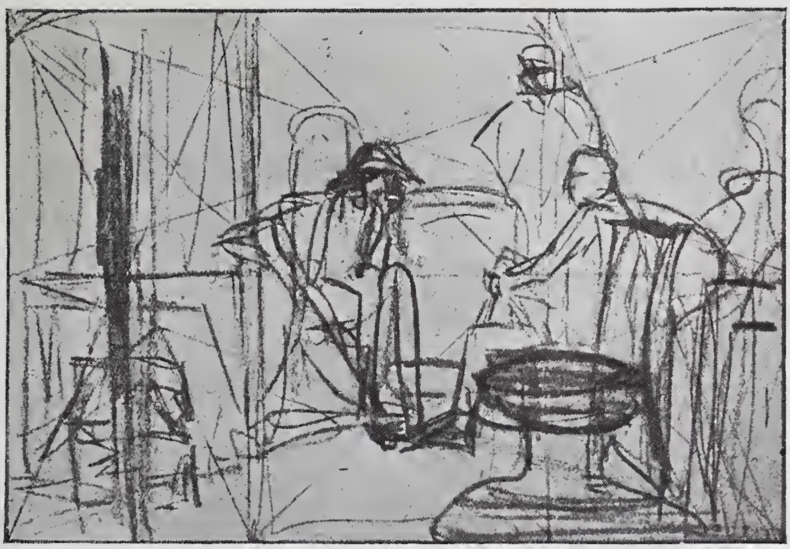


Fig. 7. The beginning of the rough plan.

thinkers can do without principles of logic. Note the rarity of figure compositions of any significance in our modern exhibitions. A few individuals by extraordinary genius may work without any principles in the same way or nearly the same way in which they would have worked had they been in possession of principles; but the bulk of mankind requires either to understand the theory of what they are doing or have rules laid down for them by those who have understood the theory. In the progress of art from its easiest to its most difficult problems, each great step in advance has had either as its precursor, or as its accompaniment and necessary condition, corresponding improvement in the notions and principles of logic received among the most advanced artists. I use the term "artists" in its truest sense, as implying an individual of creative, not only imaginative, power, because as surely as the modern architect is indebted to the engineer, so is the painter indebted to the logician within himself. Again we come in opposition to a generally accepted creed—"Art for art's sake"—a falsity which now literally means "painting for paint's sake." The objection of these devotees, reduced to a phrase, is that an acceptance of dynamic symmetry would reduce art to mathematical precision and resultant rigidity and formalism. The answer these students give is, I feel, to an open mind, conclusive. "They rejoice in the knowledge of freedom in that they conform to law."

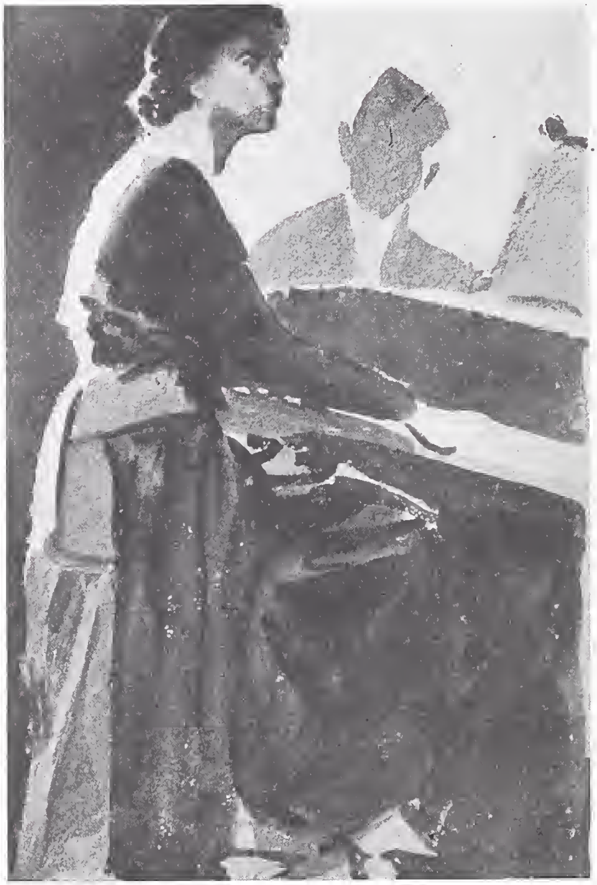


Fig. 8. Study plan for a black and white drawing.

HOWARD GILES.

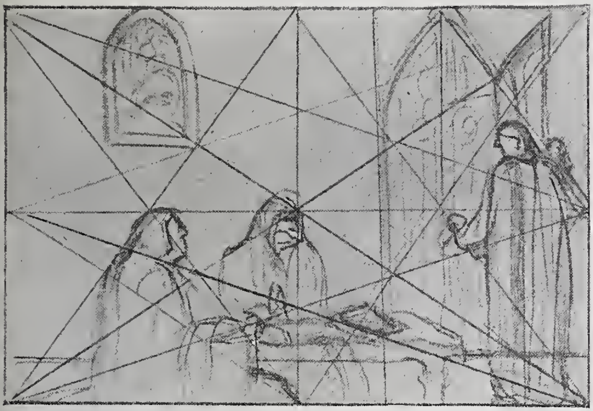


Fig. 9

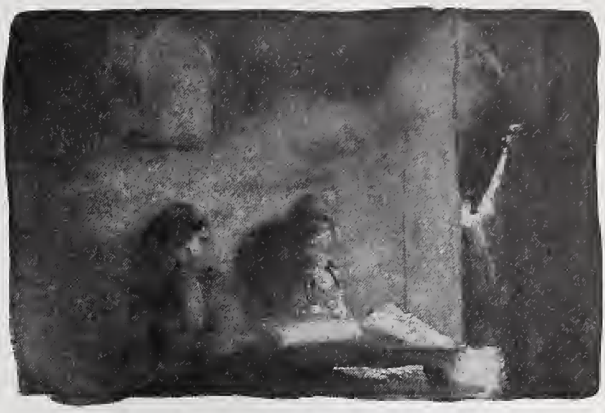


Fig. 10

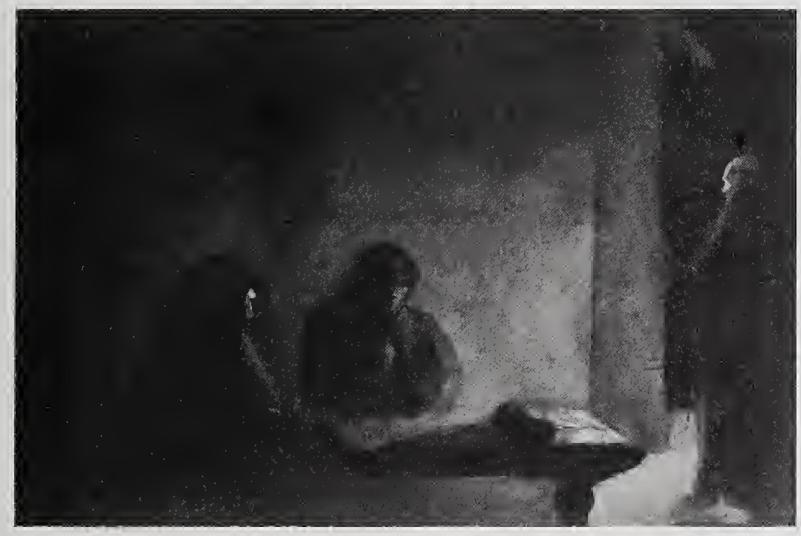
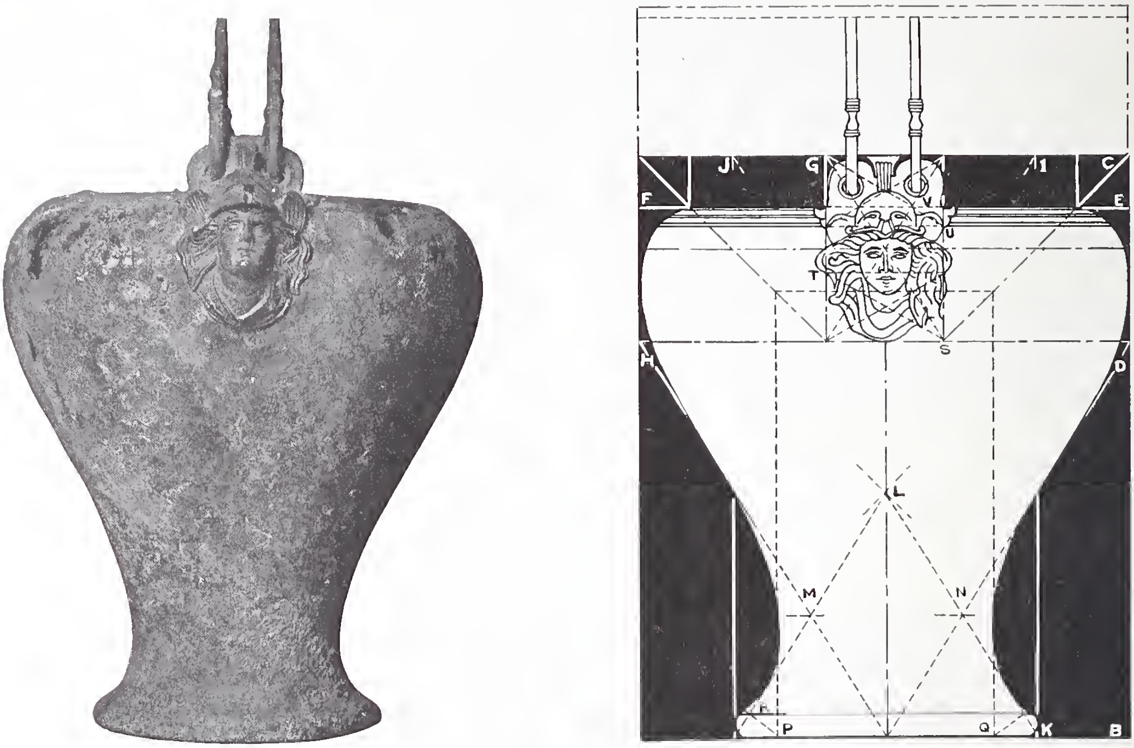


Fig. 11

Three stages in the development of a plan.



Bronze Greek Situla in the British Museum

AN ARCHAIC BRONZE SITULA IN THE BRITISH MUSEUM

THIS example, without the handles, supplies the ratio 1.191. With the handles it seems to be 1.5, *i.e.,* one and a half. As the handles are loose we cannot be certain of this ratio.

AB is a square, AC is .191; AD is also .191; *i.e.,* the area HC is twice .191, or .382.

If we subtract .191 from a square there is left the area HB or .809. This rectangle is composed of two whirling square rectangles standing side by side, as WH and WD.

The .382 rectangle HC is composed of the squares HG and SC and the whirling square rectangle SG.

Draw the perpendiculars and diagonals to the area SG. The points V and X are eyes.

Draw EF through V. This line defines the top of the situla. A side of the square AB passes through the centre of the area SG and coincides with the eyebrows of the head of Athena.

Draw a line through the point X and project it until it meets the diagonal SC. From this point, at right angles, draw a line to Q. Make the same construction for the square HG. These lines define the narrowest width of the situla.

Draw diagonals to the whole. These cut the top line of the situla at the points J and Y. Draw the line YK. This line and a similar one on the other side define the width of the foot.

On the diagonal line SC construct a square with sides equal to EC. This square defines the lip width.

L is the centre of the square AB. Draw the lines WD and WB.

Draw JK. It cuts WD at N. A similar point is M on the left side. Draw a line through these points and project it as NZ.

Draw QZ. This cuts LK to fix the height of the foot. Curve tangents are LK, ND, DE and the line which defines the width of the vessel at its narrowest point.

The angle pitch of the foot is that of a line from K to C.

TO CONSTRUCT A 1.191 RECTANGLE

Draw a square as AB, Fig. 2, and divide it into quarters.

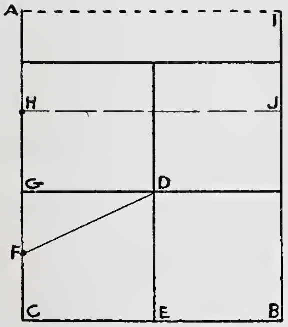


Fig. 2

Bisect one side of the square CD at F and draw the diagonal to two squares FD. Use this line as a radius to determine the point H. Draw HJ parallel to CB. HB is an .809 rectangle and is composed of the two whirling square rectangles HE, EJ, and AJ is a .191 area.

TO CONSTRUCT A 1.191 RECTANGLE WHEN THE MAJOR AREA IS A SQUARE AND A HALF

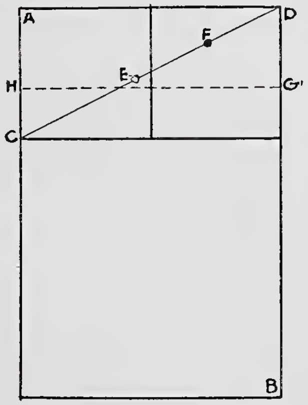
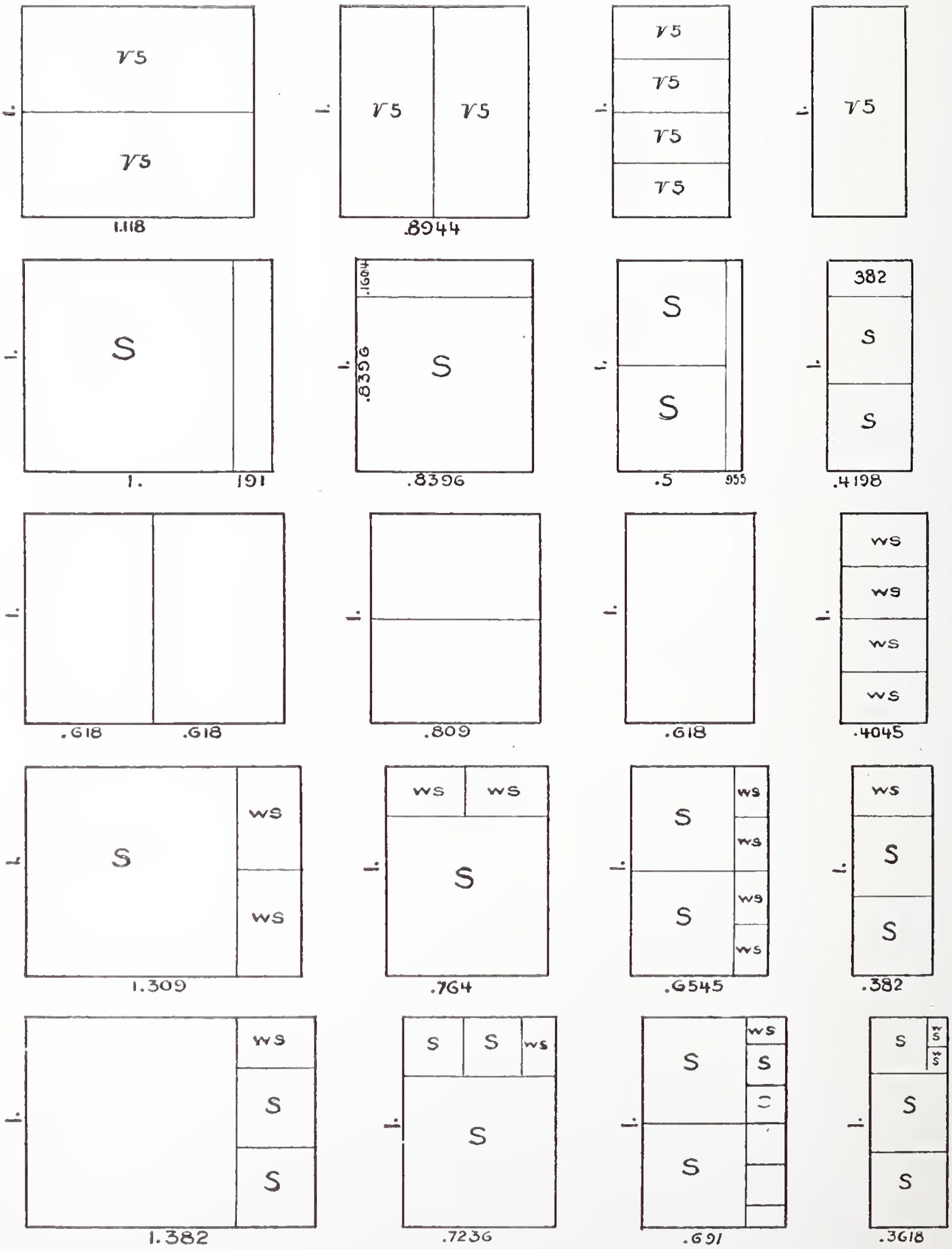


Fig. 3

AB, Fig. 3, is a square and a half. Draw CB, a diagonal to two squares. Make CE equal to CA and bisect ED at F. Make DG equal to DF. Draw GH. The area HB is a 1.191 rectangle. GC is a .191 area.

THE ELEMENTS OF DYNAMIC SYMMETRY

Much of the work upon which the Elements of Dynamic Symmetry is based was done during the year 1919 under the Sachs Research Fellowship of Harvard University. This fellowship was founded by Samuel Sachs of New York City and, under the terms of the foundation, is assigned to scholars of proved ability, whether students, instructors or others, for the general purpose of advancing the Fine Arts. This acknowledgment is but a faint expression of the writer's appreciation of the benefits enjoyed from a fellowship of a type too rare in American Universities. Jay Hambidge.



RATIOS MOST FREQUENTLY USED, THEIR RECIPROCALS AND SIMPLE DIVISIONS

THE following are some of the dynamic ratios which are most frequently found not only in nature but in Greek design:

Ratio	Reciprocal	½ Ratio	½ Reciprocal
1.118	.8944	.559	.4472
1.191	.8396	.5955	.4198
1.236	.8090	.618	.4045
1.309	.764	.6545	.382
1.382	.7236	.691	.3618
1.4472	.691	.7236	.3455
1.618	.618	.8090	.309
1.809	.5528	.9045	.2764
1.854	.5393	.927	.2696
2.236	.4472	1.118	.2236

The 1.118 ratio represents one-half of a root-five area, 2.236 divided by 2. It may be remembered as two root-five rectangles, one on top of the other.

.8944, its reciprocal, *i.e.,* 1.118 divided into 1, equals two root-five rectangles standing side by side.

.559, being one-fourth of root-five and one-half of 1.118, is equal to four root-five rectangles placed on top of one another. It is the reciprocal of 1.7888, or .4472 multiplied by 4. .4472, one-half of .8944, is equal to one-fifth of root-five, *i.e.,* it is the reciprocal shape of that area.

The ratio 1.191 may be identified by its relation to 1.382 and .809. The fraction .191 is equal to .382 divided by 2. If .382 is subtracted from 1.191 the remainder is .809. If .191 be added to 1.191 the result is 1.382. The fraction .5955 added to .4045 equals 1 or unity. The fraction .4198 is the reciprocal of 2.382.

The ratio 1.236 is equal to .618 multiplied by 2, or root-five, 2.236, minus 1. It is composed of two whirling square rectangles standing side by side. Its reciprocal .809 is also equal to two whirling square rectangles, one on top of the other. .809 is equal to 1.618 divided by 2. The .618 rectangle has been explained. If .809 is equal to two whirling square rectangles one above the other, the fraction .4045 is equal to four such shapes in the same position. Multiplied by four this fraction equals 1.618.

The ratio 1.309 is equal to a square plus two whirling square rectangles placed end to end. The fraction .309 is equal to .618 divided by 2. The reciprocal .764 is equal to .382 multiplied by 2. The difference between .764 and unity or 1 is .236, and this fraction represents the difference between root-four and root-five, 2 and 2.236.

The fraction .6545, *i.e.,* one-half of 1.309, is composed of two squares plus four whirling square rectangles, *i.e.,* .5 plus .1545—unity divided by 2 and .309 divided by 2. The fraction .6545 is the reciprocal of 1.5278, or 1.528 for short. (See Hydria in the May DIAGONAL.) If .6545 be subtracted from unity the remainder is .3455 or .691 divided by 2. The difference between .691 and unity is .309. The fraction .3455 is the reciprocal of 2.8944, *i.e.,* two squares and two root-five rectangles. The fraction .382 is the reciprocal of 2.618, or a square plus a whirling square rectangle.

The ratio 1.382 is equal to a square plus a 2.618 shape. The fraction .382 will be recognized as the difference between .618 and unity. The reciprocal ratio .7236 multiplied by 2 equals a square plus a root-five rectangle, *i.e.,* 1 plus .4472. The difference between .7236 and unity is .2764. Dividing .7236 by 2 we get .3618. The fraction .2764 is the reciprocal of 3.618, and .3618 is the reciprocal of 2.764 or 1.382 multiplied by 2, or .691 multiplied by 4.

The ratio 1.4472, a square plus a root-five rectangle, is one of the most important of the modulating forms of dynamic symmetry. Dividing the ratio by 2 we obtain .7236, the reciprocal of 1.382. The difference between .4472 and unity is .5528, or the reciprocal of 1.809. Multiplied by 2 the ratio 1.4472 equals 2.8944. The fraction .4472 multiplied by 2 equals .8944, the reciprocal of 1.118. The reciprocal .691 multiplied by 2 equals 1.382. The difference between .691 and unity is .309.

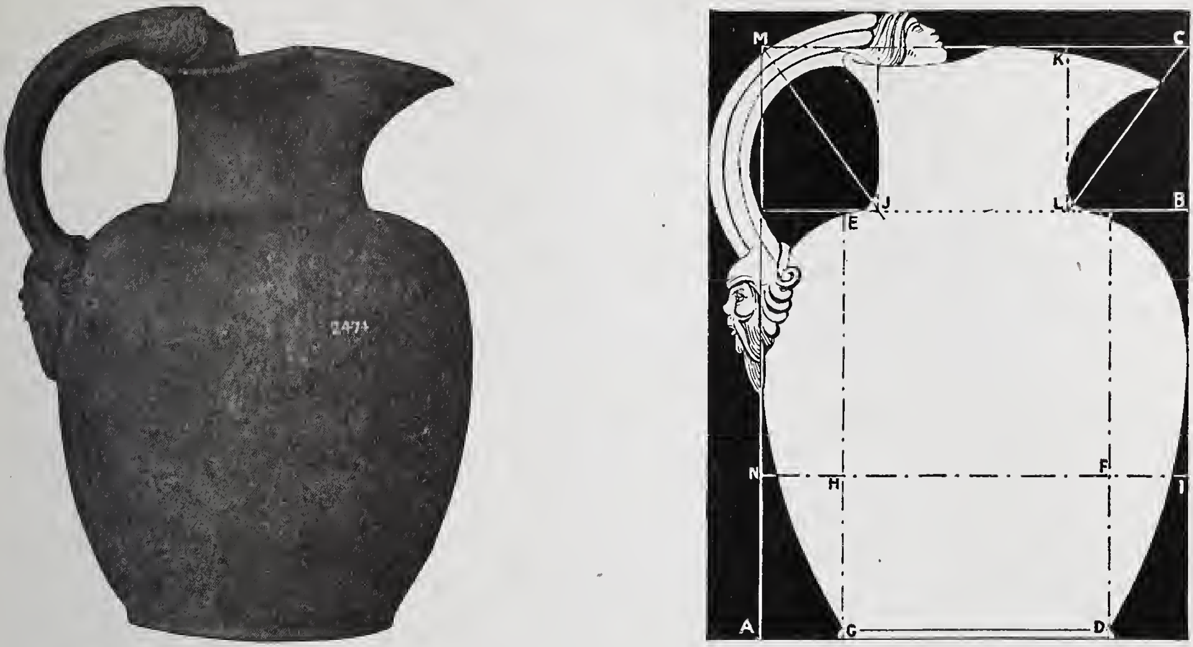
The ratio 1.618, its reciprocal .618 and the connection of both with the square root of five have been described in previous numbers of the DIAGONAL.

The ratio 1.809 is composed of a square plus two whirling square rectangles, these latter being placed one over the other. The fraction .809 is equal to 1.618 divided by 2. If we add .191 to 1.618 the result is 1.809. The difference between the reciprocal .5528 and unity is .4472. The fraction .9045 is equal to .5 plus .4045, or two squares plus four whirling square rectangles. The difference between .2764 and unity is .7236 (see above). The difference between 1.382 and 1.809 is .427; between 1.809 and root-five, *i.e.,* 2.236, is .427. A very engaging scheme and apparently a popular one with the Greeks.

The ratio 1.854 is equal to .618 multiplied by 3. If the fraction .854 be divided by 2 the result is .427. If the reciprocal .5393 be multiplied by 2 the result is 1.0786, the reciprocal of which is .927, this being composed of .618 plus .309. The fraction .2696 is the reciprocal of 3.708 or .618 multiplied by 6.

The ratio 2.236, *i.e.,* the square root of five, its reciprocal .4472 and many modulating subdivisions of the area have been frequently described in THE DIAGONAL.

[To be continued.]



Bronze Trefoil-Lipped Bronze Oinochoe in the British Museum.

BRONZE OINOCHOE NO. 2474 IN THE BRITISH MUSEUM

MEASURED, drawn and photographed by Charles O. Waterhouse of the staff of the Department of Greek Antiquities.

The ratio of the example, excluding the handle, is 1.382.

The ratio including the handle, 1.309.

Front elevation, including handle, 1.472.

AC is the 1.382 area.

AB is a square.

CN is a square.

MB and AI are .382 areas.

NB is a whirling square rectangle. EF is a square in the centre of that shape. ED is a whirling square rectangle, as is also GF.

MJ, CL are diagonals to the 1.382 area. KJ is a 1.17082 shape and MJ, CL, added, equal a square and a root-five shape, 1.4472. NG and DI are each equal to two squares.

A line drawn from C to N cuts EH at a point to determine the bottom of the satyr's head at the base of the handle. A line from K through J cuts EH at a point to determine the height of the satyr's head.

A diagonal to the whirling square rectangle ED, projected to CB, fixes the height of the front of the lip. The back of the lip coincides with EH projected to MC.

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From

The Manchester Guardian

. . . . "Mr. Hambidge is no fool. He is worth reading. His key may not unlock such mysteries as Winged Victories of the prime, but he pokes about interestingly among minor cupboards. His first number of The Diagonal would be pleasant if only for his quotation of a fascinating passage by Professor A. H. Church on the mathematics of sunflowers. Everyone who has looked into the centre of a full-blown sunflower—the place where there are usually two or three preoccupied bees—must have noticed the extraordinary beauty and delicacy of the chased pattern made on a convex surface by the tiny facets of the hundreds of ovaries. Each facet is a rhomboid—Nature is no mere Cubist,—and they are so placed as to form such an exquisite design in intersecting curves that it is difficult to take the eye away from it. Professor Church has given Mr. Hambidge a lift by showing that in sunflowers there are found to be certain constant proportions between the longer and shorter of these curves—that, in fact, you can state a mathematical basis for this lovely design in a fuller sense than you do when you say how many points there are to a shamrock." . . . .

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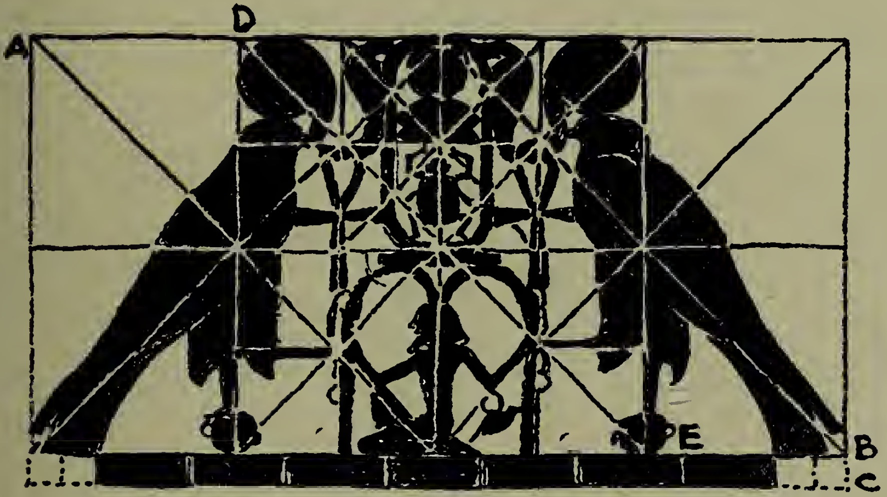

Symmetry used by Egyptian Designers of Jewelry nearly 4,000 years ago

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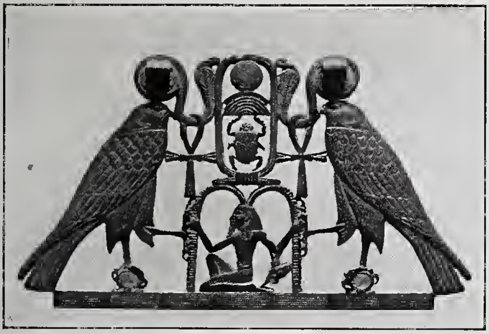

Fig. 1

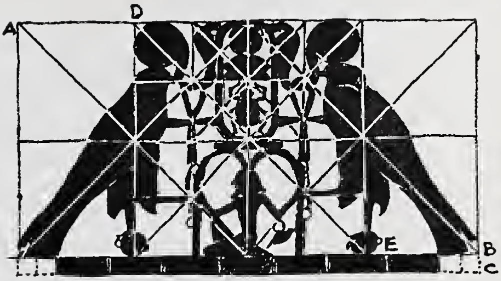

Fig. 2

AC equals 1.809.
AB equals two squares.
DE equals one square.

SYMMETRY USED BY EGYPTIAN DESIGNERS OF JEWELRY NEARLY 4000 YEARS AGO

IN the course of excavations conducted in the spring of 1914 in Egypt by Professor W. M. Flinders Petrie, there was discovered a complement of jewelry of an Egyptian Princess, a "daughter of Hathor of Dendera," supposed to date from 1906 to 1887 B. C. With the exception of certain pieces which were retained by the authorities for the Cairo Museum all the objects included in the discovery were purchased for the Metropolitan Museum of New York City in 1916. This discovery is one of the most important ever made in Egypt, not excepting the celebrated Dashur jewelry find by De Morgan in 1894.

From the symmetry point of view the pectorals, or breast plates, one of which is in the New York Museum and the other in the Cairo Museum, are the most important objects of these extraordinary examples of ancient craftsmanship.

For an illustrated and detailed description of Professor Petrie's discovery see "The Treasure of Lahun," by Albert M. Lythgoe, Part II of the Bulletin of the Metropolitan Museum of Art, New York, Dec., 1919.

Usually the symmetry of Egyptian pectorals is quite clear by inspection, though its exact character can be determined only by measurement of the original. Photographs are misleading. Through the courtesy of Mr. Lythgoe the writer was permitted to measure the example in his care. These measurements, in millimeters, are:

Full width, 823; ratio, 1.809.
Full height, 455.

The height, without the base, equals one-half the full width (plus or minus), *i.e.,* the design without its base makes two squares or a root-four rectangle. Miss G. M. A. Richter, Curator of the Greek department of the New York Museum, also measured this example and concluded these were the correct proportions.

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The 1.809 rectangle, in connection with the root-four shape of the two squares, is interesting, as centuries later these proportions were used frequently by Greek designers, especially in the skyphoi shapes they made. There is this difference, however, in the manipulation of the areas by the designers of the two nations: the Egyptians seem to have correlated the entire detail of the pectoral by the logical subdivision of the two squares, thereby creating a curious blending of the two types of symmetry, static and dynamic; the Greeks used the dynamic method almost pure. Fig. 3 shows a Greek skyphos in a 1.809 rectangle. The area of the design projection, without the foot, is composed of two squares. (See Skyphoi chapter in "Dynamic Symmetry, The Greek Vase," by the writer.)

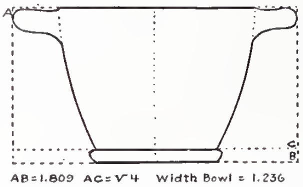


Fig. 3

The correlation of the detail of the pectoral by the two squares is very simple and obvious as the symmetry diagram shows, Fig. 2.

In connection with this example it is interesting to compare the symmetry of the pectoral in the museum at Cairo. But for this comparison we have no data except that furnished by a photograph. It is clear, however, even from this, that the symmetry flavor is quite different from that of the example in the New York Museum. In the latter example the angles formed by the limbs of the kneeling figure are angles of forty-five degrees or diagonals to squares. The angles made by the limbs of the figure of the Cairo example are clearly not those of forty-five degrees. From the photograph the symmetry of the Cairo pectoral, including its base, seems to be that of root-three. Without the base it, apparently, is 1.854, or three times .618, *i.e.,* three whirling square rectangles. If this analysis be correct then the angles made by the limbs of the kneeling figure are thirty and sixty degrees or diagonals of the root-three rectangle, see Fig. 4.

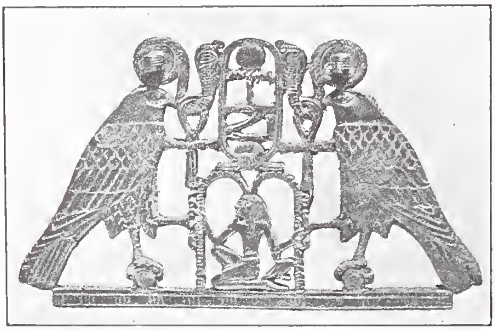


Fig. 4*a*

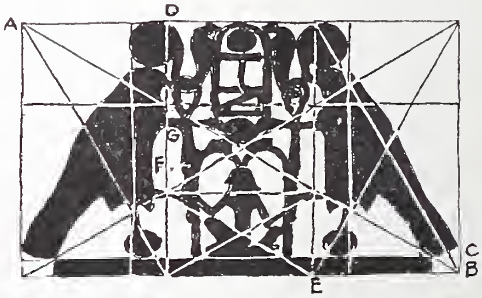


Fig. 4*b*

The Pectoral at Cairo.

AB equals root-three.
AC equals three whirling square rectangles or 1.854.
DE equals root-three or one-third of the total area.
EH equals root-three or one-third of DE.
EG equals square in DE, etc.

Perhaps, in these two pectorals, we have examples of attempts on the part of the jewelry designers of ancient Egypt to connect the proportions of static symmetry with that of dynamic, *i.e.*, the proportions of the square and the equilateral triangle with the derivative shapes of the root-five rectangle.

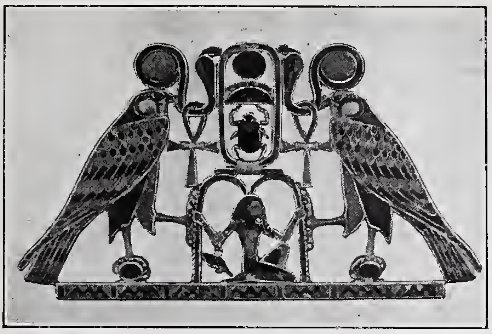


Fig. 5

Obverse of the New York Pectoral.

DYNAMIC SYMMETRY RECTANGLES AS PATTERN BASES

Willesden, Holywood, Co. Down, Ireland, June 10, 1920.

Dear Sir:

A few days ago I picked up a copy of *Drawing and Design* (New Series I), a London publication, and was interested in an article entitled "The Diagonal," which refers to an article appearing in the paper of this name edited by you and of which I would like to have a copy. Some years ago I invented a rectangle for a special purpose in which the length bears to the width the same ratio as the diagonal of a square to its side. This rectangle has many beautiful geometrical properties and half the rectangle is similar to the whole.

$$\frac{AB}{BC}=\frac{BC}{BE}=\frac{\sqrt{2}}{1}=\frac{1}{\frac{1}{2}\sqrt{2}}$$

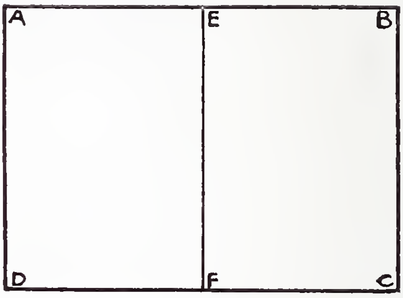


Fig. 1

Another rectangle which I have studied a little is that in which the length bears to the width the ratio of the diagonal of a regular pentagon to its side. This is I think the same rectangle as referred to in the article for in it $\frac{AB}{BC}=\frac{BC}{BE}$ then AEFD is a square.

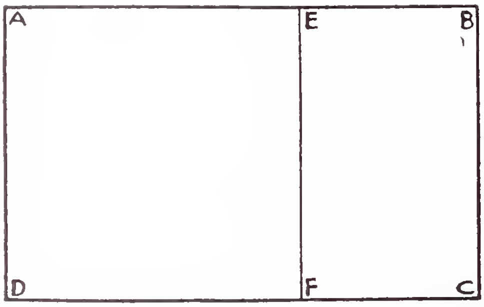


Fig. 2

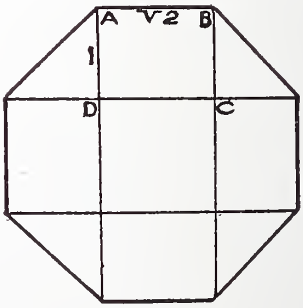


Fig. 3

I have utilized the rectangle shown in Fig. 1 in making geometrical designs (amateur) by ruling the paper into rectangles of this shape instead of into squares. This rectangle occurs naturally within and without a regular octagon in various ways, *e.g.*, ABCD, Fig. 3, and I call it the "octagon" or "perfect binary" rectangle, and I have always looked upon it as a natural fundamental artistic shape and would be glad to hear what you think of it from an artistic point of view.

Yours truly,

Arch'd H. Finlay.

Paris, July 10th, 1920,
Morgan Harjes, 14 Place Vendôme.

Dear Sir:

Your letter of inquiry of June 10th, has been forwarded to me. What you say about the root-two and extreme and mean ratio rectangles is very interesting as showing that the discovery of these shapes may be spontaneous. These rectangles belong to a series which constitute the base of the highest type of symmetry. The two shapes you mention have been discovered several times in history. The rectangle of Fig. 1 of your letter was known to the Greeks and is the first of what we may call the root rectangle series. The end and side of this rectangle are incommensurable but to the Greeks these lines were not irrational as they are commensurable in square. All the root rectangles, *i.e.,* rectangular areas wherein the end to side relationships are 1 to root-two, root-three, root-four, root-five, root-six, root-seven, etc., possess this commensurable square property.

The square on the side of a root-two rectangle is double the end square. The square on the side of a root-three rectangle is treble the end square. The square on the side of a root-four rectangle is quadruple the end square, etc., etc.

You will find that, like the root-two rectangle, all the others possess most fascinating properties for design purposes. As a matter of fact a design of the highest type is not possible unless these shapes are used. Before design can exist symmetry must be present. Symmetry in a design sense means the relation which a part bears to a whole. In design of the highest type we must have conscious co-ordination and this can best be obtained by the rectangles mentioned.

The rectangle of Fig. 2 of your letter is a direct offspring of the root-five rectangle. If to the area AC of Fig. 2 of your letter there be added an area equal to EC the new area is equal to that of a root-five rectangle. The area EC is a similar shape to AC and is the reciprocal of that area. The reciprocals of root-rectangles are even multiples of the whole and this is one of the reasons why these shapes are so superior for co-ordinating purposes. We may consider these rectangles as belonging to a measuring system wherein bounding lines, generally, are incommensurable but the areas themselves are commensurable. There are two bases for a measuring scheme of this type; one depending upon the relationship of a side to a diagonal of a square, the other upon that of a diagonal to two squares to a side of one of them. The latter is the better type. We are finding that something like eighty per cent of classic Greek design was co-ordinated by this base, about ten to twelve per cent by the diagonal and side of a square base; the remainder either showing haphazard co-ordination or one fixed by a linear system of measurement. The linear is the system used by modern designers and is the weakest. The co-ordinating scheme used by designers other than those of Greece and Egypt is based upon properties of the regular figures. This system, with the linear scheme, constitutes what I have termed "static symmetry."

The diagonal to two squares is a root-five line and the proportions derived from it are of special interest to us as modern botanists have found that the symmetry of the plant is co-ordinated upon this base. The line AB of Fig. 2 of your letter is cut in extreme and mean ratio at E. To obtain this cut we must use a construction based upon a diagonal to two squares. (See Euclid II 11, VI 30, and XIII 1 to 6.) In Book XIII you will find the geometrical proof that the side and a diagonal of a regular pentagon are in the proportion you mention. The method by which we obtain the entire series of rectangles which results from this proportion from the regular pentagon itself is explained in Chapter III of "Dynamic Symmetry, The Greek Vase."

With your permission I should like to publish your letter in The Diagonal. You might also care to tell us something more about the results of your experiments with these rectangles.

It is curious that we should both have used the word "binary" in connection with proportions of regular figures. (See a paper read by me before the Hellenic Society in London November, 1902, a synopsis of which appeared in *Nature* shortly afterwards.)

Willesden, Holywood, Co. Down, Ireland, July 14, 1920.

Dear Sir:

Your letter of the 10th inst. is indeed very interesting reading to me as I have never known of anyone else who has studied these rectangles with the exception of Mr. Watkins of Hereford, England, who has lately worked out the best shape for photographic plates —which is the root-two rectangle. I enclose a paper of his; but he seems to be only interested in it from the practical point of view. I had thought of this application the best part of twenty years ago, but never published anything about it. Some years ago I wrote to the Kodak Co. in U. S. A. Their reply was to the effect that plates and films were now standardized and could not easily be changed. The same sort of reply was received from a large firm of paper manufacturers. I knew it would be a good standard shape for sheets of paper. I had come to the conclusion that this shape had been discovered in ancient times and give my reasons later, and it is pleasant to have this view confirmed by one who knows. I may say that I have studied only the root-two rectangle to any extent. I found it much more difficult to make headway with the root-three rectangle. Though my knowledge of design is of the most elementary nature, yet I instinctively feel that what you say about these rectangles is quite true, and that beautiful results could be obtained by those who understood them and their geometrical properties. I remember when at school seeing a copybook. The correct slant for handwriting was given in it as the diagonal of a double square.

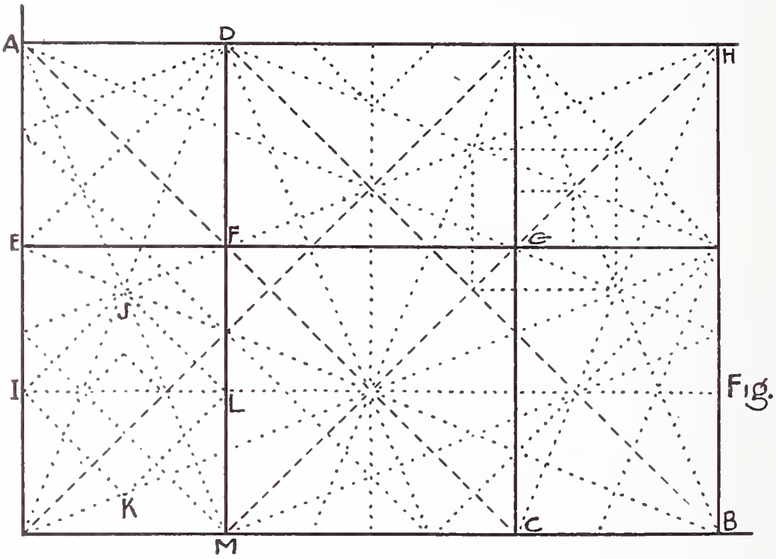


Fig. 1

My Euclid does not include Book XIII, but from IV, 10, the ratio of the diagonal to the side of the pentagon is pretty evident.

As regards the word "binary" I have only used it quite recently in this connection. It is used a good deal in another subject in which I am interested, viz., weights, measures and coinage. I have held the opinion for a long time that the subdivisions should all be binary and that all our systems of weights, measures and coinage, and arithmetic in general, should be founded on the radix of eight instead of ten.

Certainly you are at liberty to publish my letter if you think it worthy of ink and paper, and, further, I enclose tracings of the root-two rectangle, etc., to which I will now refer:

Fig. 1 is the root-two rectangle naturally subdivided into eight root-two rectangles and nine squares; and the half rectangle is divided into five root-two rectangles and six squares. It is also covered with lines all running at ¼ or ½ a right angle to the sides or ends of the rectangle. It is traced from a very much more complete combination of lines all parallel to one or other of the ones shown. When there are two squares shown, one inside of the other (as in Fig. 2), a root-two rectangle is seen to enclose them. There is a good deal to study in this figure.

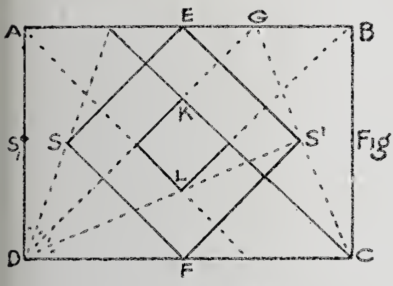


Fig2

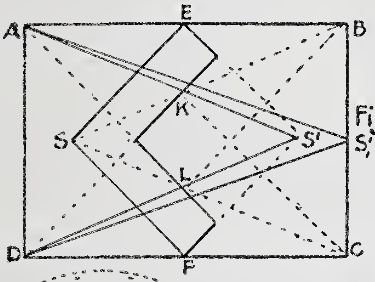


Fig3

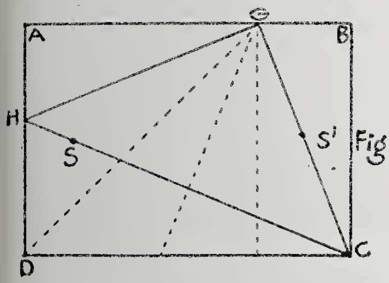


Fig5

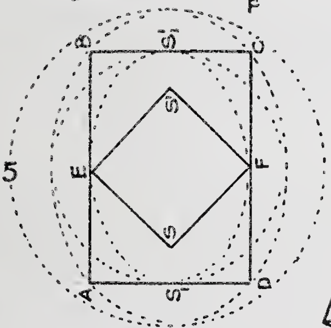


Fig 4

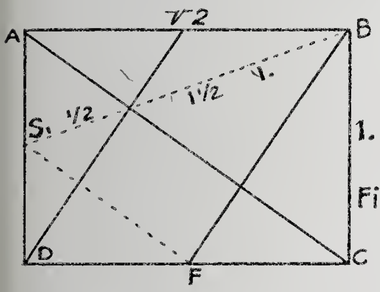


Fig6

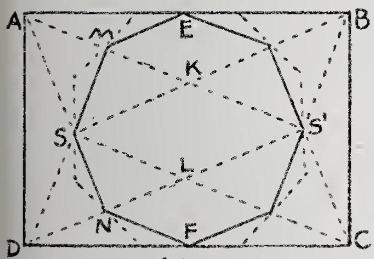


Fig 7

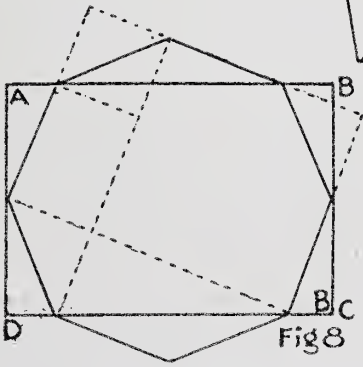


Fig8

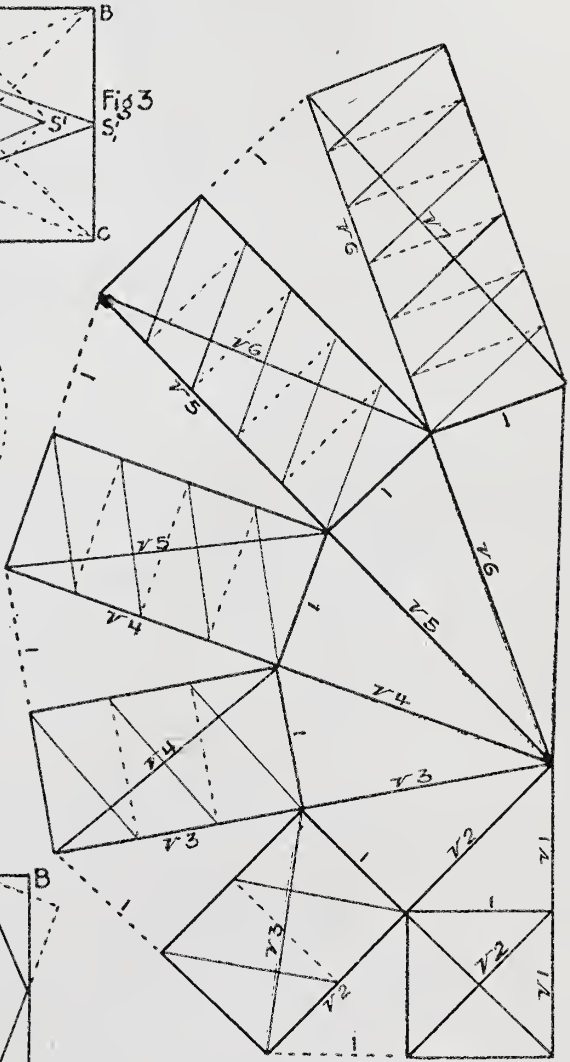


Fig 9

In Fig. 2 we have the same rectangle on a small scale, angle SDC equals ¾ a rt. angle. Angle SCD equals ¼ a rt. angle. Angle GDC equals ½ a rt. angle. AG equals AD, DG equals DC. S and S′ are the foci of the escribed ellipse, and K is the focus of the inscribed semi-ellipse touching at D, E and C.

Fig. 3. It was the discovery of this familiar figure which caused me to conclude that this rectangle must have been known long ago. In the triangles AS′D the angle at S′ bears to the angles at A and D the ratio ⅔. In the triangle AS′D the side AD bears to the side AS′ or DS′ the ratio ⅔.

Fig. 4. The width of the escribed ellipse is equal to the length of the inscribed ellipse and the length of the escribed ellipse is equal to twice the width of the inscribed ellipse. The directrix of the inscribed ellipse is tangential to the escribed ellipse.

Fig. 5. HGC is a right angle. The triangles AHG and BGC are equal in every respect and are similar to the triangle DHC, and the area DHC equals AHG plus HGC. (This is B.)

Fig. 6. E and F are the midpoints of AB and DC. ED and BF are perpendicular to AC and divide it into three equal parts. SB equals 1½ if AD equals 1. Similarly SF and AC divide DE into three equal parts.

Figs. 7 and 8 show connections between this rectangle and the octagon (see also Fig. 1). Note that the focus of the half rectangle, K, the focus of the double rectangle, and S′, the focus of the rectangle, lie on a straight line passing through A; the two partly dotted rectangles in Fig. 8 are also root-two rectangles. In this figure it is easily seen that

$$\frac{\sqrt{2}+1}{\sqrt{2}+2}=\frac{1}{\sqrt{2}}$$

Fig. 9 shows a method of drawing root-rectangles, the length of any one of the series being the diagonal of the previous one. It alsoshows that the diagonals of the reciprocal rectangles are perpendicular to the diagonal of the rectangle in each case (see Fig. 6).

Fig. 10 takes a step into three dimensions and illustrates a rectangular solid in which, when bisected, each half is similar to the whole.

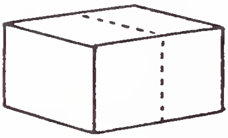


Fig. 10

$$\frac{\text{length}}{\text{width}}=\frac{\text{width}}{\text{depth}}=\frac{\text{depth}}{\tfrac{1}{2}\text{ length}},\ (\text{length})^3=2\ (\text{width})^3, \text{ etc.}$$

$$\frac{\text{length}}{\text{width}}=\frac{3}{\sqrt{2}}$$

This might be called a cube root-two rectangular solid. A whole series of such rectangular solid figures might be formed corresponding with the series of square root rectangles, but not so easily.

Have you investigated these three dimensional figures with reference to design?

Fig. 11 (sheet II tracing) is a rough straight line design based on the root-two rectangle. The paper having been first ruled into these rectangles the drawing of the design was done with very little mental effort, and yet it has a pleasing effect. It all falls in quite naturally. This idea in the hand of an expert would no doubt produce striking results.

In conclusion I would say that I hope you will find something new to you in my results and of use to you in your investigation, for it seems to me that you are doing good and permanent work for humanity in the sphere of art.

Yours sincerely,
Arch'd H. Finlay.

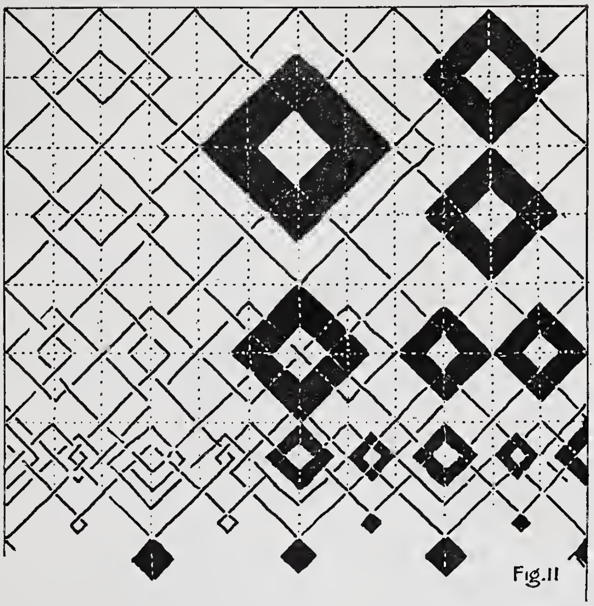


Fig. 11

Fig. 1 is a root-two rectangle. Mr. Finlay's subdivisions are very complicated. The result would be obtained by the application of a square on both ends of the area, as AC and BD. They overlap to the extent of the area DC. This operation determines the squares AF, FC and GH, and the root-two rectangles DG, EM and GB. The point J is the intersection of diagonals to AM. The line IL divides the root-two rectangle EM into two parts. These subdivisions may be continued indefinitely but the method is not the best for pattern making. It misses the major dynamic quality inherent in the root-two shape (see Lessons on Dynamic Symmetry).

Fig. 2. A root-two rectangle with a square in the center.

Fig. 3. Evidently intended to represent the old Masonic symbol of a square and a compass. It is fanciful, however. In its original form the compass seems to have represented an equilateral triangle.

Fig. 4. This is interesting, but students of Dynamic Symmetry found it while developing other curve forms in connection with the root rectangles, particularly that of curve of the equiangular spiral.

Fig. 5. DG is a square and CG a diagonal to a square and a root-two figure.

Fig. 6. Diagonal to a root-two shape and two diagonals of reciprocals.

Fig. 7. A logical development of Fig. 2.

Fig. 8. See Chapter on Static Symmetry in the writer's book on "The Greek Vase."

Fig. 9. This is a very interesting arrangement but it was worked out several years ago by the writer, also by some students of Dynamic Symmetry. It was found in each case as one of the resultant arrangements of the placing of the dynamic shapes in spiral formations. While examining these arrangements it was noticed that the whirling square rectangle was the only one whose original shape was maintained when a square was added successively to each end and side. To construct the diagram which shows this we begin with a .618 shape and add successive squares as in Fig. 16.

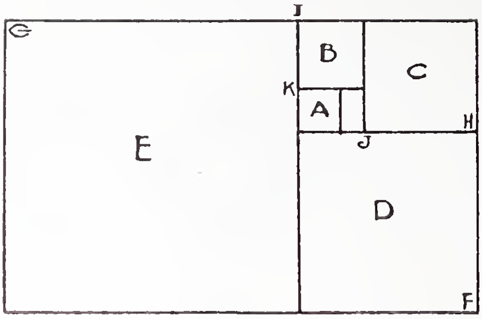


Fig. 16

The squares A, B, C, D, E, etc., maintain the whirling square rectangles FG, FI, HI, IJ, IK, etc.

If Mr. Finlay had continued his experiments with the Fig. 9 arrangement he would have found, probably, many of the other forms which are proving so valuable for design purposes.

Mr. Finlay is an original thinker, as the reader can well understand from his letters. The solid of this figure, however, is but one of the infinite series which may be obtained from Dynamic Symmetry. We do not find any solids having root-two or root-three character in phyllotaxis or the human figure; they are always based upon root-five or its derivations (see the solid derived from an actual skeleton in the March DIAGONAL).

According to the measurements of Doerpfeld, the German archaeologist, the ground plan of the Zeus temple at Olympia, Greece, and all its subdivisions, belong to the root-two class. Unfortunately the superstructure of the building is in ruins and we are not justified in accepting restorations. If this building were planned throughout on the root-two base we would have in it an example of a root-two solid.

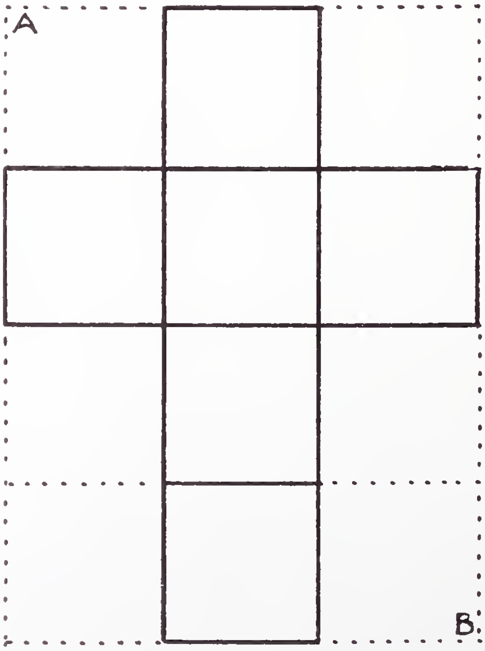


Fig. 17

The reader should bear in mind, however, that it is questionable whether it is possible to design in three dimensions. The Greek method of planning buildings seems to have been that of the old tradition of Free Masonry, which was the "opening of the building." This method appears to have been the basic operation in planning a cathedral. It was the reduction of a cube to two dimensions and was done by arranging the six sides of that solid to form a cross.

The six squares, representing the six sides of the cube, are shown in heavy lines. If the rectangle AB be completed we have a static area made up of twelve squares, three one way and four the other. If a building were developed on this base we should have the typical static structure of Gothic design.

If, instead of squares, we take the ground plan of a dynamic building and on each of its four sides place the side and façade elevation rectangles, then add to either of the side elevations one-half of the root rectangle, we should have a temple "opened up," and if this two-dimensional arrangement were folded, we should have a typical dynamic solid such as we obtain from the human figure.

This method has great advantages from the practical point of view, as it presents to the eye, in two-dimensional form, every superficial square inch of a structure. It is also the simplest method for making a model.

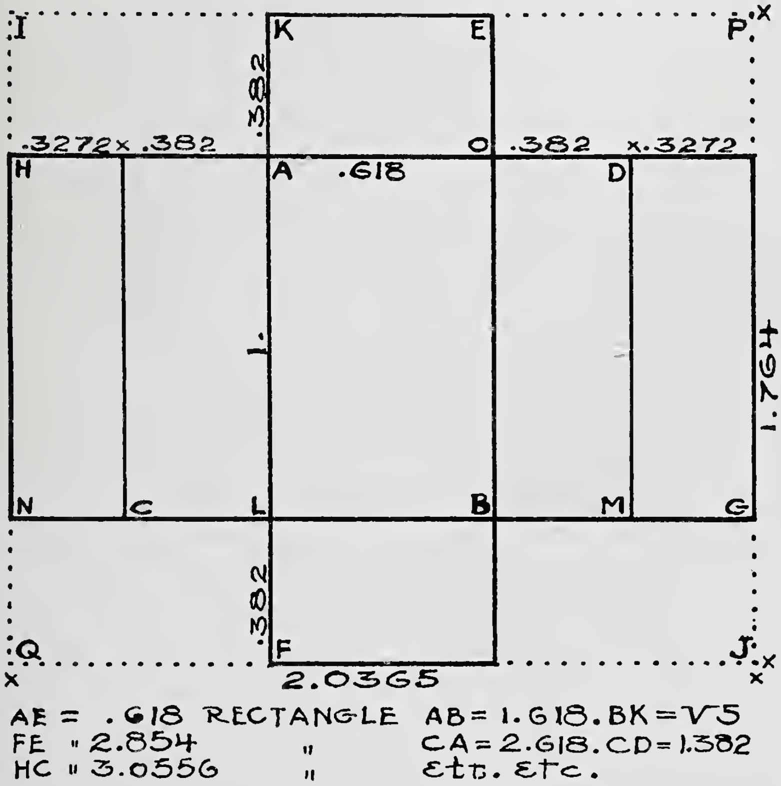


Fig. 18

As an illustration, suppose we assume that AB of Fig. 18 is a .618 rectangle and the ground plan of a building, AC, BD, two side elevations, AE, BF, two façade elevations and HC, DG, two halves of a roof. When these areas are placed in position we obviously have the rectangle IJ. AO measures .618, AL 1., AK .382, LF .382, NC .3272, MG .3272. The line KF equals .382 plus 382 plus 618 or 1.764. The line NG equals .3272 plus .382, plus 618, plus 382, plus .3272 or 2.0365. 1.764 divided into 2.0365 gives the ratio 1.1545 for the rectangle IJ, one of the dynamic seriesand equal to a square plus four whirling square rectangles. The fraction .1545 is equal to .618 divided by 4. If this diagram were drawn on a board or paper the rectangles IA, OP, LQ and MJ could be cut out and the shape represented by the straight lines folded up, thus giving the containing solid of the building.

The student will find that all the composing rectangles, AE, AB, BF, AC, OM, DG and HC, are all dynamic rectangles. According to the principles of Dynamic Symmetry any logical subdivisions of any of these composing units would result in the production of other dynamic shapes. If a Greek building of the classic age be utterly destroyed and nothing left but the ground plan we may examine the symmetry of it. If there be remaining a part of the superstructure we may determine the symmetry of that part, but the writer does not believe it either advisable or practicable to restore these classic structures.

The feature of Dynamic Symmetry which is most extraordinary is the power that its shapes possess of weaving or modulating into each other, in two or three dimensions, in a commensurable manner, but that commensurableness is of area and volume and not of line.

In Fig. 11, Mr. Finlay has hit upon the mechanical base of pattern, one of the great principles of all design. Since the discovery and development of Dynamic Symmetry the writer has marvelled that designers have not been quicker to see that the shapes of this symmetry furnish an infinite series of new pattern bases.

When it is realized that practically all pattern bases are today, and for several centuries have been, correlated by the regular two-dimensional figures, usually the square and the equilateral triangle, it will be seen that an extraordinary opportunity for the making of new pattern forms has been created by this symmetry discovery.

The history of design teaches us that no new decorative fabric can be created unless it is founded upon purely mechanical bases. No matter what principles of construction are developed, in efforts to create new architectural types, the structures themselves cannot possess sufficient character unless they are enriched by decorative forms in keeping. This will be understood if we examine the pattern enrichment forms of any architectural type such as Gothic, Saracenic, Persian, Chinese or Aztec. This is a fundamental principle to which there is no exception in history. So true does the writer feel this to be that he is inclined to consider the development of new pattern bases of as great importance as the architectural structure itself. History teaches us that if this principle is not recognized efforts to create new decorative forms are futile. Rococo and Nouveau art designs are examples of this futility.

With the root-two form before him, and inspired with the enthusiasm of discovery, Mr. Finlay harks back to the threadbare square as a decorative unit. In this he is simply playing a variation on an old theme, and the possibilities of the root-two area for the creation of new pattern shapes are practically lost. Mr. Finlay explains, however, that he is not a designer, so his neglect to take advantage of his opportunities may be understood. Nevertheless he is entitled to much credit for what he has done.

A few expert designers in America who have studied Dynamic Symmetry and realize something of the possibilities of its shapes for the creation of new pattern bases are working upon nonconventional decorative forms.

Every shape of the dynamic series has possibilities of decorative development on new design lines that are practically infinite and until these possibilities are worked out, at least in some measure, there exists little hope for a stable new art movement. A pattern base, to be of any value, must possess power of subdivision into similar or proportional figures like those of the equilateral triangle, the square or the other regular two-dimensional figures. But these figures have become commonplace to an extreme; they are exhausted. Every nation since history began has used them. Moreover they are fundamentally static and almost always cause a design fabric woven on them to have a heavy, stoggy quality. The dynamic forms, on the contrary, are fresh and new and possess greater arrangement possibilities than any of the older shapes.

It is suggested that the student try working out some of the old motives of decoration on the dynamic bases. Mr. Albert Southwick, head of the design department of Tiffany & Co., tells me that the dynamic shapes are superb for Gothic art. He frequently has occasion to design Gothic cases for large clocks. He also says that no matter what decorative style is developed on the dynamic rectangles it is practically impossible to obliterate the classic flavor. A Gothic clock case shown to the writer recently confirms this. But it was also quite apparent that the new base gave the old Gothic form another lease of life.

If the dynamic areas are substituted for the conventional figures it will be found that new design forms begin to develop at once and in such variety and profusion that the old sources appear barren by comparison.

Figures 12, 13 and 14 are suggested pattern motives worked on root-two rectangles. Figure 15 is an .854 shape. These patterns are new and, even as roughly indicated, could be used for almost any design purpose. They would make good inlays, tiles, embroideries, laces, fabric patterns, carvings, grilles, or architectural course enrichments. These units could be developed in a thousand ways. In fact dynamic shapes supply us with not only pattern bases but decorative units which transcend invention.

Mr. Watkins, mentioned in Mr. Finlay's letter, is a well-known English investigator of problems connected with photography. An article by him, in the *British Journal of Photography* for January 2, 1920, suggests root-two as a shape for photographic plates, as it is

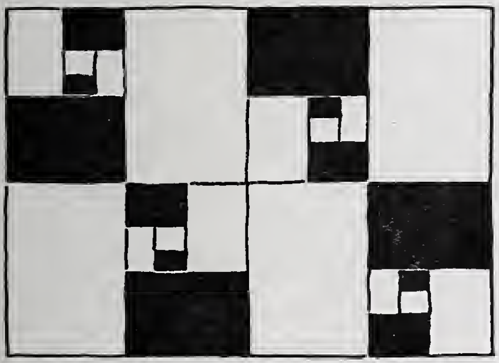


Fig. 12*a*

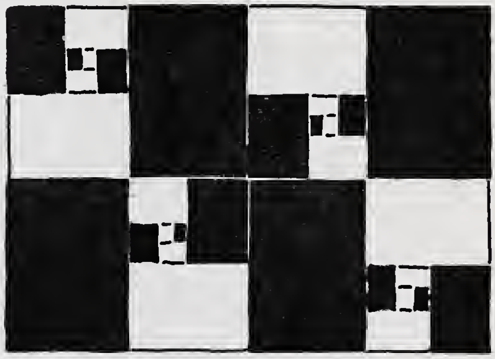


Fig. 12*b*

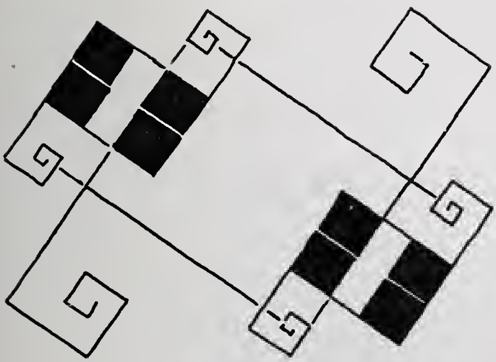


Fig. 13

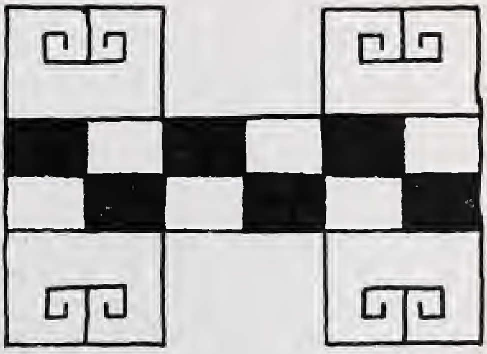


Fig. 14

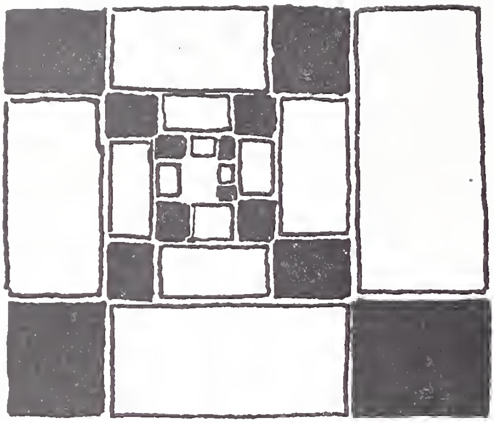


Fig. 15*a*

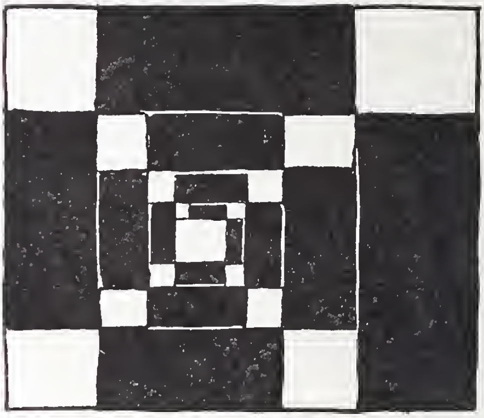


Fig. 15*b*

the only area which, when cut in half, remains similar to the whole. The shape is already in use in America by paper makers for writing paper and envelopes. It has been, for some time, a popular shape with American book designers who have studied Dynamic Symmetry. A book made on this proportion has the same shape whether open or closed. Its use, however, is limited, and, as has been mentioned in The Diagonal, probably constitutes, with the root-three rectangle, an intermediate type of symmetry between the static and the dynamic. If this be true then the dynamic is the class made by the root-five area as the parent shape and its logical derivatives. Or, perhaps, we have in root-two and root-three a minor dynamic class. At any rate the root-two rectangle was the shape first adopted by the Greeks when they changed from a linear to an area system of measurement for design purposes, or a system based upon the relationship of a side to a diagonal of a square. And this was superseded by a system which depends upon the relationship of a diagonal of two squares to a side of one of the composing units. It may be said in passing that several years ago Dr. Ostwald, the German scientist and author of "Letters to a Painter," worked out a plan for a great German World Museum and storehouse of all human knowldge which was to be erected on a bank of the Rhine. The index system of this storehouse was to consist of cards of root-two shape because the half, the quarter, the sixteenth, etc., of this form were similar in shape to the whole. Like Mr. Watkins, however, Dr. Ostwald does not seem to have understood any of the other principles of the root-two form.

According to Vitruvius (see the proportions of a Roman Atrium), the root-two shape was used in architecture in his day. Gwilt says the proportion was used during the Middle Ages for windows. (See Gwilt's "Dictionary of Architecture," Chapter on Proportion.)

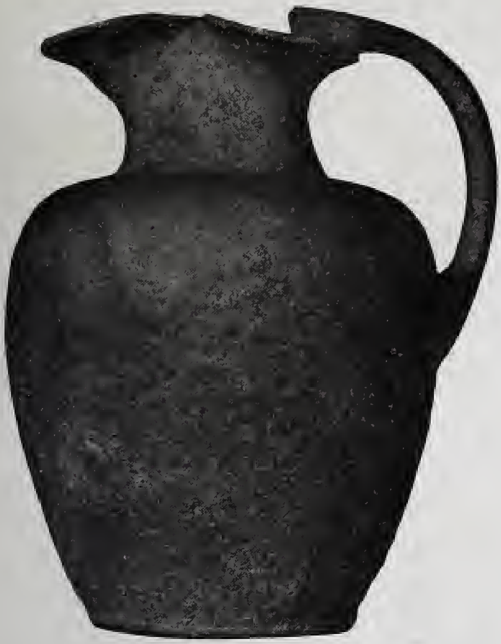


Fig. 1

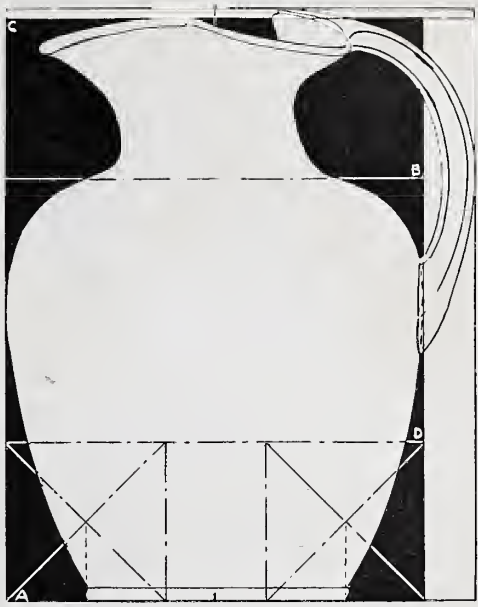


Fig. 2

BRONZE OINOCHOE FROM THE BRITISH MUSEUM

MEASURED, drawn and photographed by C. O. Waterhouse of the British Museum Staff.

A small bronze oinochoe in the British Museum supplies the following proportions:

Full height and full width, including the handle,	1.25
Height and width, without the handle,	1.382
Height with handle and width without the handle,	1.4045

AB, Fig. 2, is a square.
BC is a .382 area.
CD is a square.
AD is a .382 area.

The centers of the two small squares of this latter area fix the width of the foot. When AD is subtracted from the square AB the excess area is that of a whirling square rectangle.

ANALYSIS BY DIAGONALS

AB or CD equals a square.
AF or CE equals a 1.382 area.

Diagonals to these two shapes at intersections as X, P, O, W, Q, U and V furnish points which enable us to determine all the minor proportions of the pitcher including the tangents of the curves. This method of analysis is general; that is, any dynamic theme may be fixed by diagonals of the whole and diagonals of the composing units of the major area. As a matter of fact this is an important process for the manipulation of dynamic form synthetically and, in all probability, was the one most generally employed by the Greek designers. It is dangerous to use this method exclusively in analysis, however, as we are apt to lose the thread of reasoning which an understanding of the nature of the composing units of a dynamic area makes clear. In the next number of THE DIAGONAL the writer will explain something of the method of using diagonals for creative purposes.

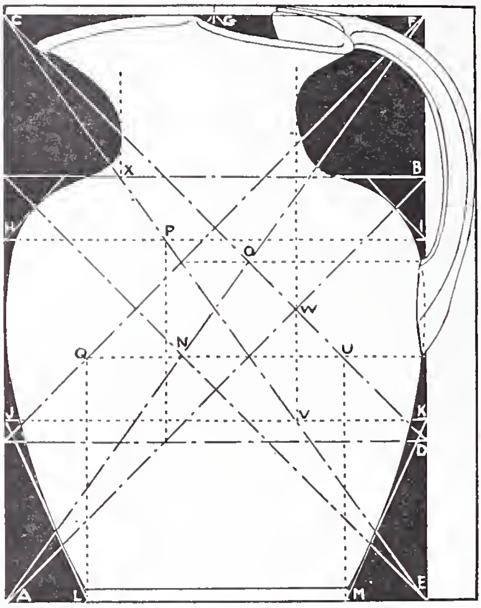


Fig. 3

THE ELEMENTS OF DYNAMIC SYMMETRY

Much of the work upon which the Elements of Dynamic Symmetry is based was done during the year 1919 under the Sachs Research Fellowship of Harvard University. This fellowship was founded by Samuel Sachs of New York City and, under the terms of the foundation, is assigned to scholars of proved ability, whether students, instructors or others, for the general purpose of advancing the Fine Arts. This acknowledgment is but a faint expression of the writer's appreciation of the benefits enjoyed from a fellowship of a type too rare in American Universities. Jay Hambidge.

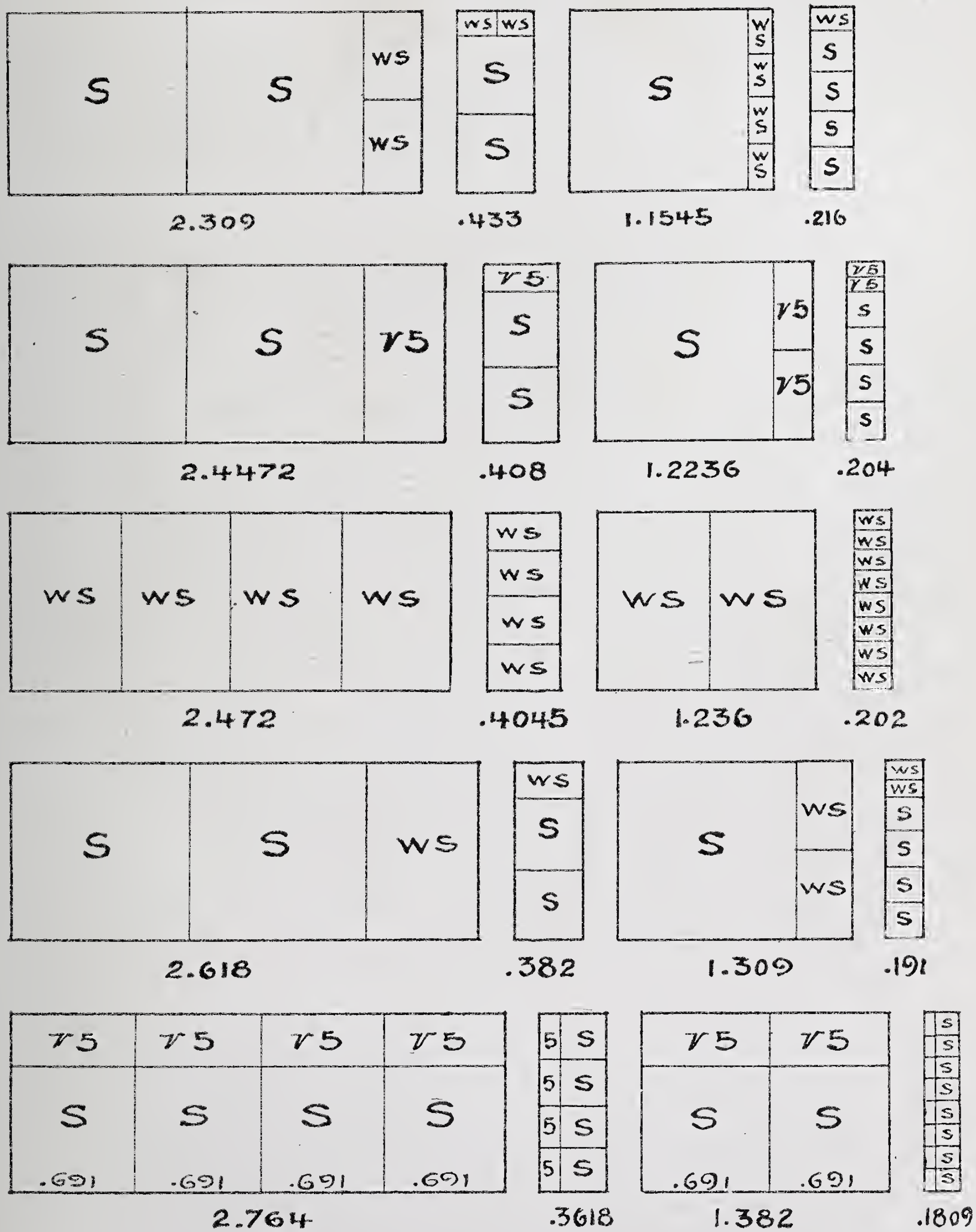
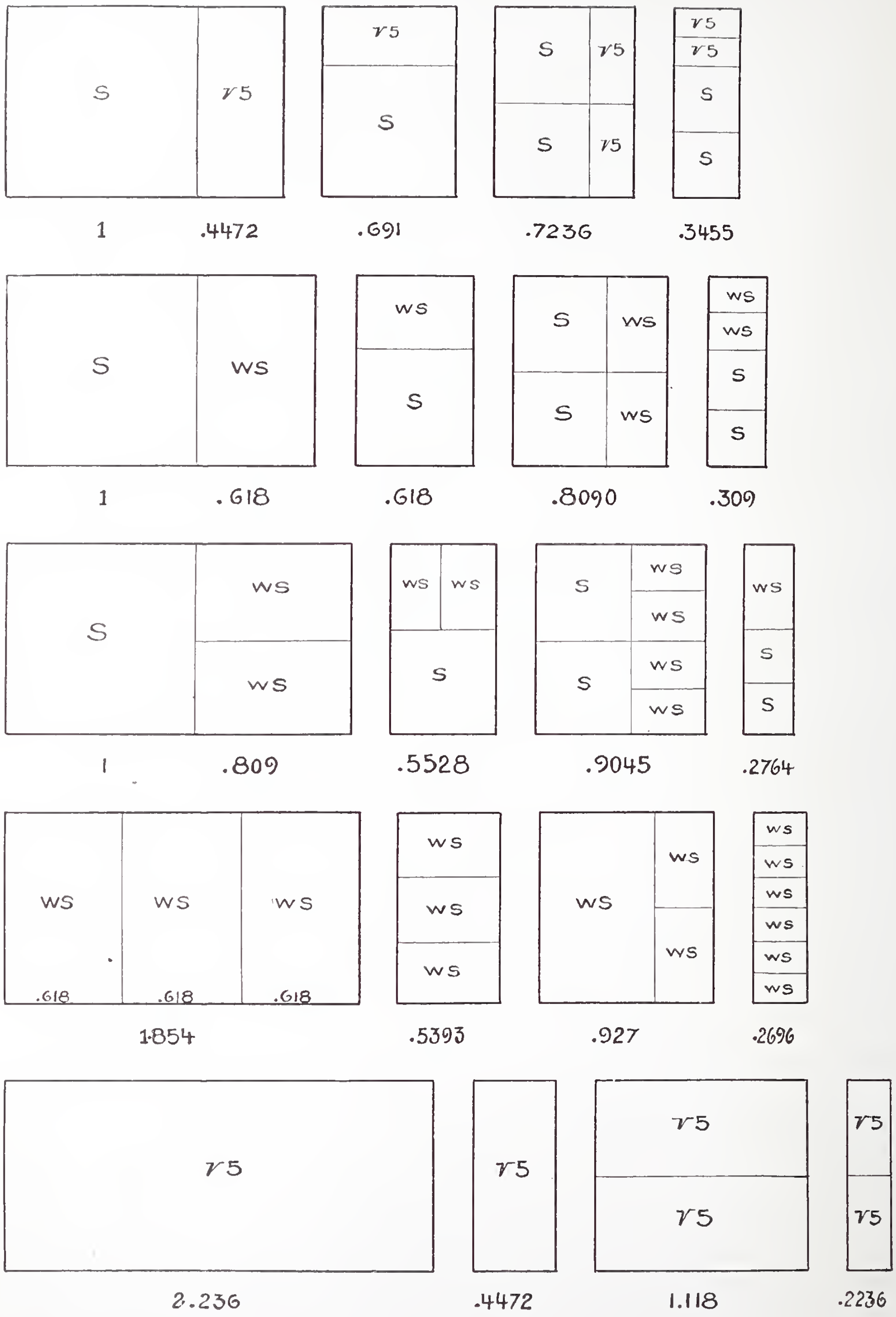


Fig. 2


S
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.309
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WS
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.809
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.5528
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WS
S
S
.2764
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.618
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.2696
√5
2.236
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.4472
√5
√5
1.118
√5
√5
.2236

Fig. 1

RATIOS MOST FREQUENTLY USED, THEIR RECIPROCALS AND SIMPLE DIVISIONS

Ratio	Reciprocal	½ Ratio	½ Reciprocal
2.309	.433	1.1545	.2165
2.4472	.408	1.2236	.2041
2.472	.4045	1.236	.2022
2.618	.382	1.309	.191
2.764	.3618	1.382	.1809
2.809	.3559	1.4045	.1779
2.8944	.3455	1.4472	.1727
3.236	.309	1.618	.1545
3.427	.2918	1.7135	.146
3.618	.2764	1.809	.1382

THE ratio 2.309 will be recognizable as 1.309 plus a square; or .309 plus two squares. The fraction .309 is identified as half of .618. In diagram form this fraction is simply one whirling square rectangle over another, end to end, as in Fig. 3.

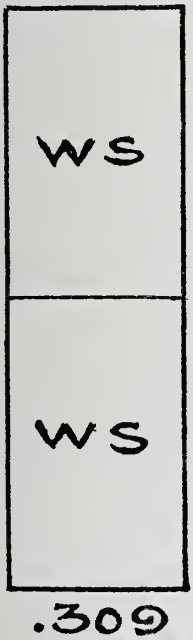

Fig. 3

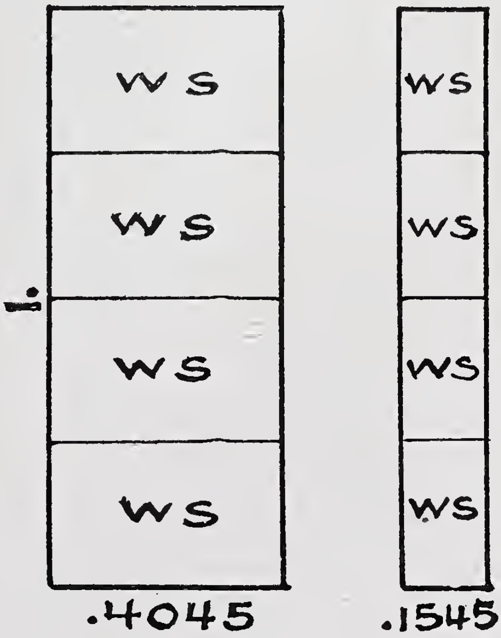

Fig. 4

If this ratio is divided by two the result is represented diagrammatically by one square plus four whirling square rectangles standing end to end on top of each other, and the new ratio is 1.1545. If these four shapes were placed on top of each other, side to side, they would be represented by the reciprocal ratio .4045, *i.e.*, one-fourth of 1.618.

Of course the 2.309 area may be divided in many other ways, but the one described is the direct or simple subdivision.

It will prove instructive to the student to work out the following subdivisions of the 2.309 shape.

.382	plus	1.927	equals	2.309	
.500	"	1.809	"	"	
.618	"	1.691	"	"	
.764	"	1.545	"	"	
.691	"	1.618	"	"	
.809	"	1.500	"	"	
.882	"	1.427	"	"	
.927	"	1.382	"	"	
.8944	"	1.4146	"	"	
1.118	"	1.191	"	"	
1.236	"	1.073	"	"	
1.4472	"	.927	"	"	etc., etc.

The ratio 2.4472, *i.e.,* two squares plus the root-five reciprocal .4472, may be subdivided by any dynamic combination which equals two; the .4472 area will take care of itself in such combinations. In its simplest form it is composed of a square plus a square and a root-five area, *i.e.,* 1 plus 1.4472. We may have combinations like the following:

.1382	plus	2.309	equals	2.4472	
.8944	"	1.5528	"	"	
1.309	"	1.1382	"	"	
1.618	"	.8292	"	"	
1.7236	"	.7236	"	"	
1.8944	"	.5528	"	"	
2.0652	"	.382	"	"	
2.17082	"	.2764	"	"	etc., etc.

The ratio 2.472 occurs frequently in Greek design. It is easily recognizable as .618 multiplied by four, or 1.236 multiplied by 2. Other combinations are:

1.382	plus	1.090	equals	2.472	
1.545	"	.927	"	"	
1.618	"	.854	"	"	
1.708	"	.764	"	"	
1.927	"	.545	"	"	
2.045	"	.427	"	"	
2.236	"	.236	"	"	etc., etc.

The most natural subdivision of the 2.618 area is that of a square plus a whirling square rectangle. Other arrangements or composing elements of this shape are:

1.309	plus	1.309	equals	2.618
1.382	"	1.236	"	"
1.427	"	1.191	"	"
1.4472	"	1.1708	"	"
1.472	"	1.146	"	"
1.5278	"	1.090	"	"
1.691	"	.927	"	"
1.7236	"	.8944	"	"
1.764	"	.854	"	"
1.809	"	.809	"	"
1.854	"	.764	"	"

2.000	plus	.618	equals	2.618	
2.236	“	.382	“	“	
2.309	“	.309	“	“	
2.4472	“	.1708	“	“	
2.472	“	.146	“	“	etc., etc.

The 2.618 ratio is one of the most engaging of the dynamic series. It, apparently, is closely connected with the leaf distribution phenomena. Also, it is the square of 1.618, *i.e.*, this latter ratio multiplied by itself equals 2.618.

The 2.764 area will readily be recognized if divided by 2. This operation cuts it into two 1.382 shapes. Some of its composing units are:

1.382	plus	1.382	equals	2.764	
1.472	“	1.292	“	“	
1.528	“	1.236	“	“	
1.691	“	1.073	“	“	
1.764	“	1.000	“	“	
1.809	“	.955	“	“	
1.854	“	.910	“	“	
2.000	“	.764	“	“	
2.236	“	.528	“	“	
2.309	“	.455	“	“	
2.472	“	.292	“	“	
2.618	“	.146	“	“	etc., etc.

[To be continued.]

SEE ALSO LESSONS ON THE ROOT-TWO RECTANGLE

Editor of The Diagonal.

Dear Sir: I am very much interested in the theory of Dynamic Symmetry and should be grateful if you would explain some of the simple subdivisions of the root-two rectangle.

J. C. Aiken, London, England.

Root-two or 1.4142 may be subdivided in many interesting ways. For example, we may use the applied squares on the ends, or an applied square on the side. It is extremely easy to construct this shape, as all that is needed is a square and one of its diagonals, the side of the root-two area being this diagonal line.

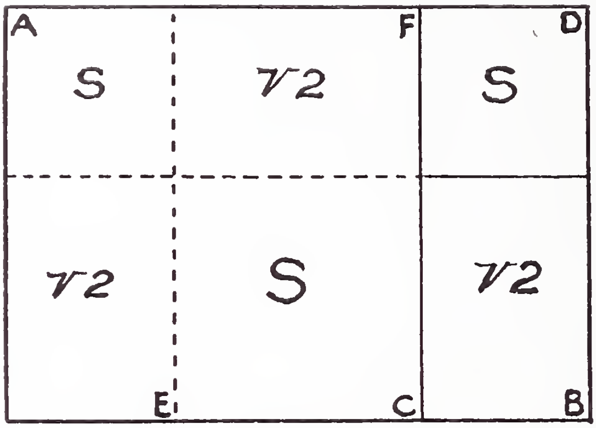

Fig. 1

In AB, Fig. 1, AC is a square as is also DE. The excess areas are CD and EA and each of these is composed of a square and a root-two figure. EF is composed of a square and a root-two form. Any one of the subdivisional areas or composing units may again be subdivided in the same way as was the overall shape and the defining lines of such subdivisional areas may be produced into a neighboring area as:

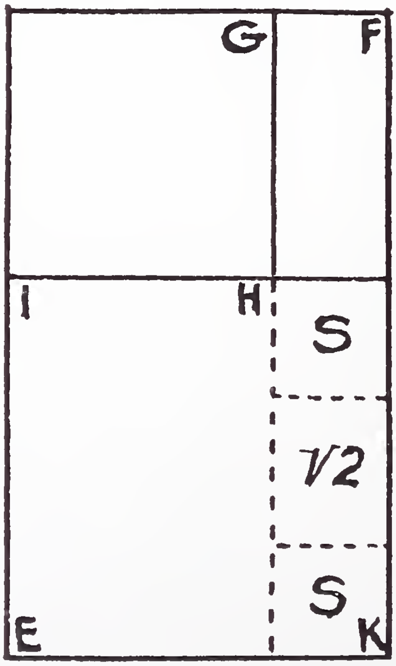

Fig. 2

IG is a square applied to the root-two area IF. If GH is produced to J then IJ is a root-two figure in the square IK. Figures in IK or IF reverse themselves in a neighboring area. This is a very important principle.

If a square is applied to the side of a root-two area, *i.e.,* a square equal to a square on an end, another interesting subdivision follows.

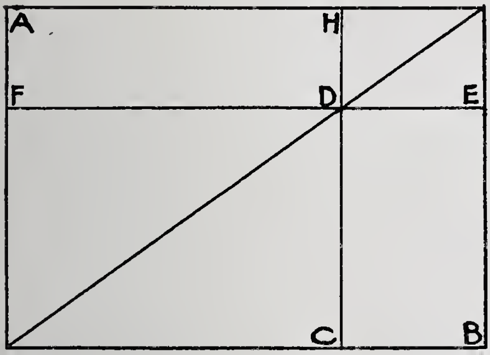

Fig. 3*a*

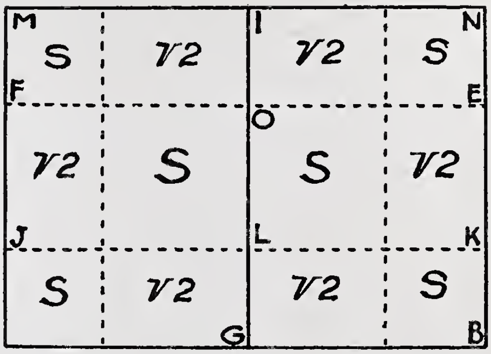

Fig. 3*b*

AC, Fig. 3*a,* is an applied square on the end of the root-two area AB. A side of this square, as HC, cuts a diagonal of the whole at D. The line FE, through D, defines FB, an area equal to the area of the square AC. But FB, while equal to the area of the square AC, is composed of two squares, as FG, GE in Fig. 3*b*. The line IG divides MB into two equal parts, *i.e.,* MG, GN are two root-two figures and each has a square applied as FG, GE. The reciprocal of a root-two rectangle is equal to half of the major area. MG, GN are reciprocals to MB and each is subdivided, as was the major figure AB, Fig. 3*a*.

If we apply the area FB to the line MN, Fig. 3*b,* the area MK results and this overlaps the area FB to the extent of the area FK. This is composed of two root-two figures and two squares.

If we use half of a diagonal of a square applied to the end of a root-two rectangle to determine a root-two area within the square this root-two form is equal to half of the major area, *i.e.,* it is a reciprocal.

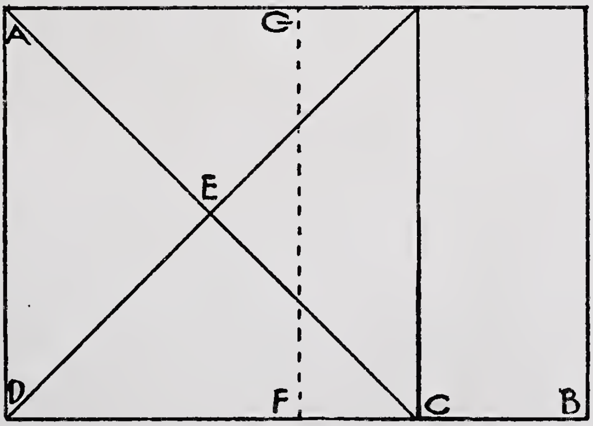

Fig. 4

AB, Fig. 4, is a root-two rectangle and AC is a square applied on an end. DE is half a diagonal of the square. DF is made equal to DE. From F draw FG, parallel to DA. The area DG is a root-two area equal to half of AB and therefore is a reciprocal. The area GC is composed of two squares and a root-two rectangle.

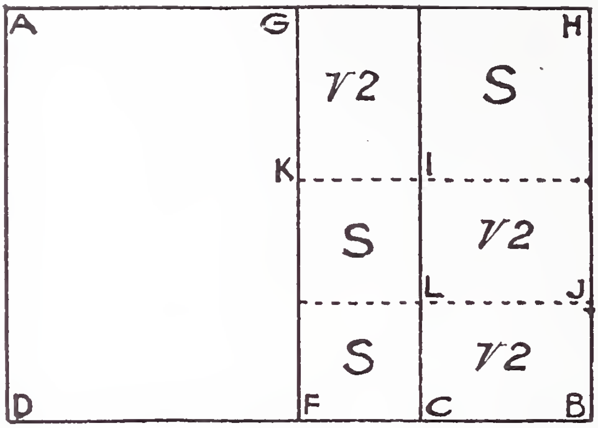

Fig. 5

In Fig. 5 the area CH is composed of the square IH and the root-two rectangle IB. The area CG is composed of the two squares FL, LK and the root-two rectangle IG. If the sides of the squares FL, LK are produced to the line HB the reciprocal area GB is divided into three squares and three root-two rectangles.

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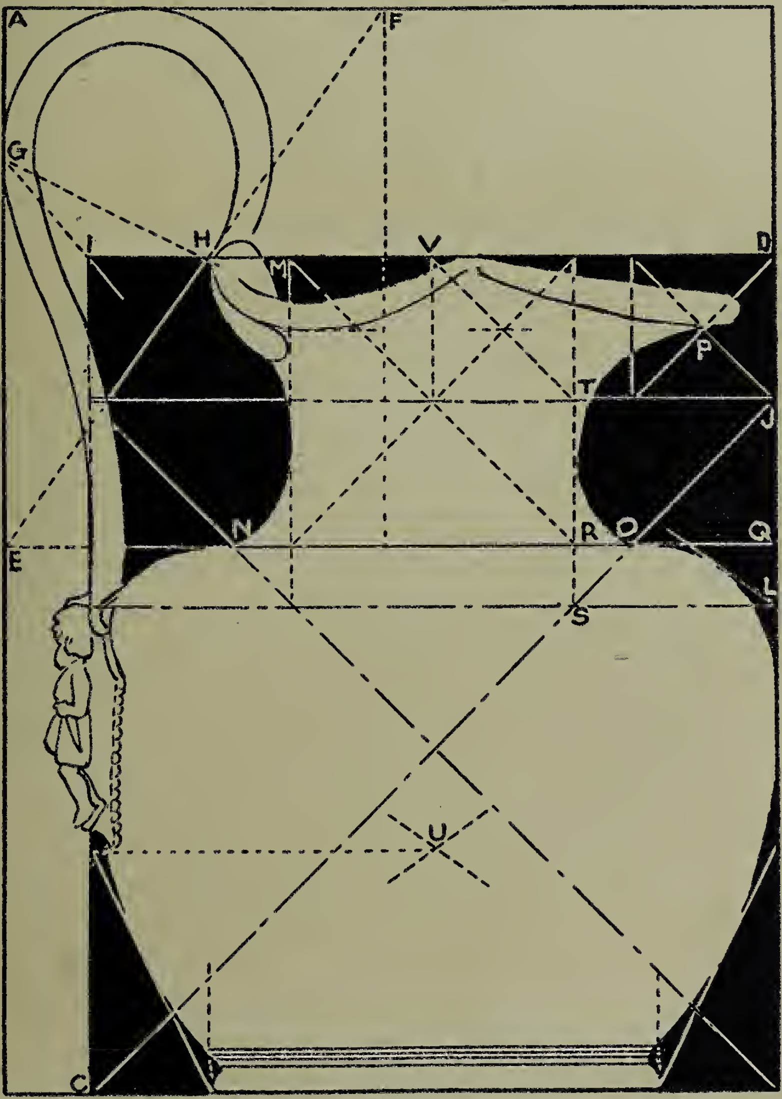
AN ILLUSTRATED MONTHLY MAGAZINE DEVOTED TO THE EXPLANATION OF THE REDISCOVERED PRINCIPLES OF GREEK DESIGN, THEIR APPEARANCE IN NATURE AND THEIR APPLICATION TO THE NEEDS OF MODERN ART

EDITED BY JAY HAMBIDGE



Small Bronze Oinochoe, British Museum, W. T. 656

NO. 10 AUGUST, MDCCCCXX VOL. I

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By JAY HAMBIDGE

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THE DIAGONAL

THE TRANSITIONAL DESIGNERS of CLASSIC GREECE

BEFORE we can appreciate the astonishing products of Greek design of the great period, it is necessary that we should consider with some care the signed works of those men who mark the transition from the Archaic Greek to the time of full classic flower. The men whose work we must especially consider in this connection are Amasis, Exekias, Tleson, Nikosthenes and Andokides. In the Louvre there is an extraordinarily handsome oinochoe signed by Amasis and in the Bibliothèque Nationale a remarkable amphora. The oinochoe in the Louvre has, on one side, a mysterious mark which, so far, has baffled the archaeologists. As we shall see later, this mark supplies a possible means of recovering an actual Greek method for the determination not only of general proportions but the subtle and beautiful curve forms. The Louvre also possesses one important amphora by Exekias, three by Andokides, one unsigned, and some twenty-five signed works by Nikosthenes. The latter we shall consider first, as they furnish us the best record so far obtained of the work of a man who changes his symmetry method. Nikosthenes apparently began his career by using static proportion and later changed to dynamic. There is no question about the superiority of this latter work. The archaic designer Tleson has already been discussed in THE DIAGONAL.

The measurements of these designs were made by Mlle. Jeanne Evrard of the Louvre at the suggestion of M. Pottier, Keeper of Greek vases. Mlle. Evrard knew nothing of Dynamic Symmetry. She made the measurements while the writer was absent from Paris, consequently we may regard her work as quite without prejudice.

The study of these amphorae by Nikosthenes in the Louvre, shows us that he was constantly varying his proportions, without, however, departing far from a general base or form recipe which seems to have been original with him. Across the body of these amphorae we notice three emphasized lines which appear to have been the subject of much thought. One of these lines marks the juncture of the neck with the bowl; the other two are raised bands formed by the fabric of the vase body itself. In every Nikosthenes amphora so far inspected, these lines are part of the general proportioning scheme. The distance from the base to the line made by the juncture of the neck with the bowl in F 106 is equal to the width, including the handles. This width is .204. The height measurements are given by Mlle. Evrard at I and show that the height of this line from the base, with the full width, produces the square IJ. The line from I to O is a diagonal to two squares. It cuts the topmost band at P. The construction shows that the height of this band, from the base, with its width, produces another square. The width of this band varies for four diameters from .166 to .168; the heights from .166 to .171. The variations in the figures show us the exact error of the architectural construction.

KJ is a diagonal to the major area. Two such diagonals cut each other at L, the center of the shape. The second band-line passes through this point.

The overall or major area is composed of a square and a third of a square. If the full width is .204 then the height is .272. In diagram 1*a*, the height measurements made by Mlle. Evrard are shown at the left; the widths at the right. Even slight inspection will show us that these figures, without a geometrical plan, furnish the exact symmetry of the amphora.

The width of the foot and its height up to the base of the ring, which marks the juncture of foot and bowl, give us the side and end of three squares.

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In Fig. 1*b* the relationship of these three squares with the rest of the design is made more apparent. HG is a square and BP three squares or one-third of HG. The line QR is equal to RS. The point E is the center of the area PQ. This area, equal to one-half of the area of three squares PB, is itself equal to a square and a half. EQ is equal to one-fourth of PQ and consequently is a square and a half or a similar figure.

AB is a square and HO one-third of that area or three squares. IJ is one of these squares. It was doubtless intended that the width of the neck at its narrowest should be one-third of the total width. The points L and K are fixed by diagonals to the area DN, *i.e.*, an area

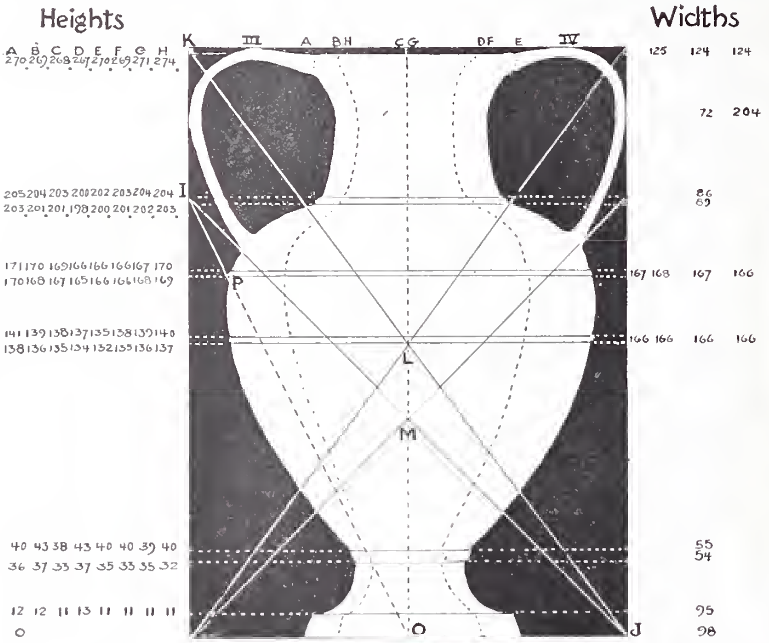


Fig. 1*a*

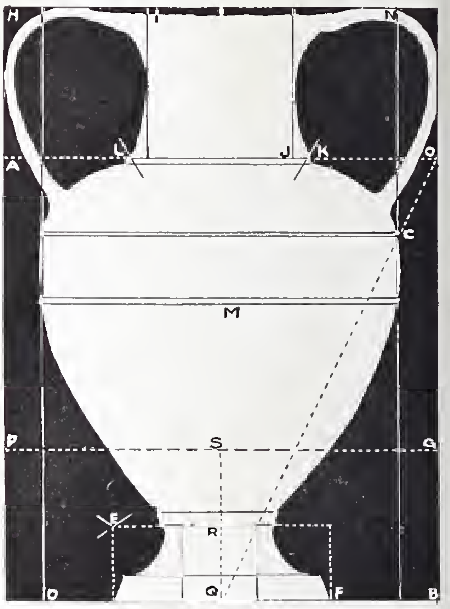


Fig. 1*b*

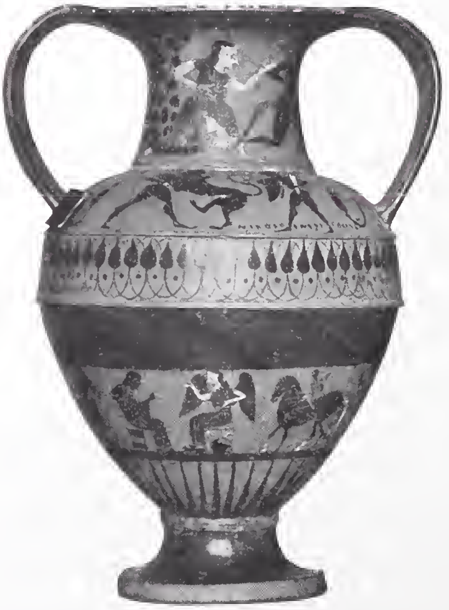


Fig. 1*c*

defined by the height of the vase and its width, minus its handles, or an area evidently intended to be one and two-thirds.

Amphora F 101, like F 106, has the neck and bowl juncture fixed by the total width. BT is a square and BR a diagonal to its half. This line cuts the upper body-band at Q. This band is the side of another square.

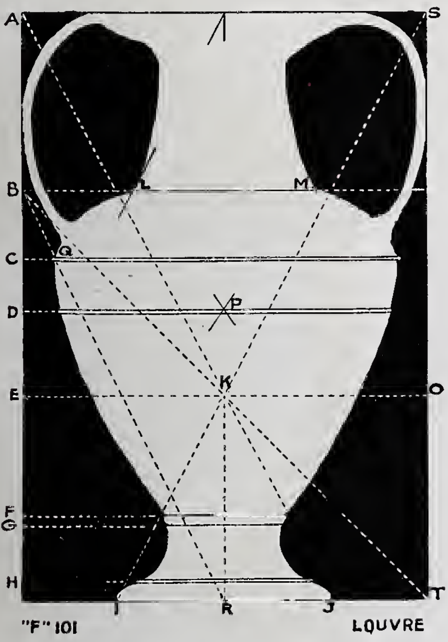


Fig. 2*a*

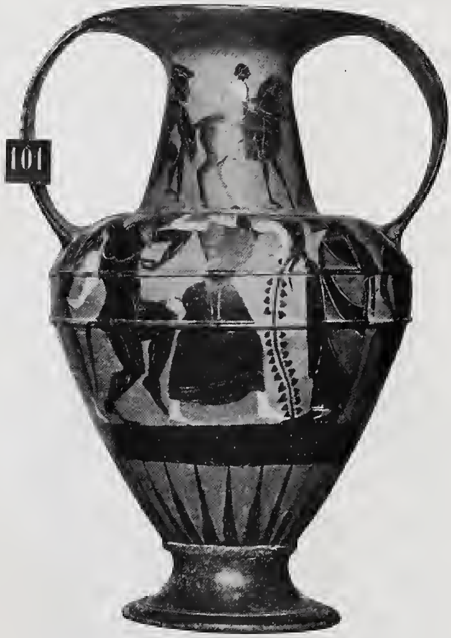


Fig. 2*b*

The point P is the center of the major area. This fixes the height of the lower body-band.

K is the center of the square BT. Lines drawn from A and S through K and continued to I and J determine the width of the foot. The height of the foot, to the base of the ring joining it with the body, is one-third of the foot width; or this height and width give us three squares, see F 106.

From the center of the lip of the vase draw lines to E and O. These lines fix the width of the neck and body juncture on a side of the square BT, at L and M.

Judging from the other similar amphorae in this collection, the overall shape is composed of the square BT and the area BS, this latter consisting of two squares and a third of a square.

The measurements of the vase are:

Heights A .309, .309, .310, .307, .308, .306, .310, .309.
" B .209, .209, .210, .207, .215, .203, .210, .210.
" C .177, .178, .182, .179, .184, .181, .184, .182.
" D .156, .153, .153, .153, .153, .156, .157, .152.
" F .046, .048, .044, .045.
" G .042, .037, .038, .040, .034.
" H .013, .010, .010, .011.

Widths A .216.
" B .096.
" C .187.
" D .181.
" F .071, .071, .071, .071.
" G .064, .062, .061, .062.
" H .096 (see neck with bowl).

Foot width .114.

Note the unusual arrangement of figures on the bowl.

Amphora F 107 shows us a peculiar if not successful experiment. The handles are compressed to equal the width of the body.

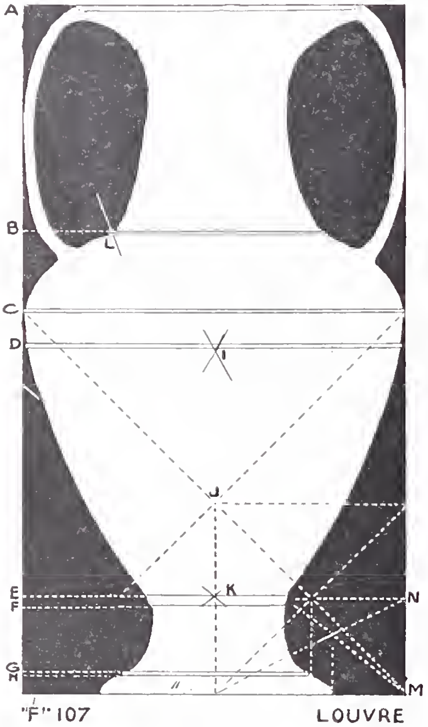


Fig. 3*a*

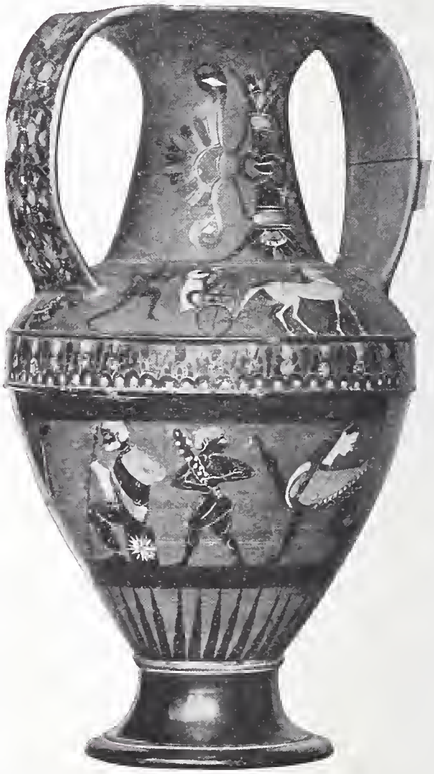


Fig. 3*b*

The upper body-band defines the square CM. I is the center of the major area and fixes the height of the lower body-band.

The square JM fixes the proportions of the foot.

The proportions of the neck and body juncture are determined in the same manner as F 103.

The overall proportion is one and four-fifths, .315 divided by .177. A diagonal to the four-fifths area above the square CM cuts a line from J to A at L.

Mlle. Evrard's measurements for this vase are:

Heights A .318, .318, .316, .315, .318, .314, .316.
" B .212, .212, .211, .214, .211, .215, .211, .213.

" C .178, .178, .183, .178, .181, .178.
" D .158, .160, .151, .161, .157, .158, .156, .153.
" E .040, .041, .041, .046, .045, .041.
" F .038, .035, .039, .038, .039.
" G .007, .010, .010, .009, .011.
" H .006, .008.
Widths A .177.
" B .098, .098, .097, .100.
" C .178.
" D .175.
" E .067.
" F .063.
" G .086.
Foot widths .107, .108, .108, .107.

The overall area of amphora F 103 is composed of a square and two-thirds, .185 divided into .3082.

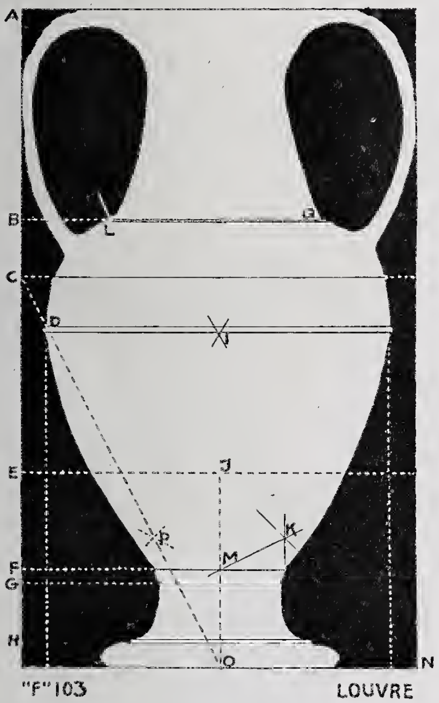


Fig. 4*a*

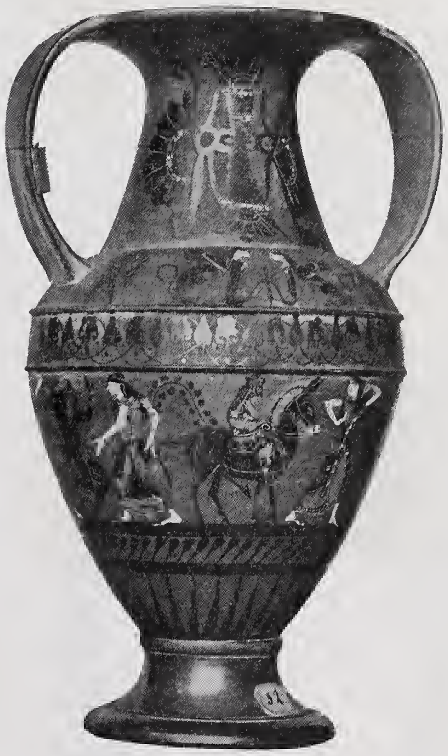


Fig. 4*b*

Height of the upper body-band, with the full width, gives us the square CN.

CO is a diagonal to half this area; it cuts the lower body-band at D. This band, with the line IO, furnishes another square. I is the center of the major shape.

EN is composed of two squares. M is the center of this area. A line from E to N cuts CO at P. The points P and K fix the width of the foot and bowl juncture.

The foot width was probably intended to equal one and two-thirds of the full width.

J is the center of the square CN. Lines from J to A cut diagonals to the two-thirds area above the square CN, at L and Q, to fix the height and width of the neck and bowl juncture.

Mlle. Evrard's measurements for this vase are:

Heights A .308, .310, .307, .310, .307, .309, .307, .307.
“ B .210, .211, .208, .215, .209, .210, .208, .212.
“ C .184, .187, .183, .186, .182, .185, .180, .183.
“ D .160, .161, .158, .161, .158, .161, .157, .159.
“ F .043, .045, .042.
“ G .037, .040, .036, .036.
“ H .012, .013, .013, .013.

Widths A .185.
“ B .100.
“ C .159.
“ D .163.
“ F .060.
“ G .055.
“ H .86.

Foot width .112.

In amphora F 104 we see a characteristic variation on F 106. The square CJ is made by the top of the upper body-band. This band, as will be seen by the photograph, is considerably distorted, a defect probably caused in baking. One of the heights, however, is given by Mlle. Evrard as .203, the exact full width.

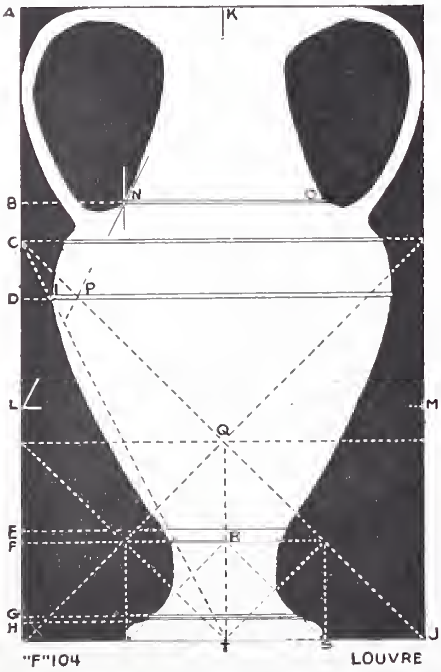


Fig. 5*a*

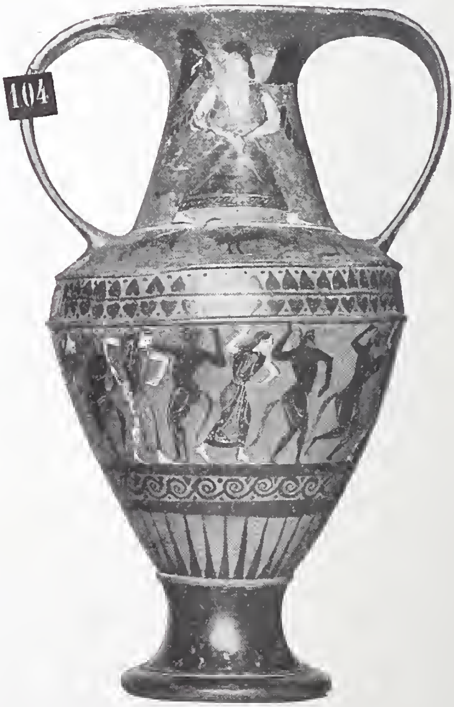


Fig. 5*b*

The line CT is a diagonal to half a square and it cuts the lower band-line at P. The heights of this line from the base are:

.176, .172, .172, .173, .175, .173.

The widths: .173, .173, .175, .173.

The height of the foot to the bottom of the ring covering the juncture of the foot and bowl is: .050, .053, .050, .053, .053.

The foot width is .101. This height and width are the end and side of an area composed of two squares.

The lower band-line length, with its height from the base gives us another square.

The full width is .203. The foot width was undoubtedly intended to equal one-half the full width.

The major area, in excess of the square CJ, is equal to a square and three-quarters. This is interesting as it shows a method of handling static proportion analogous to that employed with dynamic shapes shortly after the transitional period. When Dynamic Symmetry appeared in Greece the ground was apparently thoroughly prepared for its reception.

The widths of the juncture line of neck and bowl at the base are given as .102, .103, .103, .102. This line is equal to the width of the foot.

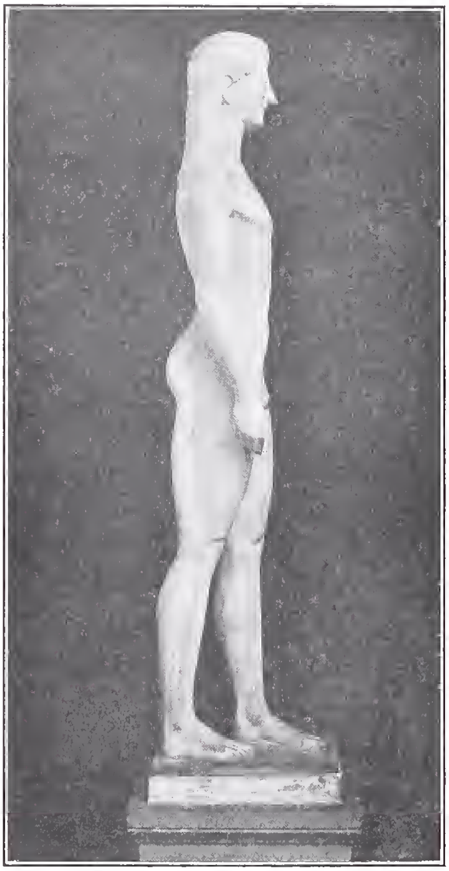
AM is a square applied from the top. KL and KM are diagonals to half this square. These lines cut the neck and body juncture at N and O and fix its width. Mlle. Evrard's measurements are:

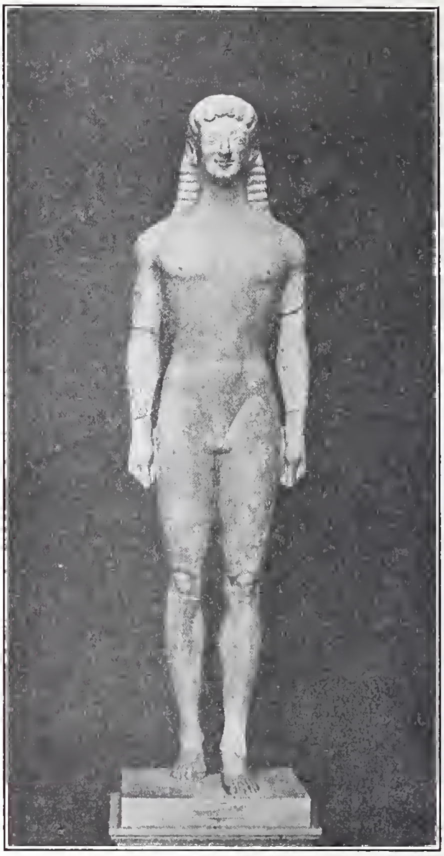
Heights A .319, .321, .318, .322, .319, .321.
" B .219, .217, .220, .218, .222, .221.
" C, bottom of band, .200, .196, .202, .201, .197, .197.
" D .176, .172, .176, .172, .173, .175, .173.
" E .057, .055, .058, .062, .061, .058.
" F .050, .053, .050, .052, .056, .053.
Widths B .102, .103, .103, .102.
" C .165, .168, .166, .163.
" D .173, .173, .175, .173.
" F .064, .064.
" G .079.
" H .083.
Foot width .101.
Full width .203.

[To be continued.]

From an unsigned and unnumbered figured cup in the Louvre.

DYNAMIC SYMMETRY OF THE HUMAN FIGURE FOR ADVANCED STUDENTS





OF all the examples of very early Greek archaic sculpture of the figure the celebrated Tenea Apollo in the Museum at Munich is in the most perfect condition and furnishes, therefore, an admirable subject for symmetry analysis. The statue was found in 1846 on the site of Tenea, near Corinth. It was originally in the possession of Baron Prokesch von Osten, from whom it passed, in 1854, to the Glyptothek, Munich, where it now is. The measurements on which the present analysis is based were taken from a full-sized cast in the Museum of Fine Arts, Boston. When found the statue was in almost perfect preservation, only a piece of the right arm being missing. The figure, however, as we learn from the abundant remains of similar statues, belongs to a type the original of which is supposed to be Egyptian. The statue of Amenophis IV of Egypt is such a prototype. In both cases we have the same general attitude of a man standing stiffly upright, both hands held against the body and the left foot in advance of the right. According to the Sicilian historian Diodorus, the early Greek sculptors acquired knowledge of their canon for figure proportions from the Egyptians. The Glyptothek catalogue gives this description—"Apollo of Tenea,

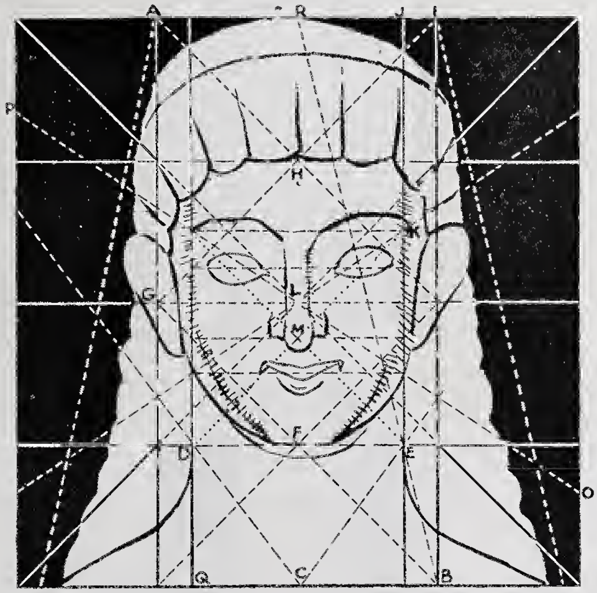
Height 1.53 m.

Length of face 0.142 m.

"In the determination of the bodily proportions the foot of the figure seems to have served as the basic measurement. The lower legs are two feet long, the height of the head is one foot. From the end of the chin to a line connecting the nipples, from there to the navel, and from the navel to the fork, is each one foot long.

"The statue is to be dated in the neighborhood of 600 B. C. Probably a work of Dipoinos and Skyllis, or of their school." Others date it considerably later.

We cannot say that the length of the foot was the basic measurement, as the feet are of different lengths. The right is .2020 while the left is .2075. When we measure the full height and greatest width, we find that the lesser measurement is contained in the greater three and two-thirds times. The height is 1.5352 while the width is .4187.* Assuming that the statue was cut from a prepared rectangular block of marble this block, if it exactly contained the figure, would have as front elevation an area composed of three squares and two-thirds of a square; the side elevation would be a proportional part of the front elevation, as shown in III of Fig. 1. When we divide the front elevation area as in I, Fig. 1, we find that we can actually draw the figure with great precision. The area AD is composed of the three squares, while DE is two-thirds of a square. The top line of the three squares AD passes through the base of the neck.



Head of the Tenea Apollo

This is a square taken from the center of the two-thirds area of the front elevation. AB is composed of two squares. DC, CE are diagonals to the half of the two-thirds area. Lines through E and D fix the two-square area DJ. DK is one of these squares and M is its center. F and H are apparent. P and O and its companion line in the other direction are diagonals to the entire two-thirds area. The complicated lines of this diagram are really not necessary, as the proportions could be fixed with a few of them. They are given to show that there is a persistent interrelationship of proportion.

If we apply the square EO to the top of the façade rectangle, the center of this square is the point of the chin. The diagonals of this square cut the top of the three squares AD at G and H. These two points enable us to divide the area ED into three equal parts, each

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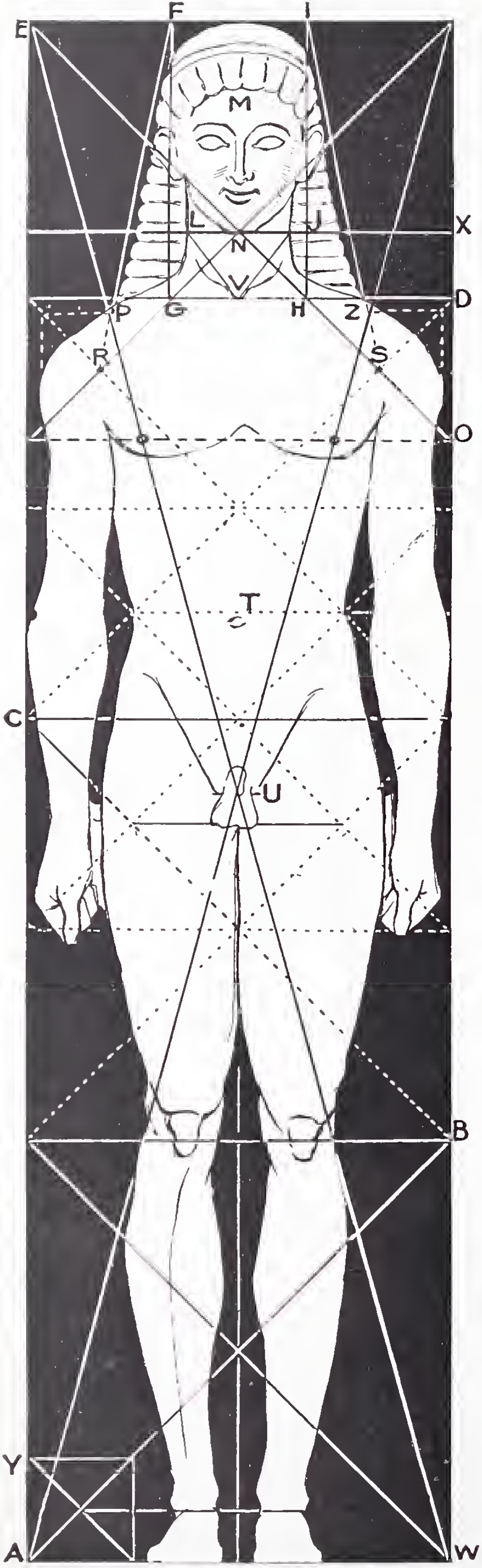
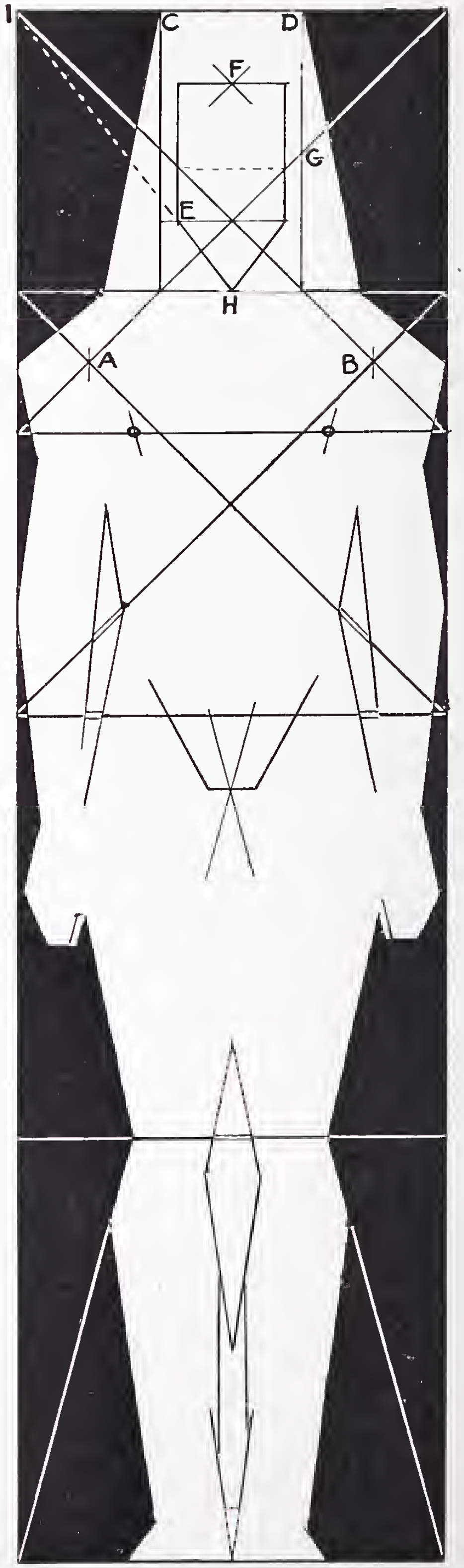
* The slight variation between the Munich height measurement and that on which this analysis is based is justified by the fact that the figure is not standing level. The 1.5352 height is more correct than 1.53 of the Munich catalogue.

I



II



consisting of two squares, EG, GI and ID. The area GJ is composed of two squares, JF a square and a half. M, the line of the hair on the forehead, is the center of one of the squares of FH. The center of the other is N. The lines BL and BJ are diagonals to half of the area ED. The points L and J give us the width of the neck and enable us to fix the proportions of the face. The large scale head of Fig. 2 shows this in detail. AB is the two-square area of I, Fig. 1, and DC, CE are diagonals to half of the two-thirds area. Lines through these points fix the new double-square area DJ of Fig. 2. The simplest method for determining this latter area would be to draw a line from B to R, this point being midway between A and I. The line RB is a diagonal to four squares. EB is also composed of four squares. RB was evidently intended to pass through the center of the eye. One eye is a trifle higher than the other, so this point must remain unsettled. When the figure façade is charted out as described we notice the following points:

The nipples are fixed on the base of the square EO by diagonals to the whole cutting this line. The exact center of the major rectangle is at U. The waist, at its narrowest, is equal to half the width of the figure or the full length of the head from the chin to the crown. The hands and arms are unequal, the right being slightly longer. The hands were probably intended to reach a line through the center of the square CB. The lower legs are equal to the full width of the rectangle, a side of the square AB passing through the patella. The width of the shoulders is fixed by P and Z. It was undoubtedly intended that the length of the foot should be equal to half the full width or to half of a side of a unit square, but the statue from the knees downward is overworked. From the knees upward the statue rigidly follows a static canon. From the knees to the base it would seem that the sculptor studied an actual model. Both feet and lower legs have the characteristic appearance of a slightly timid overworking, as we find in the arm of the Delphi Charioteer. This overworking would account for the length difference between the feet.

Fig. 1 is an illustration of a possible marking off of the façade of the marble preparatory to sawing off or pointing out excess material. It is apparent from this arrangement that the finishing of the statue would have been a fairly simple matter.

The statue furnishes us with all the characteristic features of a canon of proportion or symmetry but it is static. The statue of Amenophis IV, undoubtedly a prototype, is dynamic. The ratio for the Tenea Apollo is 3.666; for the Egyptian prototype it is 3.618. These two statues show us clearly that the Greeks, in the Tenea figure, had only the shell of the Egyptian system. It must have been soon after this, however, that they acquired full knowledge. When this happened it did not take them long to go far ahead of their Egyptian masters.

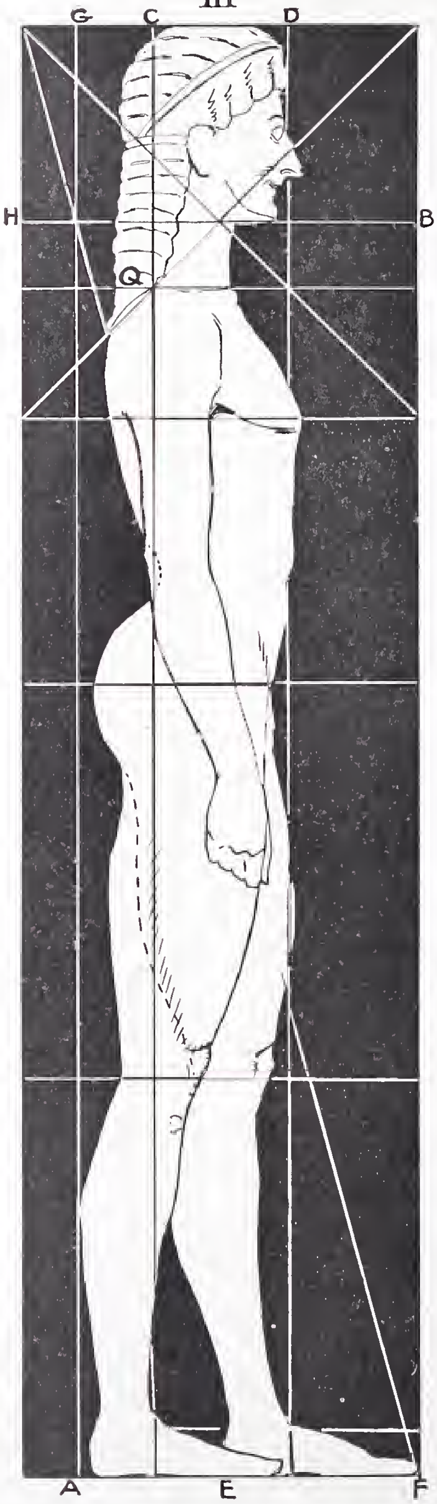
In III, Fig. 1, we have a side elevation of the Apollo. A diagonal to the whole, *i.e.*, the three and two-thirds area of 1, cuts the line BH to fix the area GF. The area AB is a similar figure to the whole.

In IV and V, Fig. 1, the front and side are shown in their relationship to what would be a modern subdivision of the overall area into thirty-three squares or an area three by eleven. Very little of this arrangement seems to have been used by the Greek sculptor.

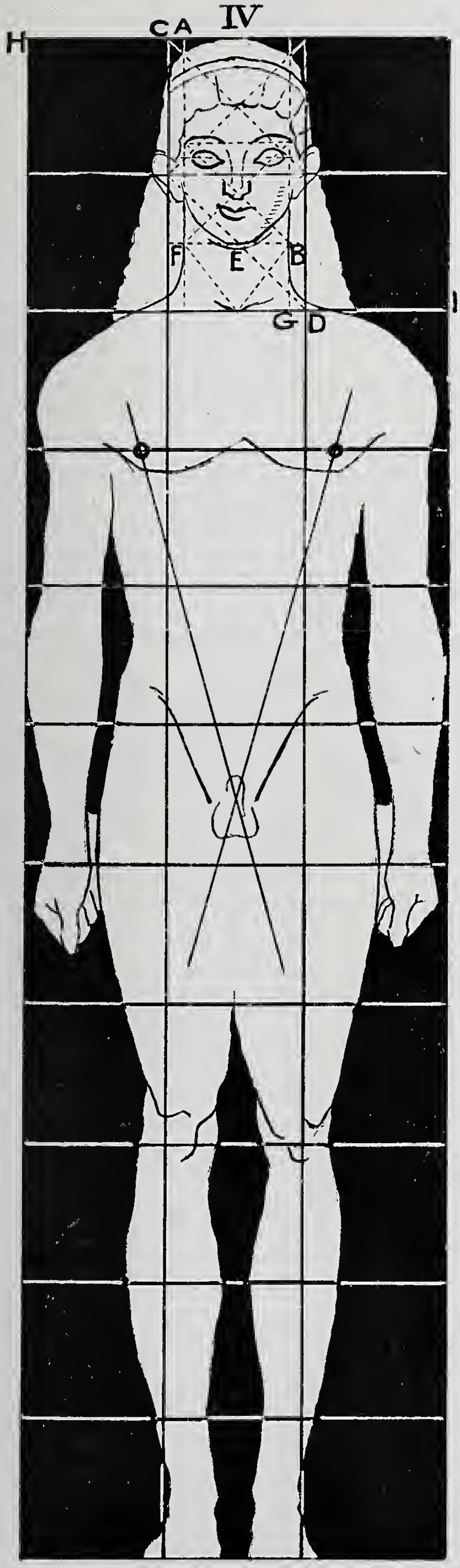
Artistically the Tenea Apollo can have but slight interest for us, but historically it is most valuable. It is from sources such as this that we shall learn the ways and means employed by the Greeks when they started on the road which led to such astonishing achievement in art. It will be noticed that this early archaic sculptor apparently used a process of area subdivision of a like character to that which we find in the work of the designer Nikosthenes or Tleson. In fact, the method of area manipulation which we find in the old Greek static design shows a high degree of ingenuity and design cunning. Nikosthenes, in his early work, is apparently constantly alert for subtle and hidden squares. These early designers seem to have thoroughly understood the effect of area union on generating forms. In fact, the cunning we find in Greek static design is quite of a piece with that observable in the dynamic fabric. The ground was well prepared for the reception of the great idea later.

[To be continued.]

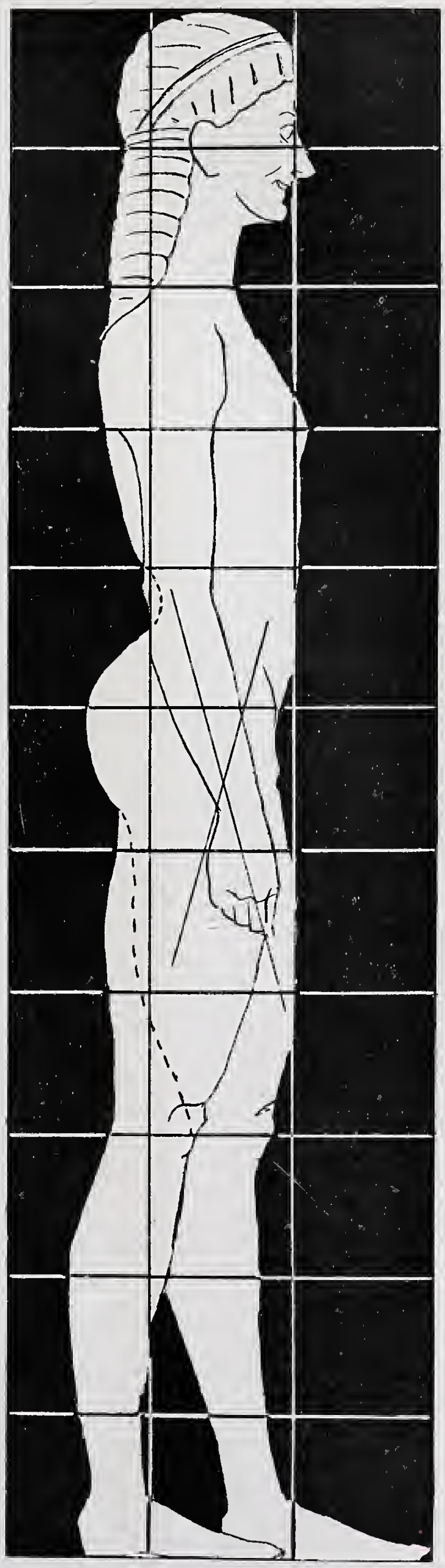
III



IV



V



THE ELEMENTS OF DYNAMIC SYMMETRY

Much of the work upon which the Elements of Dynamic Symmetry is based was done during the year 1919 under the Sachs Research Fellowship of Harvard University. This fellowship was founded by Samuel Sachs of New York City and, under the terms of the foundation, is assigned to scholars of proved ability, whether students, instructors or others, for the general purpose of advancing the Fine Arts. This acknowledgment is but a faint expression of the writer's appreciation of the benefits enjoyed from a fellowship of a type too rare in American Universities. JAY HAMBIDGE.

RATIOS MOST FREQUENTLY USED, THEIR RECIPROCALS AND SIMPLE DIVISIONS

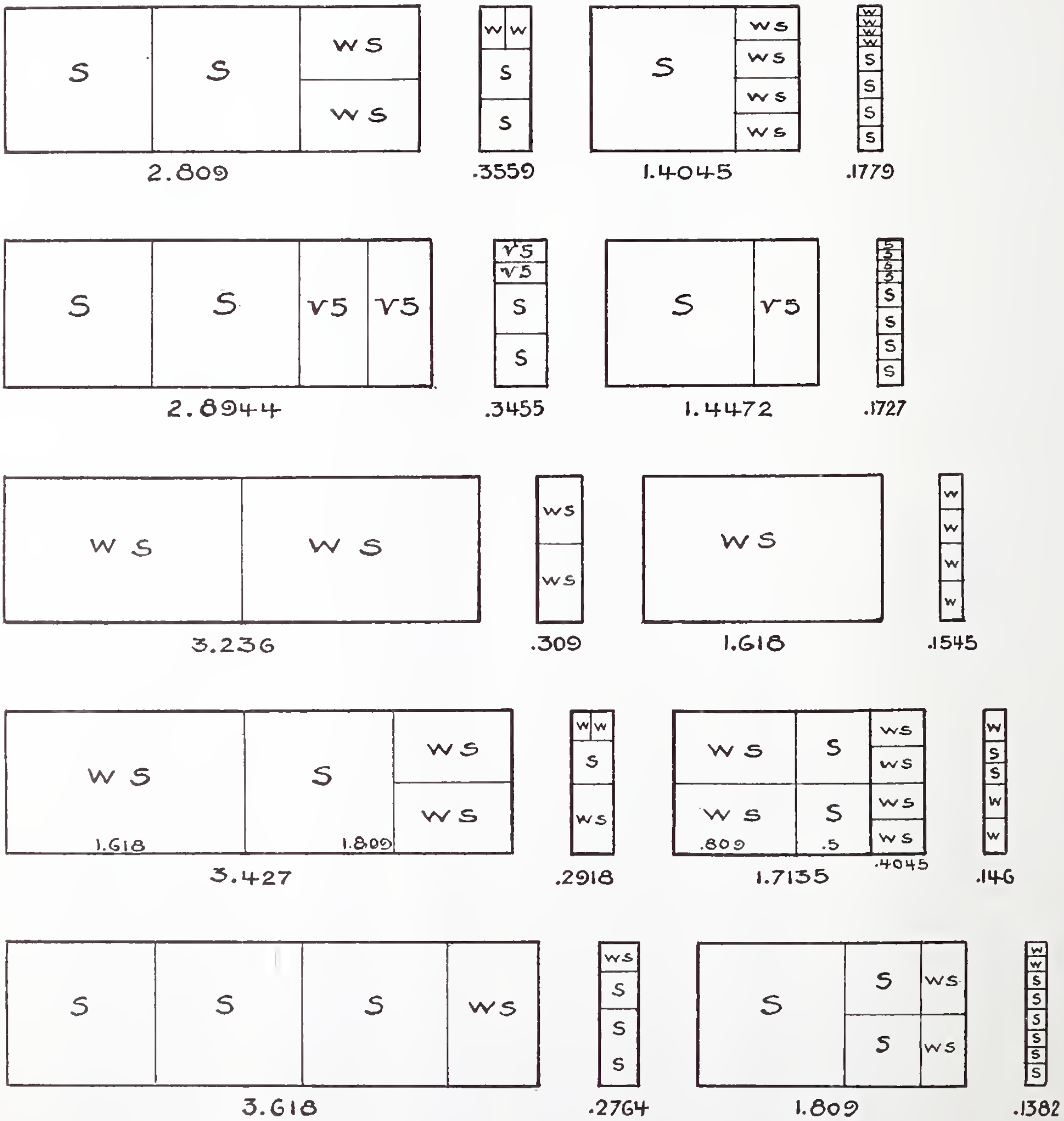


Fig. 2

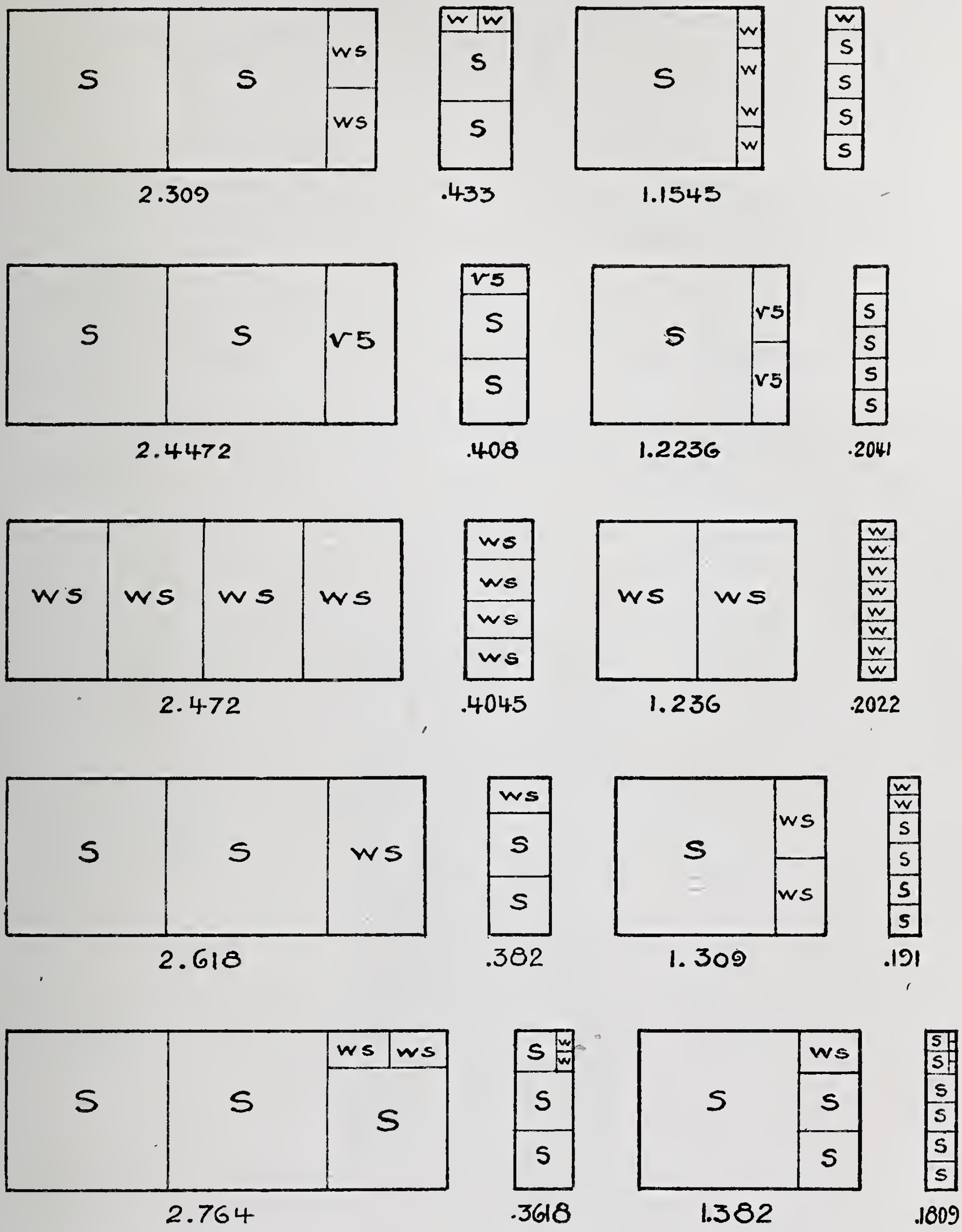


Fig. 1

See July DIAGONAL for a description of the ratios in the above diagram.

FREQUENTLY it is necessary to construct odd compound rectangles within a square, especially so when turning corners. The simplest and most direct method for this purpose is that of constructing the required figure, minus a square, outside of a square, and then drawing a diagonal to the entire area. For example, if it is required to fix a 2.309 rectangle within a square the process is to draw a 1.309 shape outside a square as in Fig. 3.

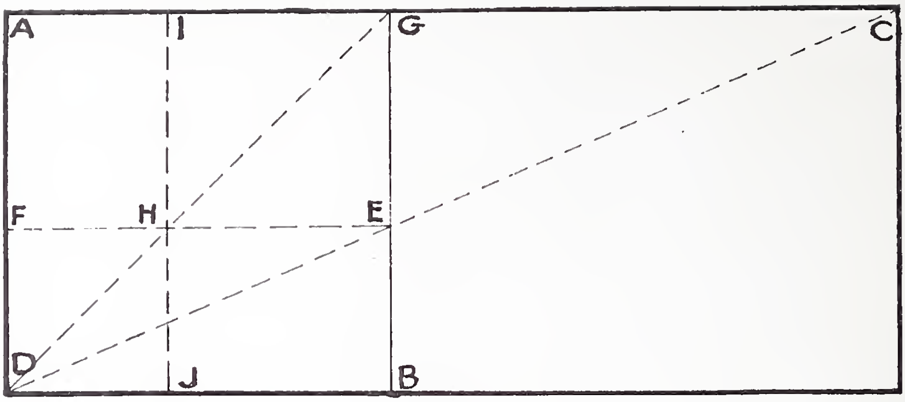


Fig. 3

AB is a square and BC a 1.309 rectangle. DC is a diagonal to a 2.309 area; *i.e.,* 1.309 plus 1. This diagonal cuts a side of the square AB at E. Draw EF parallel to DB. The area DE is a 2.309 rectangle within the square AB, *i.e.,* DE is the reciprocal of DC.

The nature of the area AE is found when we fix the numerical value of the line DF. This is the reciprocal of 2.309 or .433. In Fig. 2 the line AD equals 1, DF .433 and AF the difference between .433 and unity or .567; this is a reciprocal ratio, being that of 1.764 minus. The fraction .764 is the reciprocal of 1.309. Therefore the area AE is equal to a square, HG, and a 1.309 rectangle, AH.

AJ is a 2.309 area, being the reciprocal of DC. If the square DH is subtracted from AJ then AH equals 1.309.

GJ is a similar figure to AE and JE is similar to AH.

We frequently find in Greek design curious compound figures such as 1.691. This ratio, for example, is composed of a square plus a square and a root-five rectangle, .691 being the reciprocal of 1.4472. If we had a plan wherein .691 areas were arranged along the bounding lines of a dynamic area as in Fig. 4, AB might be such a rectangle, CD and EF .691 rectangles.

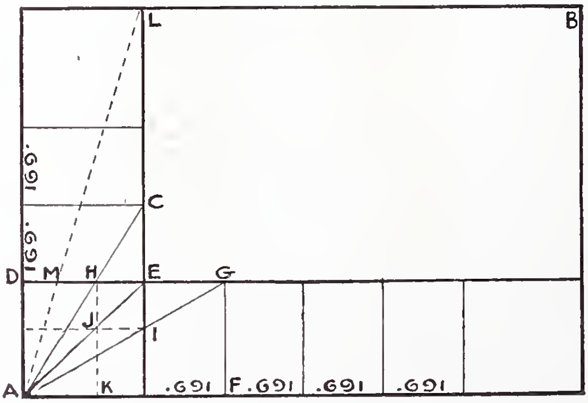


Fig. 4

AC and AG are diagonals to 1.691 rectangles, AH and AI are 1.691 areas within the square AE, and DI, IK are .691 shapes. AJ, JE are squares.

AL is composed of two 1.691 areas or 3.382. If we draw a line from A to L it cuts DE at M. The area AM is a 3.382 rectangle and the reciprocal of AL.

If we wish to determine a 2.4472 area inside a square we construct a 1.4472 area outside the square and draw a diagonal to the whole.

Angle square	Area outside	Resulting ratios	
1	1.618	2.618	
1	.691	1.691	
1	.7236	1.7236	
1	.764	1.764	
1	.809	1.809	
1	1.809	2.809	
1	2.809	3.809	
1	1.191	2.191	
1	2.191	3.191	etc., etc.

[To be continued.]

A Modern Vase in the Luxembourg without Design or Proportion.

THE SPIRAL AND OTHER CURVES OF DYNAMIC SYMMETRY

IT will be necessary for students of Dynamic Symmetry to construct geometrically the logarithmic or constant angle spiral. As was mentioned in early numbers of THE DIAGONAL, this curve appears to be the growth curve in plants. Inasmuch as the middle one of any three radii vectors, equiangular distance apart, is a mean proportional between the other two, we are able to draw this mathematical curve with ease and accuracy.

The first geometrical discovery made by the Greeks, in fact the first general law discovered by man, of which there is record, was that the angle in a semicircle is a right angle. Fig. 1 makes this clear.

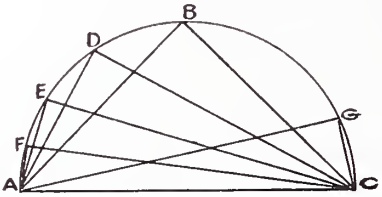


Fig. 1

ABC is a semicircle and AC is a diameter. BAC is a right angle. So also are DAC, EAC, FAC, GAC and so on.

Another great discovery, made later, was that a line dropped from the juncture of the two legs of a right-angled triangle to meet the hypotenuse was a mean proportional between the two segments of the hypotenuse as cut by this dropped line.

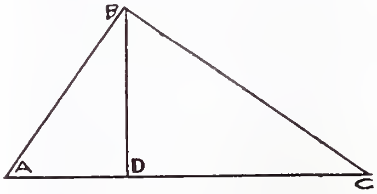


Fig. 2

BAC, Fig. 2, is a right-angled triangle and BD is a mean proportional between AD and DC. By the principle inherent in Fig. 1, the line AC of Fig. 2 is a diameter to a circle and the points A, B and C lie on the circumference of half of this circle.

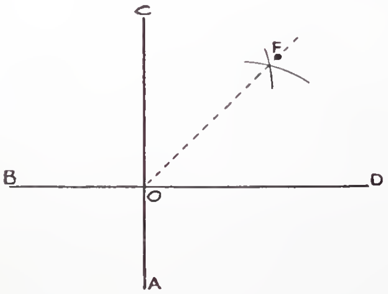


Fig. 3*a*

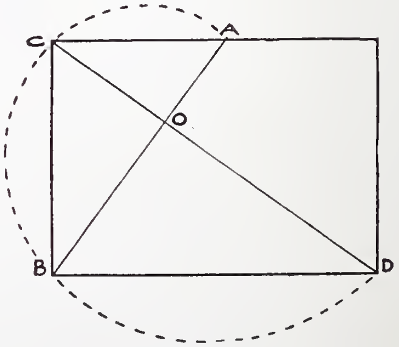


Fig. 3*b*

Suppose A, B, C and D, Fig. 3*a* and *b,* are points on the curve of a logarithmic or constant angle spiral and OA, OB, OC, OD are radii vectors equiangular distance apart. By definition OC is a mean proportional between OB and OD. Or OB is a mean between OA and OC. But suppose we wish to find points on the curve other than A, B, C and D; say between C and D. In that case we must find a mean proportional between OC and OD.

This is done by bisecting the angle DOC. A line is drawn equal in length to DO plus OC, Fig. 4.

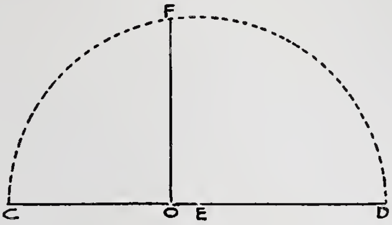


Fig. 4

This line is bisected at E. Then CE is equal to ED. With radius equal to CE or ED describe the semicircle CFD and, perpendicular to CD, draw the line OF. This line OF is a mean proportional between CO, OD, and F, Fig. 3*a,* is a point on the curve.

To find other points on the curve bisect angles like DOF or FOC, then add together the lines DOF or FOC, bisect them as described in Fig. 4, construct on these added lines as diameters a semicircle and draw the required mean proportional line as described. By this process any number of points may be found on the spiral curve.

In geometry a mean proportional line is equivalent to a square root. By definition a mean proportional line is the side of a square equal in area to the rectangle contained by the two extreme lines. If we construct a square on the line OC of Fig. 3*a,* this square is equal in area to the rectangle made by OB as end and OD as side. That is, the line OC is the square root of DO, OB.

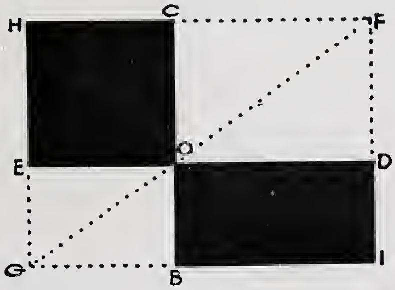


Fig. 5

The simple geometrical proof that the square CE is equal in area to the rectangle OB is obtained by completing the rectangle of which GF is a diagonal.

The triangle HGF is similar and equal to GIF.

ODF is similar and equal to OCF.

GEO is similar and equal to GBO.

Consequently the area CE must be equal to the area DB.

In Fig. 3*b* we have a rectangle with a diagonal to the rectangle and a diagonal to the reciprocal; they cut each other at right angles at O. Because of this OA, OC, OB, OD, are points on the curve of a logarithmic or equiangular spiral. OC is a mean proportional between OA and OB, and OB bears the same relationship to OC and DO. We find other points on this curve by the process described above. By definition the lines OA, OC, OB and OD are in continued proportion. That is, OA is to OC as OC is to OB and OC is to OB as OB is to OD, and OC, OB are two mean proportionals between OA and OB. In other words, these lines furnish us with a graph of the simple rule of three. If we substitute numbers for these lines their relationship will be more obvious. Suppose we give OA the value of two and OC four; then OB must be eight and OD sixteen. Four and eight are means and two and sixteen are extremes. In four terms in continued proportion the product of the means is equal to the product of the extremes. Four multiplied by eight equals thirty-two; so does two multiplied by sixteen. OA, OC and OB are three terms in continued proportion. According to the numerical value given to these lines OC multiplied by itself or squared is equal to OB multiplied by OA. Four times four equals sixteen and twice eight equals sixteen. But we may draw a logarithmic spiral within a rectangle.

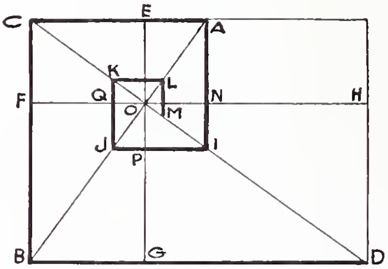


Fig. 6

In Fig. 6, O is the pole of a logarithmic spiral determined by the line BA crossing CD at right angles. Through this pole we may draw the lines EG and FH. H, G, F, E, N, P, Q, etc., are points on a logarithmic spiral curve contained in the rectangle CD and other points on this curve may be found as described above.

It is suggested that the student draw and redraw these simple diagrams until he thoroughly understands the principles involved, as he will find they will prove of great help to him in future work.

During the entire period of Greek classic design the construction of the spiral curve seems to have been a matter of constant experiment. A comparative study of the Ionic volute makes this fact clear. Fig. 7 shows the construction of volutes of different kinds by straight lines. Having a satisfactory volute arrangement in straight lines the construction of the curve is a very simple matter; it may be made free hand, by circle arcs of varying radii or by geometric construction as previously described. Of course the actual Ionic volute was in a rectangle which was some logical division of the rectangle which contained the entire column and this again was a logical part of the shape which contained the entire building.

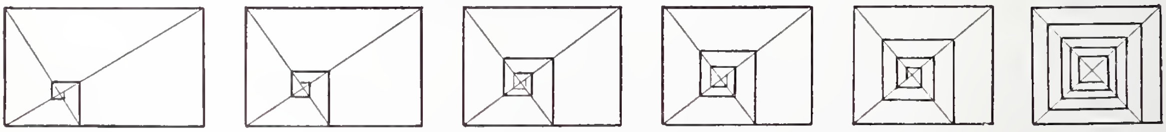


Fig. 7. Spiral and Other Curves.

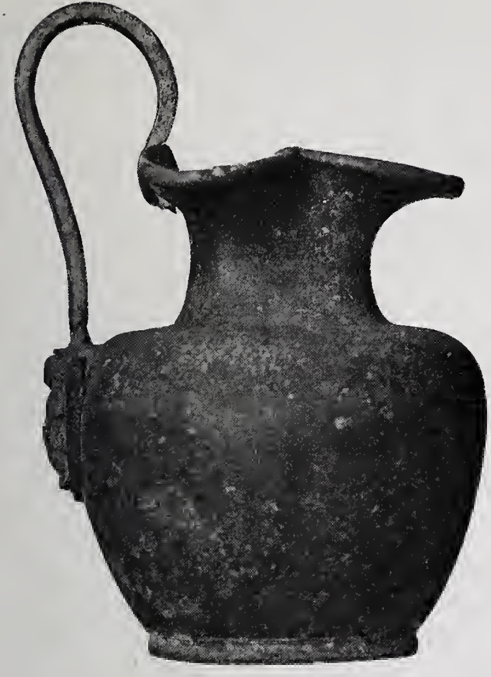


Fig. 1*a*

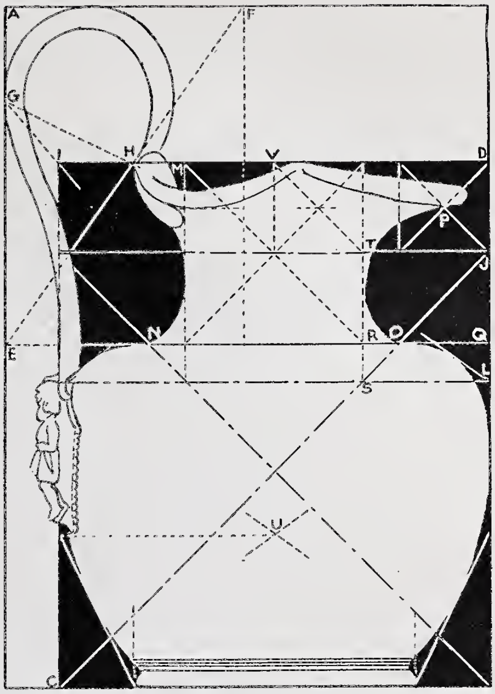


Fig. 1*b*

Small Bronze Oinochoe, British Museum, W. T. 656.

H. —H.	.1400±	1.2071	H. +H.	.1820±	1.4142
W.—H.	.1160—		W.+H.	.1300±	

Measured, drawn and photographed by C. O. Waterhouse of the Museum Staff.

BRONZE OINOCHOE IN THE BRITISH MUSEUM

ANOTHER bronze oinochoe in the British Museum furnishes ratios which show a theme in root-two (see June and July DIAGONALS).

Height including the handle	.1820	ratio 1.4142
Width including the handle	.1300	
Height minus the handle	.1400	ratio 1.2072
Width minus the handle	.1160	

The details of this example, in at least one particular, seem to have been correlated by a blending of the proportions of the two root-two forms.

The overall rectangle, including the handle, is a root-two rectangle. The line EQ divides this overall area into two equal parts. Consequently the area AQ is a root-two rectangle. But EQ defines the height of the bowl of the oinochoe.

A 1.2071 rectangle of the required proportion for this example is constructed within the major root-two area as follows: within the root-two rectangle AQ defines two squares. GQ is a diagonal to these areas and it cuts a diagonal to the root-two shape EF at H. Through H, parallel to EQ, draw ID. The line GW is a diagonal to a root-two rectangle and two squares; *i.e.*, it is a diagonal to a 1.2071 rectangle. The composition of this ratio is .7071, a root-two area, EW, and .5 or two squares, GQ. .7071 plus .5 equals 1.2071. The line GQ cuts ID at I, consequently IW is a 1.2071 area because IW is common with GW.

CJ is a square and IJ is composed of two squares and two root-two rectangles.

VQ is composed of two squares and a root-two rectangle. RD is the root-two area and RV the two squares.

IQ is double VQ or IJ.

CL is a root-two rectangle and U is its center.

IL is composed of two squares.

MR is a square.

ML is a curve tangent. Other curve tangents are apparent.

S and U fix the height of the figure at the base of the handle.

Points O, N, H and P are apparent.

The line ID, minus IH, multiplied by two gives the width of the foot.

The Editor wishes to announce that hereafter he will be pleased to answer questions relating to "Dynamic Symmetry" or to the matter which has appeared in THE DIAGONAL. *It is requested that correspondents make their communications as short as possible.*

Willesden, Holywood, Co. Down, Ireland, 4th July, 1920.

Dear Sir:

Since writing you a few days ago I have been looking for some reason why so large a percentage of Greek art is, as you say, based upon the $\sqrt{5}$ rectangle; and I have found a rectangle, based upon the $\sqrt{2}$ rectangle, which is very nearly a $\sqrt{5}$ rectangle and it seems to me that possibly, at least in some cases, what has been attributed to a $\sqrt{5}$ rectangle might really belong to this other rectangle. I estimate that for the same width the difference in length is about ¼ per cent. The new rectangle is formed from a $\sqrt{2}$ rectangle as follows:—add to each end the difference between the length and width. Then $\frac{\text{width}}{\text{length}}\,\frac{1}{\sqrt{2}+2(\sqrt{2}-1)}=\frac{1}{3\sqrt{2}}-2=\frac{1}{\sqrt{5}}$ approx.

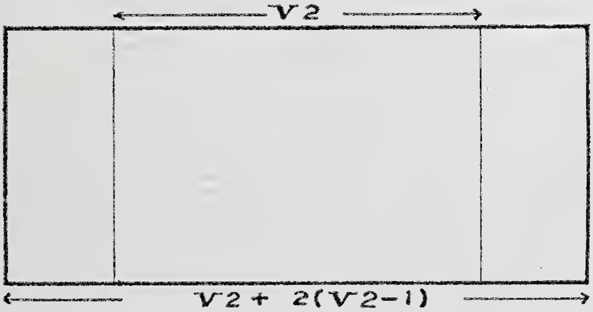


Fig. 1

I enclose a tracing of this rectangle to show the connection with the $\sqrt{2}$ rectangle and its wonderful properties.

Yours very truly,

ARCHD. H. FINLAY.

Jay Hambidge, Esq., 14 Place Vendome, Paris.

The reason why such a large percentage of Greek design is based upon the root-five rectangle apparently lies in the fact that a diagonal to two squares was preferred by the majority of classic artists to a diagonal to one square as a means of producing areas which could be subdivided easily without resulting in incommensurability. It should be remembered, however, that the root-five rectangle, *per se,* is seldom found. This applies also to the whirling square area, which is a derivative of the root-five shape. This latter area is a base to which the multitude of compound forms of this highest symmetry type may be referred. Occasionally compound arrangements of the root-two base are found which very closely approximate ratios of the root-five base, but so far no confusion has resulted from this, as the general overall shape of a design is considered of no particular importance unless all the details reveal a theme. A case in point was furnished by a vase in the Boston Museum of Fine Arts. Dr. Caskey, of the Museum, measured this vase and concluded it was a root-two shape; but the example failed to respond to any type of root-two analysis. The example was given to the writer for further inspection. The successful analysis showed that the shape, instead of being 1.4142 was 1.4045 or a square plus four whirling square rectangles. This point of approximating ratios was ably discussed by the American artist, Mr. Rutherford Boyd, before a meeting of the American Institute of Architects in the spring of 1919. He showed that areas so close to each other in shape that the eye could scarcely distinguish their difference, were yet absolutely different when subdivided into their logical units.

The ratios 1.7082 and .7082 are hardly to be distinguished from 1.7071 and .7071 as plain shapes; yet as subdivided they are unmistakably different.

In previous numbers of The Diagonal the work of the sixth century B. C. Greek designer Tleson was discussed. This designer, at one time, used static symmetry in the form of a square or squares divided into even multiples very much as modern designers do. He changed from this system to one based upon root-two and examples of his creation show the process of the change. The designer Nikosthenes, one of the most prolific producers of all the sixth century B. C. designers, if we are to judge by the number of examples of his work which have survived, began by using static symmetry in a much more subtle and complicated manner than did Tleson. Nikosthenes also changes his method. There are twenty-five examples of this designer's works in the Louvre, part of which are static and part dynamic, and there is a large kylix in the Metropolitan Museum, New York, which is transitional. So far there is no evidence that Nikosthenes used the root-two base. His early work is static, based upon relationships of squares and even multiples. His later and best efforts are formed upon the root-five base. So far nothing of a root-two flavor has been found. There are two signed works of Amasis, a Greek meteque with an Egyptian name, in Paris. They are superlative works. One example is a pure whirling square arrangement, the other a 1.309 shape. Amasis is supposed to antedate both Tleson and Nikosthenes. The handsome amphora by Exekias in the Louvre is a simple root-two arrangement. Three wonderful amphorae in the Louvre by Andokides, two of which are signed, are based upon the root-five or double-square symmetry. Exekias is supposed to be later than Amasis and earlier, slightly, than Nikosthenes. Andokides is probably somewhat later than any of those mentioned, though he produced black-, white-, and red-figured designs and therefore is a true transitional artist.

It would be pleasing to the writer, and from expressions he has heard, to many American artists, if it could be proven that the Greeks did not use the double-square symmetry, as we may temporarily call the root-five base. So far, however, the evidence seems to be overwhelmingly the other way. If we found more pure root-five shapes in Greek design a confusion between the shape suggested by Mr. Finlay might at times exist. Readers of The Diagonal will appreciate the closeness of the two areas if they are expressed arithmetically. The square root of five is 2.23606. The compound root-two form is 2.2426; *i.e.,* 1.4142 plus .8284. If it could be shown that root-two was the predominating base of Greek design, the fascinating natural growth base, as furnished by the law of phyllotaxis or leaf distribution, would be removed. The writer confesses that the relationship of this superlative design to natural form is, to him, most important. In his estimation it would be preferable to abandon Greek design rather than to give up a design based upon ratios found in nature.

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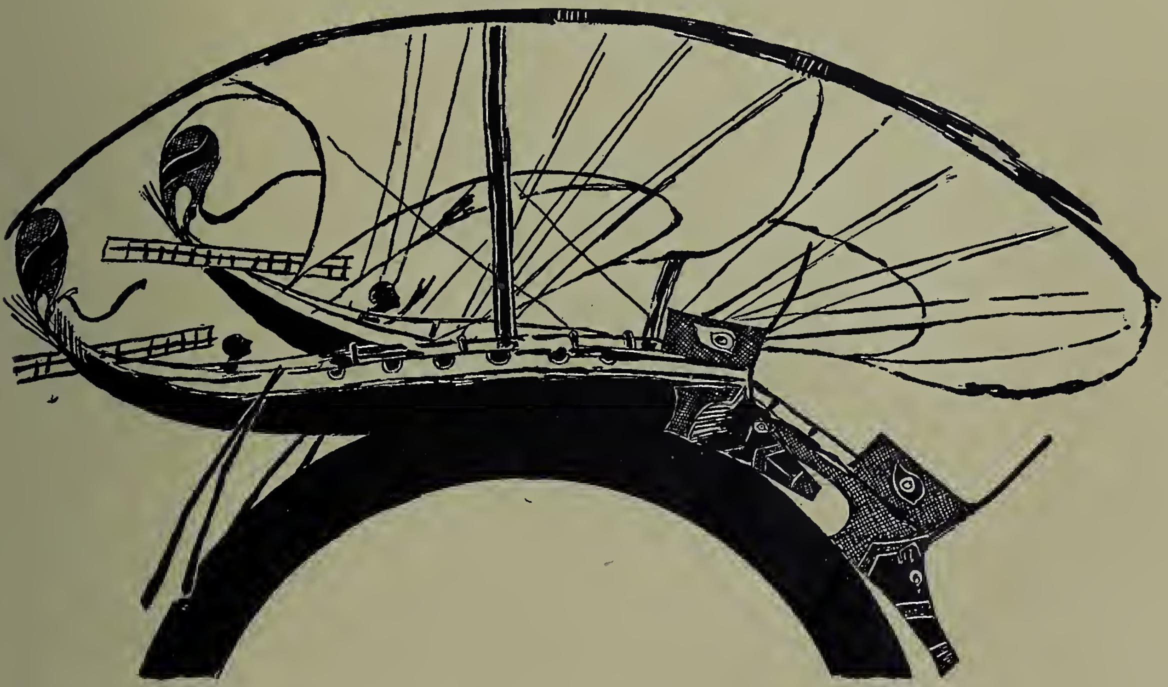
THE DIAGONAL

AN ILLUSTRATED MONTHLY MAGAZINE DEVOTED TO THE EXPLANATION OF THE REDISCOVERED PRINCIPLES OF GREEK DESIGN, THEIR APPEARANCE IN NATURE AND THEIR APPLICATION TO THE NEEDS OF MODERN ART

EDITED BY JAY HAMBIDGE



Greek Boat Race from a Kylix in the Louvre by Nikosthenes

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DYNAMIC SYMMETRY: THE GREEK VASE

By JAY HAMBIDGE

THE DISCOVERY OF DYNAMIC SYMMETRY MAKES POSSIBLE THE RECOVERY OF THE DESIGN PRINCIPLES USED BY THE CLASSIC GREEK ARTISTS

THE GREEK VASE SUPPLIES US WITH A DETAILED HISTORY OF THE METHODS USED BY GREEK ARTISTS TO OBTAIN THEMES IN DESIGN DURING THE CLASSIC AGE. THESE METHODS ARE EXPLAINED IN DETAIL WITH MANY DRAWINGS *and* PHOTOGRAPHS IN

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By Jay Hambidge
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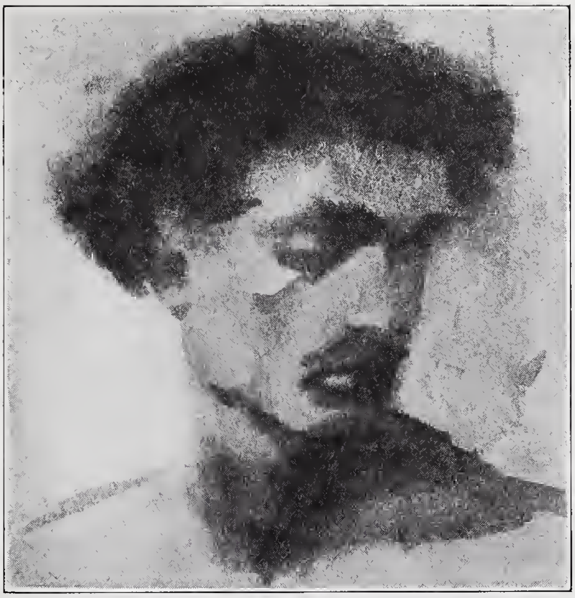
THE DIAGONAL



DYNAMIC LIFE WORK

THE drawings of heads in this number of THE DIAGONAL were made by life students working with Dynamic Symmetry under Mr. Howard Giles at the New York School of Fine and Applied Art.

The drawings speak so well for themselves that comment would seem to be unnecessary, but it would not be out of place to mention the professional character of the work. They suggest the efforts of gifted artists of long training. Indeed very few "arrived" artists could do better. They also disclose a quality which has been commented upon before in THE DIAGONAL. That is an abandon of technique which, usually, is only acquired by those who are such masters of their mediums of expression that they have forgotten that such a thing as technique exists.

It is well known that artists put into the faces of the subjects they draw only that measure of intelligence which they themselves possess. If an artist possessing a good technique but little brain paint a portrait of a subject who has brain capacity, it is almost a truism that such an artist's presentation of such a subject will lack the appearance of the intelligence of the original. It is this phenomenon that mars the work of almost all students while they are groping and floundering and trying to find their way through the usual academic swamp of training. The work of Mr. Giles' students is a refreshing exception to the callow and immature efforts of beginners. And the faces of these drawings show exactly the right degree of mental development that the subjects should possess.

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AN EXPLANATION

THE reason for the delay in this number of THE DIAGONAL is due to the writer's absence in Greece, where he has been getting the final evidence of the use of dynamic symmetry. When he arrived at Athens he found that the postal service was so unsatisfactory that it was practically impossible to be sure of getting manuscripts, drawings and photographs through to America on schedule, not to mention proofs for correction. He found also that the labor attending an inspection of the Greek buildings was so great that it practically prevented him from giving attention to anything else. But the delay has been beneficial, probably, in at least one way—it has given readers a chance to digest the material already presented.

Inasmuch as THE DIAGONAL is more a bulletin of research than it is a journal of entertainment, it is felt that its readers will not be too exacting about strict adherence to regularity of appearance. It has always been the conviction of the writer that the artist needs more accurate impersonal data and less personal opinion and influence upon which to build his creative superstructure. We could sit in our studio in America and figure out a plausible scheme of Greek design practice. But this would not do. When the writer undertook an inspection of Greek pottery design he would not trust his own measurements, even though he had unrestricted access to the originals, but whenever possible used data obtained by others. A similar course was pursued in the matter of Greek buildings. There are no new measurements of these structures, but there are plenty of old ones made by men of undoubted ability and probity. But these old measurements needed checking and the originals to be seen.

The writer left America for Greece in June, 1920, to re-examine classic buildings and to make a frank interpretation of their symmetry. The search began at Paris, led to Florence and Rome, Athens, Olympia, and then to Bassae in Arcadia. At Paris were found a number of early signed masterpieces of Greek pottery, the creations of men whose work antedated that of the designers of the best Greek temples. Men like Amasis, Exekias, Andokides and Nikosthenes, the very greatest masters of architectural pottery design. These works exhibit the use of dynamic symmetry beyond any question.

At Florence, Italy, the writer was fortunate in finding original drawings by Michaelangelo with unerased pencil lines upon them showing exactly how he employed his static symmetry. These drawings are interesting because, if we accept their author as the most representative artist and architect of his age, and as we have obtained the procedure of the classic Greeks, they enable us to compare the planning methods employed by the masters of the two superlative periods of art supremacy.

At Athens months were devoted to a re-examination of the Parthenon. This work showed that measurements made by Francis Cranmer Penrose during the middle of the last century are accurate, except in some quite minor particulars. The most difficult part of the Parthenon problem was the fixing of the symmetry scheme in the line plan and its modification caused by the substitution by the builders of curved for straight lines. But a trip to Bassae in Arcadia, the home of the celebrated Apollo temple designed by Iktinos, the leading architect of the Parthenon, helped in making clear the symmetry scheme of the latter building. The Apollo temple is a straight-line building, there being no curves except those of circular columns and their heads. In these two buildings by the same master we have a perfect foil or background, one for the other. And of all the works of formal design which disclose dynamic symmetry, so far examined, the Apollo temple at Bassae is the simplest.

Both the Parthenon and the Apollo temple show consistent dynamic symmetry schemes of like character throughout, or so far as the surviving remains of the buildings justify the examination. Considering the great size of these buildings, in comparison with pottery and decoration and the accuracy of workmanship of the Parthenon, the symmetry schemes are so perfect that we may be sure that we have recovered the method of their planning.

As a result of this visit to Greece we have recovered the plans of the Parthenon, the Apollo temple at Bassae, the Zeus temple at Olympia, the temple at Aegina and the temple at Sunion. In addition much Greek decoration was inspected and plans made for a future work on Greek sculpture. But it is felt that our most immediate need is for a book on what Greek art is. The obtainable books on the subject are misleading and inadequate. The critical inspection of Greek design has been left largely to literary men and others who seem to have neither feeling nor understanding of design.

THE TRUE INWARDNESS OF MR. HAMBIDGE'S THEORY OF DYNAMIC SYMMETRY

By Cloudesley Brereton.*

SOME time ago Mr. Jay Hambidge gave a lecture on dynamic symmetry to the Art Workers' Guild and, judging by the attitude of the audience, the presentment of his views made a considerable impression on a large number of them while inspiring the remainder with a keen desire to hear more. He was particularly insistent on the point that he was not trying to develop a new theory of aesthetics, which many of his English critics seemed to think, although to the present writer his work, whatever its aim, appears fraught with the highest significance. Much less did he pretend to give a sort of recipe for manufacturing beauty—the research for which, one imagines, would be as profitable as that for the philosopher's stone. In fact, he was not so much concerned with the artist as with the craftsman; out of whom he rather maintained the artist should develop. In reality he seemed to have little use for the artist who did not seek to be a super-craftsman. His theory, to put it in a paradox, seemed to be "Take care of the crafts and the art will take care of itself." He held very strongly that the more consummate the craftsmanship of an age the greater its artistic masterpieces would be, and he instanced Phidias as the most consummate of craftsmen.

From Mr. Hambidge's point of view we cannot have too much knowledge of technique provided that the person who acquires it recognizes that technique is but the *generalized* experience of the craft and always regards it, not as a substitute for, but as an ancillary aid to creative work. Its presence in an artistic masterpiece can only add greater richness and ripeness to it.

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* From the *Journal of the Royal Institute of British Architects,* September, 1920.

All this seems very true. To impart technique, whether on Montessori or other lines, is in fact one of the major objects of all education. Technique from this aspect is the self-imposed yoke whose service is perfect freedom, for without it freedom is mere license; or, in other words, the greatest artist consciously submits to technique in order to triumph over it, being ready to break any one of its principles to achieve greatness, but not to break them all at once, for that is mere anarchism—ready again, when he breaks them, to explain why he did so. Every supremely great artist is a potential Antigone whose breach of the highest conventions is justified by the results, and all the more readily explainable by the artist himself the more profoundly he is acquainted with the existing canon of conventions which, like a judge of the Supreme Court, he is perpetually interpreting. The greatest artists, the Leonardos and the Phidiases, are those who have pondered the deepest on their art or turned the searchlight of their consciousness on the productions of their subliminal self.

Of course, without inspiration there can be no true art. But *vis consili expers mole ruit sua,* and inspiration even with its happiest and apparently most spontaneous moments depends on the calling into play that precious experience of a lifetime, external or internal, of which Whistler spoke. If the painter or the poet is to utter adequately the thoughts that arise in him, the forces he mobilizes for self-expression, for creating the outward and visible symbol of his passion, are just the technical experience he has been amassing consciously or unconsciously during the preceding part of his life.

This is the true element in the saying that genius is the infinite capacity for taking pains. Genius always does take infinite pains, though the result may only be ultimately revealed by the shortening of the artist's life. Qualitatively a genius may live ten years in ten days and die at thirty. But whether those pains be the work of the brooding spirit within or outward attempts at self-expression, in either case or both they may be lightened and alleviated by studying the experience of the particular art or craft concerned, always bearing in mind that one cannot get hold of too much of this experience if one looks at it not as a set of immutable laws but as so many starting points for fresh discovery, truths up to date and nothing more, though some of them may remain unaltered for centuries, like the law of gravity, till an Einstein comes along.

But this defence of technique as a necessary acquisition in education of every sort and kind, though Mr. Hambidge is naturally in no wise responsible for it, will in the course of the article assist, one hopes, in justifying what he is trying to do, or rather has been trying to do for the last twenty years.

And what is that?

To put it in a nut shell, he is apparently out to recover what one may call some of the *lost grammar of ancient craftsmanship.* I use the word "grammar" advisedly because any act of self-expression must have its self-evolved rules, be it oratory, poetry, painting, sculpture, pottery, basket-making or what not. Moreover, in order to give this grammar a rational basis, he has analyzed it and found that like all true grammars it possesses an essential element of logic in it.

To take the "grammar" itself first. If we examine its function in the literary sphere, we find it dealing with the mere carpentry and joinery of sentences, ensuring thereby verbal correctness, while in its highest development as rhetoric it concerns itself with the problems of style. These two stages are also apparent in the other arts. The simple grammar of the ordinary stone-masons or furniture-maker will not produce Gothic cathedrals or Chippendale chefs-d'œuvres, but it will ensure that the house that is being built will not topple over and that the chair that is being made will support an average weight.

But such things are the mere A B C of the craftsman's technique. When a great school of building or a great school of furniture-making has been in existence some time, then the technique of the master craftsman will evolve into an elaborate canon of style, codifying the common experience of the guild, just as the French language through the countless labors of several centuries of grammarians and writers has been worked up into the wonderful organ of expression it is to-day, while still preserving its liberty to progress.

Now it is just one of the most important canons of style that Mr. Hambidge has recovered to-day after twenty years of patient, exhaustive experiment. For his principle of dynamic symmetry (personally one would have preferred the term *natural* as opposed to artificial, since the symmetry is found in Nature) is, as he has proved by thousands of actual measurements, the general principle that underlies the creations of the finest productions of Greek art at its most flourishing epoch. That is, that the Greeks, whether plotting out a temple or hewing a statue or moulding a vase, selected for the ground-plan of their designs areas whose two sides, in contrast to Mr. Hambidge's so-called Static Symmetry, were always incommensurable, this particular category of areas being obviously regarded by them as the most fitting space within which to set out their masterpieces.

But by an absolutely natural evolution every technician begins at a certain stage to try to find out what are the laws underlying the lore or "grammar" that has gradually grown up round his cult, to divine the reason or reasons which led him, if he were (say) a poet or an orator, to look below the dictates of "use and wont," till by dint of thinking he discovers how large an element of *logic* his grammar contains. It is probably here, however, that, language being more abstract than most of the arts, the logical analysis of its grammar was posterior to that of the grammar of the other arts. But every civilized people must sooner or later, like M. Jourdain, make the discovery that it has been talking prose!—that is to say, the linguistic medium it uses possesses certain definite rules. Perhaps the clearest instance, however, of this emergence of the logical factor in any art or craft is the history of Euclidean Geometry, originally evolved in Egypt as the scientific element underlying the art of land surveying, itself in turn rendered necessary by the obliteration of landmarks in the Nile Valley by the yearly floods. The codified practice of the land-surveyors—their "grammar" in fact—was found to repose, allowing for, or rather ignoring, minute personal differences in measurements, on certain definite, logical and rational bases. And what was true of the ancient land-surveyors was true of the ancient builders and stone-masons. They, too, discovered the mathematical laws underlying right construction, and, as their sense of beauty in the constructive arts developed, they also found out certain higher formulae that summarized their ideas of proportion and design.

The mathematics employed by Mr. Hambidge are therefore merely the logical exposition of the scientific basis of one of the most fundamental principles in Greek art. This scientific basis was in all probability known to the Greeks, but had already been superseded, when the Romans came into close contact with them, by a development in art which was really a step towards decay.

Mr. Hambidge believes that in re-introducing this principle of Dynamic Symmetry into design as far as craftsmanship is concerned he will re-introduce into it one of the most *revitalizing* elements, by giving us from the Montessori point of view the right *milieu* to work in. He feels sure it will also have a beneficial effect in turn on the fine arts, which in the opinion of some to-day are still too much divorced from the crafts from which they sprang. Like Antaeus they must seek a renewal of strength from the prime source of their origin if they are to escape the strangle-hold of the many-headed crowd that, like Hercules, threatens in the near future to crush to death any *Intelligentsia* too divorced from actual life.

The extraordinary subdivisions of labor that the industrial revolution has inflicted on us alike in science, art and craft have with their minute water-tight compartments made us forget that life is *one,* that *tout se tient,* and that everything is in everything. Many to-day, especially the votaries of art for art's sake (a kind of Super-Pragmatism), are amused or irritated at the suggestion that there is a scientific or intelligible element in Beauty, though the man of science knows there is one of beauty in Truth and the religious man knows there is an element of both in Goodness. The fact is that in the trinity of Beauty, Truth and Goodness each *dominant* element contains a subordinate element of the other two. Our age, analytical to absurdity, has not hesitated to divorce Beauty from Goodness and Goodness from Truth; and yet, in all reverence, we would ask what meaning in its theological counterpart would God the Son have if completely sundered from God the Father! Surely that august Triad, the highest pinnacle of Western thought, must stand or fall together.

The work of Mr. Hambidge in retrieving one of the most vital principles of all the arts that work in two or more dimensions has liberated a vital sap that will not cease to circulate till it has mounted to some of the topmost branches of human activity.

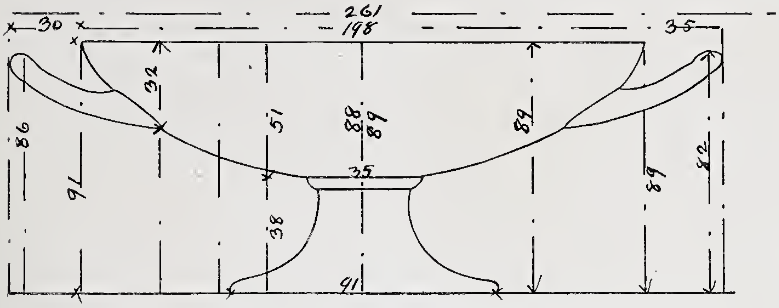
DYNAMIC EXAMPLES OF THE WORK OF THE EARLY GREEK MASTER DESIGNER NIKOSTHENES

OF the twenty-five examples, in the Louvre, of the work of the early Greek potter Nikosthenes, fifteen are static and ten are dynamic in their symmetry.

As the dynamic examples are much superior in design it seems fair to assume that these were made later. In fact, Nikosthenes seems to be a real transitional designer. He straddles the static and the dynamic periods, it being not improbable that he began his design career by using static symmetry, possibly then the only kind known, and afterwards learned the more powerful means of design assembly.

At Rome, in the collection at the Villa Giulia, there are three Nikosthenes examples, two static amphorae similar to those in the Louvre, and one dynamic pyxis of extreme beauty. The two static examples have an overall ratio of one and one-half. The pyxis is a square and root-five area (1.4472). A very handsome Nikosthenes kylix at Florence is pure dynamic.

As has been mentioned in a previous number of The Diagonal the Nikosthenes examples in the Louvre collection were measured and drawn by Mlle. Jeane Everard. Knowing nothing of the dynamic idea she was simply asked to make the most accurate measurements possible. It would be tiresome to go through the entire mass of this material, but a few examples may not be out of place, particularly as masterpieces of pottery design furnish such splendid preliminary study of the products of the architectural masters. Some of Nikosthenes' static work was published in the last number of The Diagonal. I will give here the first six examples of his dynamic work as they came to me. Unfortunately Mlle. Everard is not skilful in draughting, so the real quality of the fine Greek curve is missing in her drawings. I can vouch for the accuracy of her measurements, however, as I checked them with the originals later.

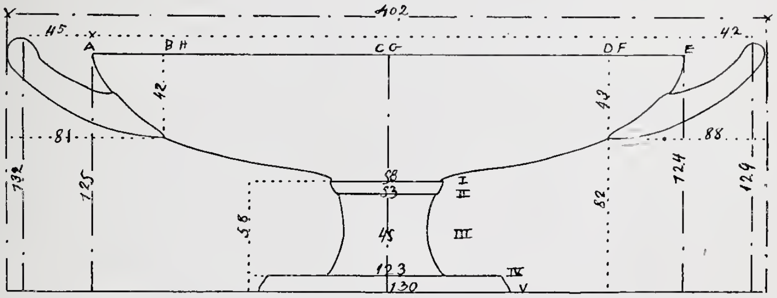


Kylix F 124, Louvre.

Height of bowl: .091, .088, .089, .089, .089, .087, .088, .086.
Height of handles: .086, .082.
Full width, including the handles: .261.
Full width of bowl: .198, .198, .198, .198.
Height of foot: .038.
Width of foot: .035, .91.

The full width of the kylix is .198 millimeters. If this is divided by root five (2.236) the result is .0885 for the height of the cup as it stands, that is to say, the projection of the kylix without its handles is a root-five rectangle.

The width, including the handles, I found a shade less than .261. The ratio is 2.927. Root five, or 2.236, subtracted from this leaves .691.



Kylix 125, Louvre.

Height of bowl: A .125, B .128, C .121, D .126, E .124, F .125, H. 128.
Height of handles: .137, .129.
Full width, including the handles: .402.
Full width of bowl: AE .312, DF .313, CG .315, DH .315.
Height of foot: I .058, II .053, III .029, IV .008, V .000.
Width of foot: I .059, .058, .058, .059, II .053, III .045, IV .123, V .130.

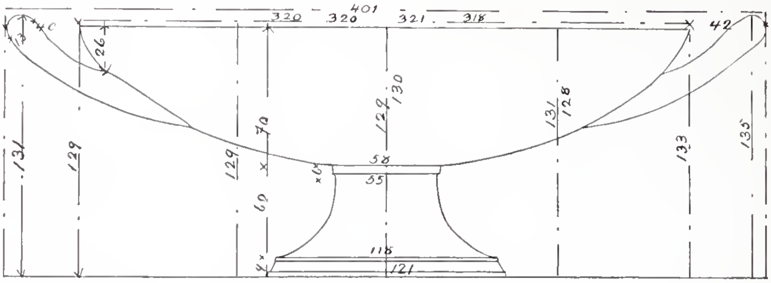
The ratio for the cup, without the handles, seems to be 2.472, *i.e.*, .618 multiplied by four. This gives a bowl width of 3.139 and a height of .127.

The overall ratio, .402 by .127, is 3.163, or 2.472 plus .691.

There is an indication in this example that the foot is equal to the height. The top of the foot is the line, or a point between the top and the bottom. We are here speaking of the thickness of the foot. Mlle. Everard includes the stem in the foot height.

The width of the foot at the bowl juncture is .058, the height .058, and this makes a square.

.058 multiplied by root five (2.236) equals .12968. This area was probably intended to be root five. (See the measurements .059).

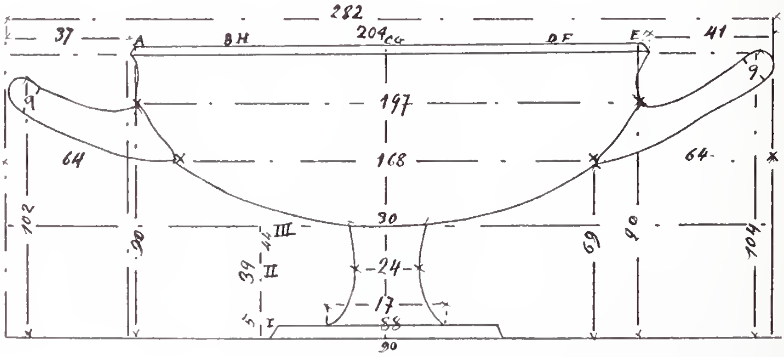


Kylix 126, Louvre.

Height of bowl: .129, .129, .130, .131, .133, .128, .129, .127.
Height of handles: .131, .135.
Full width, including the handles: .401.
Full width of bowl: .320, .320, .321, .318.
Height of foot: .060.
Width of foot: A .058, B .055, C .117, D .121.
Width of bowl .320, height .1292, ratio 2.472, or .618 multiplied by 4.

The overall ratio seems to be 3.090, or .618 multiplied by 5. For this the full width should be .3992, instead of .401, a negligible error, especially for the handles of a kylix.

The height of the foot and stem, .060, and the width, .121, is two squares, or root four, with a millimeter error. The width of the foot to the full height of the cup is probably a geometrical relation difficult to show with this type of drawing.



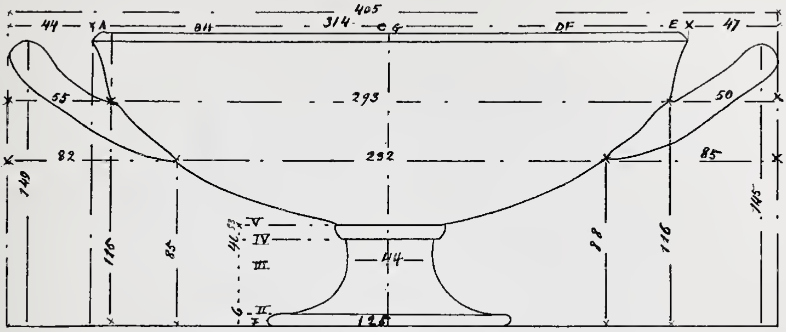
Kylix F 80, Louvre.

Height of bowl: A .115, B .115, C .115, D .115, E .114, F .113, G .112, H. 115.
Height of handles: .102, .104.
Full width, including the handles: .282.
Full width of bowl: AE .204, BF .204, CG .204, DH .205.
Height of foot: III, .043, .044, .044, .044, II .039, I .005.
Width of foot: o .090, I .088 and .047, II .024, III .030.

Assuming that the height is .114 and that the width including the handles is .282, the overall ratio is 2.472, or .618 multiplied by 4.

If the width of the bowl is .204 and the height .114, the ratio is 1.7888, or root five (.4472) multiplied by 4.

If the height of the foot and stem is .044 and the width of the foot at its top .088, we have two squares, or root four, for this element.



Kylix F 81, Louvre.

Height of bowl: A .150, B .153, C .155, D .150, E .150, F .148, H .145.

Height of handles: .149, .145.

Full width, including the handles: .405.

Full width of bowl: AE .314, BF .310, CG .315, DH .312.

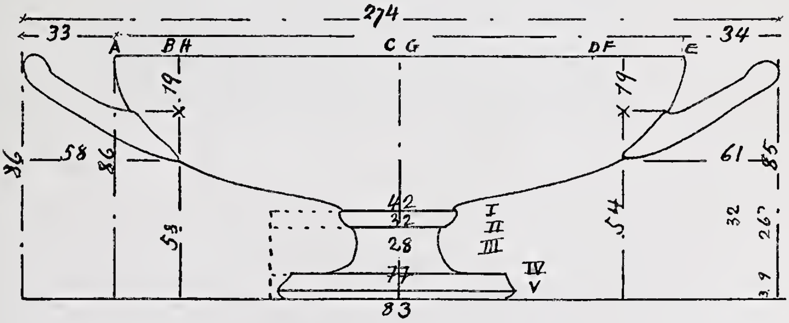
Height of foot: V .054, .053, .053, .052, IV .046, .045, .045, .046, III .036, II .006.

Width of foot: II .125, III .044, IV .046, V .056.

If the overall width is .405, or a fraction of a millimeter less, and the height .1547, the ratio is 2.618, or 1.618 plus 1, a whirling rectangle plus a square.

The width of the bowl varies between .310 and .315. The ratio of the height to this is probably root four, or two squares.

If the width of the foot is .1251 and the height of the bowl .1547 or .155, the ratio of the rectangle is .1236, or .618 multiplied by 2.



Kylix F 120, Louvre.

Height of bowl: A .086, C .086, E .087, G .087.

Height of handles: .086, .085.

Full width, including the handles: .274.

Full width of bowl: AE .206, BF .207, CG .206, DH .206.

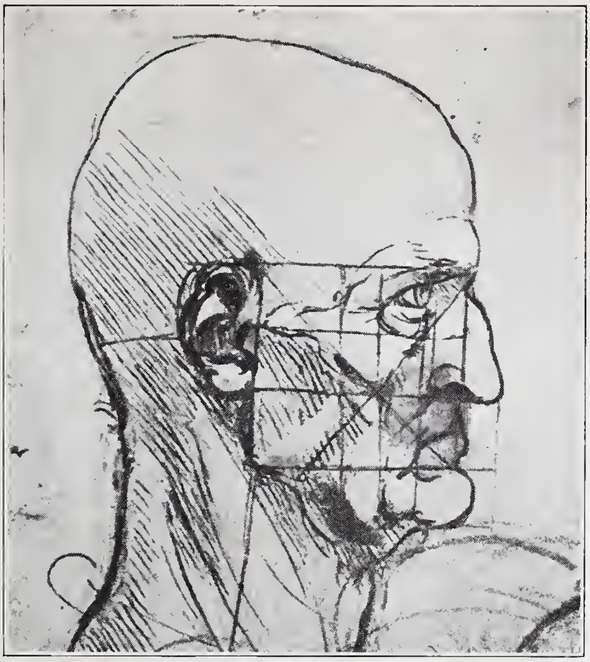
Height of foot: I .032, II .026, III .020, IV .009.

Width of foot: I .042, II .037, III .028, IV .077, V .083.

Apparently this design is a theme in root two. If the height is .086 and the bowl width .2076, the ratio is 2.4142, or root two plus a square.

If the height is .086 and the overall width .2735, the ratio is 3.1813, or 2.4142 plus .7071, the reciprocal of 1.4142.

Other combinations of these areas may be figured by the student from Mlle. Everard's measurements.



THE ELEMENTS OF DYNAMIC SYMMETRY

IN the earlier parts of these lessons the reciprocal of a shape was explained in some detail. There is, however, another important element which has not been mentioned. This is the complement of a shape. We may compare a form complement to a color complement.

If we select any color, the complement of it is that quantity of color necessary to balance or neutralize it. For example: if a blue be selected its complement will be an orange; if a green, then a pinkish crimson; if a yellow, then a violet, etc. We may reverse the process and select crimson, orange or violet. Then the complements will be blue, green and yellow. This complement balance is well understood by color authorities. It is not so well known, however, that if the selected color is debased with light or dark, the complement must also be debased in reverse order.

For example: if we debase a blue with light, its complement, orange, must also be debased with dark in due proportion. The exact percentage of debasement is beautifully illustrated by a simple experiment in photography. If we wish to determine the exact complement, say, of a blue debased with light, this color is photographed on a color plate, preferably a Lumière plate. After the exposure this plate is then developed as a *negative.* Ordinarily it is developed as a positive and used as a transparency.

This negative color plate will show the exact complement of the blue color which was debased with light, *i.e.,* it will appear as an orange, properly debased with dark to make the precise complementary balance for the blue. In other words, a negative is a complement. Students are all familiar with an ordinary black and white negative. Artists of course make necessary complementary mixtures in pigment by eye. The student, however, must train his eye, and for this purpose I know of nothing better than a number of color-plate negatives wherein the original color has been debased by light or dark in various ways. For the artist's palette "white" is substituted for "light."

The complement of a shape is that area which represents the difference between the selected figure and unity.

For the purpose of finding a complement it is first necessary to reduce the ratio of the selected area to its reciprocal form, that is, divide it into unity.

For example: the ratio of a root-five area is 2.236 and the reciprocal .4472.

.4472 subtracted from 1 leaves .5528. This is approximate but accurate enough for working purposes.

.5528 is the reciprocal of 1.809.

The complement of a root-five rectangle, whether expressed by 2.236 or .4472, is .5528 or 1.809.

A diagram will help to emphasize the relation.

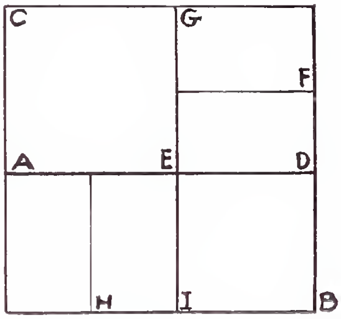


Fig. 1

AB of Figure 1 is a root-five rectangle.

CD is a 1.809 rectangle and CB is a square.

AD equals 1, ED .4472 and CA .5528.

CD is composed of the square CE and the two whirling square rectangles EF and FG.

AB is composed of the square BE and the two whirling square rectangles EH and HA.

BG is also a root-five rectangle composed of BE, EF and FG.

Also CI is a 1.809 rectangle composed of CE, EH and HA.

We learn from this arrangement of an area and its complement that a certain part of each area is common to both, as, in this case, DG and AI, while the two remaining areas are similar in shape but are different in size.

This complement arrangement applies to any rectangular area.

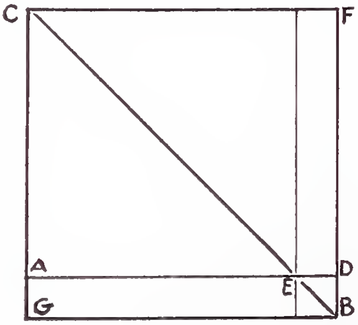


Fig. 2

In Figure 2 the area CD is the complement of AB, and EF is similar and equal to EG, while CE and EB are similar figures of unequal area.

The point for us to bear in mind is that there is an intimate relation between areas and their complements, and whatever arrangement is made in one necessitates a complementary or reverse action in the other.

A division in a 1.809 rectangle, for example, automatically produces a reverse arrangement in a root-five rectangle.

In painting, if we wish to secure force and emphasis, whatever we do with a color we also do in reverse order to its complement. It will be impossible for the artist to be master of his means of expression unless he understands the nature of this automatic action. His feeling and sense of fitness will help him to overcome many difficulties, but he will not come into his full heritage until he comprehends what he is doing.

The photographic color plate shows us that, if we add or subtract a color element to or from another color, we must subtract or add the complement of such addition or subtraction to or from the complement of the original color.

We apply the same idea to areas.

It will be noticed that any rectangle, with its complement, forms a square, and that subdivisions of either the rectangle or its complement may be considered as subdivisions of this square.

Those familiar with the primary steps of geometry will recognize the similarity between a square divided in this manner and most of the problems of the second book of Euclid.

Authorities agree that most of the substance of this book had an early origin in Egypt. The ancient idea as it has survived to us was the dividing of a line and the fixing of the relationship between the areas of the squares and rectangles on this line and its segments.

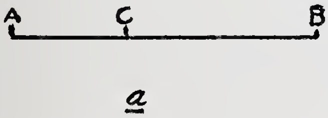


Fig. 3*a*

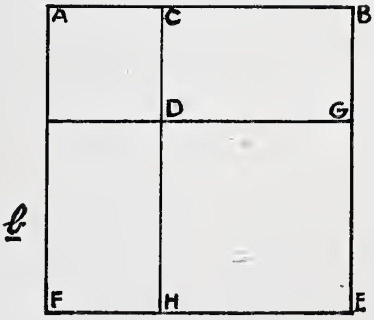


Fig. 3*b*

AB, Figure 3*a*, is any line cut at any point, say C.

AE, Figure 3*b*, is a square constructed on AB.

AD, DE are squares on the segments AC, CB.

BD, DF are rectangles formed by the segments AC, CB.

The square on AB is equal to the squares on AC and CB plus twice the rectangle on AC, CB, etc., etc.

Let us suppose, for example, that a painter is using a rectangular canvas, as CE. The complementary shape to this canvas is an imaginary area.

It would not occur to many that the canvas CE could be divided dynamically in terms of the imaginary area AH.

It will prove profitable practice for the student to divide the areas of his designs in terms of their complements.

As simple as the arrangement of areas is in *b*, Figure 3, it will nevertheless require some ingenuity to figure out arithmetically the exact relationship of the areas when they are made by dynamic cutting.

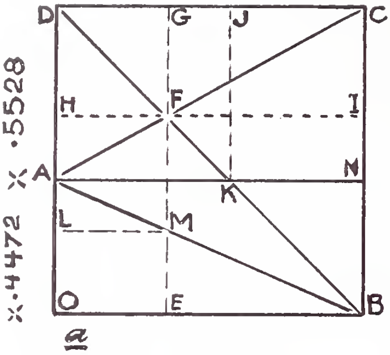


Fig. 4*a*

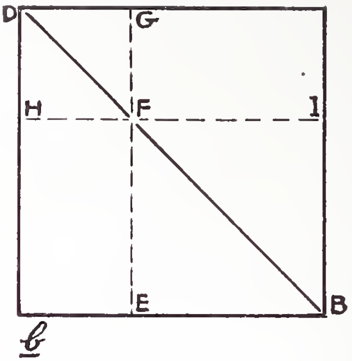


Fig. 4*b*

Let AB, Figure 4*a*, be a root-five and AC a 1.809 rectangle.

BD is a diagonal to the major square.

AC is a diagonal to the 1.809 and AB a diagonal to the root-five rectangle.

F is an intersection of AC and DB.

Through F draw GE and HI.

What are the areas AE, LE, HE, GI, GN and JI?

It is clear that GI and HE of *b* are equal and similar figures, and that DF and FB are squares of unequal area.

In *a,* DF is a square, while AF is a similar figure to AC because they have the same diagonal.

GA therefore must be equal to a square plus a .5528 area, that is, GA must be a 1.5528 rectangle. Or we might say a square plus a square and an .809 rectangle.

We now need the actual numerical length of DG.

AD we know is .5528, while DC is 1.

To obtain the length DG we must multiply DA, or .5528, by the reciprocal of 1.5528. This is .644.

We thus obtain .356 plus for the length DG, while GC is 1 minus .356, that is .644.

We may obtain this result in another way.

The area DI of *a* is equal to the square DF plus the 1.809 area GI, *i.e.,* DI is a 2.809 rectangle. The reciprocal for a 2.809 area is .356 or the length DH or DG.

We know that the length AO is .4472, because AB is a root-five rectangle. The area AE, .356 by .4472, is found by dividing the lesser into the greater number.

The result of this division is 1.2562, or .809 plus .4472, a compound of part of the complement and the root-five area.

The area AM is this root-five area while LE is the .809 figure. JN and LE are similar shapes, each composed of two whirling square rectangles.

Suppose we fix a reciprocal in a complement of a root-five rectangle.

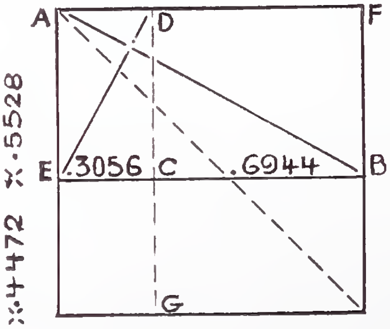


Fig. 5

AC of Figure 5 is the reciprocal of AB.

To find the length AD we must multiply AE by .5528, that is, we must square this number. The result is .3056.

We might also divide .5528 by 1.809.

DF equals 1 minus .3056 or .6944.

.4472 divided into .6944 equals 1.5528 for the area BG.

EG is the difference between 2.236 and 1.5528, *i.e.,* .6832, or .4472 plus .236.

Of course it is not necessary for an artist in his practice to resort to this arithmetical calculation any more than it is necessary for him to figure out precisely his perspective, but, as is also necessary with perspective, before he can use symmetry freely he must understand it, to the end that his intention may be rational. In fact *intention* is the dominant factor in artistic as in other forms of expression. If the artist has not a clear conception of what he intends to do, his acts will lack force and distinction no matter how strong his intuition may be.

In practice the artist's greatest dependence will rest upon his use of diagonal lines.

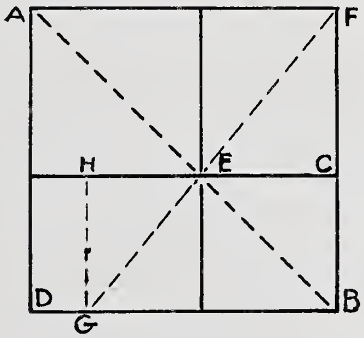


Fig. 6

AB of Figure 6 is a square; DC is a root-five and AC a 1.809 rectangle.

AE is a square, consequently EF must be an .809 shape.

FE is a diagonal to this area. Produced from E to G the area EG becomes an .809 shape and EB is a square. CG is a 1.809 rectangle applied to the root-five area CD.

DH is the difference between 2.236 and .809, or .427.

If the .427 area DH is applied to the other end of the root-five shape, the area in the middle will be a 1.382 figure.

This simple operation divides the root-five area into terms of its complement in a very suggestive manner. This process applies to any rectangle and its complement where the latter is greater than one-half the major square.

[To be continued.]

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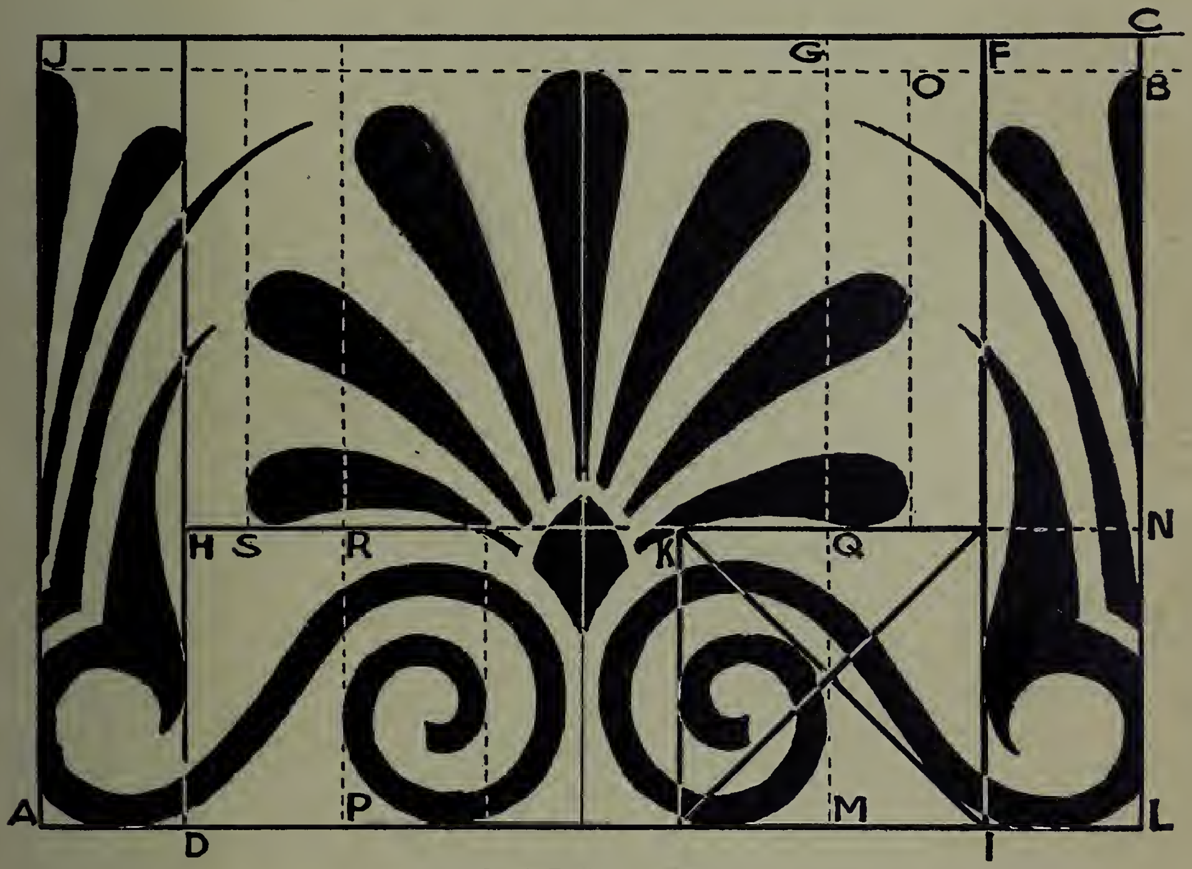
THE DIAGONAL

AN ILLUSTRATED MONTHLY MAGAZINE DEVOTED TO THE EXPLANATION OF THE REDISCOVERED PRINCIPLES OF GREEK DESIGN, THEIR APPEARANCE IN NATURE AND THEIR APPLICATION TO THE NEEDS OF MODERN ART

EDITED BY JAY HAMBIDGE



Greek Design Patterns

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THE DIAGONAL

THE END OF THE DIAGONAL

IT has been decided to make this, the twelfth number of THE DIAGONAL, the last, and hereafter to publish the twelve numbers in one volume.

The reason for this is that archaeological research can be of use to the artist only to a limited extent, and it is believed that enough of this sort of investigation, for the present purpose, has been done. Dr. L. D. Caskey, Curator of the Greek Department of the Boston Museum of Fine Arts, has completed his examination of the classic pottery in his care and his book on "The Geometry of Greek Vases" is now in press. It is felt that Dr. Caskey's work is a sufficient verification of the dynamic theory as far as it applies to architectural pottery.

Also the writer's work in Greece during the past year has resulted in securing overwhelming evidence of the use of Dynamic Symmetry in classic temple planning. The results of this work are now being put into shape for early publication.

It may not be inappropriate to mention now that the writer's interest lies entirely in the field of creative effort, and that research has been incidental only to his general aim. He was impelled to take up the study of symmetry because he could not entirely agree with the modern tendency to regard design as purely instinctive. As the trend of the individual and of society seems to be toward an advance from feeling to intelligence, from instinct to reason, so the art effort of man must lead to a like goal. The world cannot always regard the artist as a mere medium who reacts blindly, unintelligently, to a productive yearning. There must come a time when instinct will work with, but be subservient to, intelligence. There have been such periods in the past, notably that of classic Greece. The Greeks were by no means perfect but, for at least a generation in their history, they had a clear understanding of law and order and a passion for its enforcement. Life to them seemed a sublime game, but one meaningless without rules. Indeed, this is the distinction between the Greek and the barbarian.

The Christians struggled for a moral, the Greeks for an intellectual, law. The two ideals must be united to secure the greatest good. As moral law without intellectual direction fails, ends in intolerance, so instinctive art without mental control is bound to fail, to end in incoherence. In art the control of reason means the rule of design. Without reason art becomes chaotic. Instinct and feeling must be directed by knowledge and judgment. It is impossible to correlate our artistic efforts with the phenomena of life without knowledge of life's processes. Without mental control, instinct, or feeling, compels the artist to follow nature as a slave a master. He can direct his artistic fate only by learning nature's ideal and going directly for that as a goal.

The present need is for an exposition of the application of Dynamic Symmetry to the problems of today. The indications are that we stand on the threshold of a design awakening. Returning consciousness is bound to be accompanied by dissatisfaction with the prevailing methods of appropriating the design efforts of the past. When it is realized that symmetry provides the means of ordering and correlating our design ideas, to the end that intelligent expression may be given to our dreams, we shall no longer tolerate pilfering. Instead of period furniture and antique junk we shall demand design expressive of ourselves and our own time. The oriental rug, the style house, the conscious and the artificial

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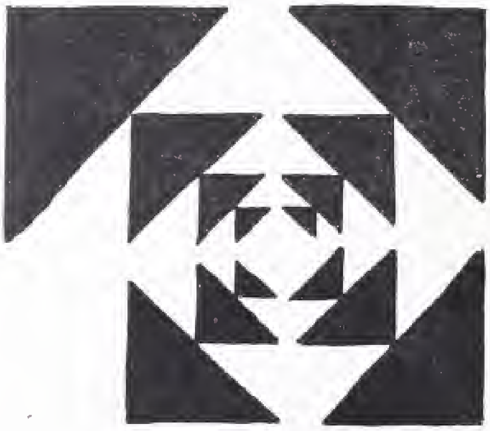
craft products, all echoes of other times and other peoples and sure evidence of design poverty, must give place to a healthy and natural expression of the aspirations of our own age.

But an exposition of the application of Dynamic Symmetry to the many different aspects of design is clearly beyond the province of THE DIAGONAL. This may be done best by illustrated lectures which will consider in practical detail the use of this symmetry in the different departments of design. Such lectures are now in preparation. They will be delivered to classes and made available to subscribers in printed form. The plan embraces symmetry application to architecture, domestic, civil and monumental; sculpture in the flat and in the round; painting, landscape, historical and portrait; bookmaking, printing and advertising; craftsmanship, pottery, metal work and weaving; interior decoration and furniture making; engineering design. For information about these courses write to Hambidge, 681 Fifth Avenue, New York City.



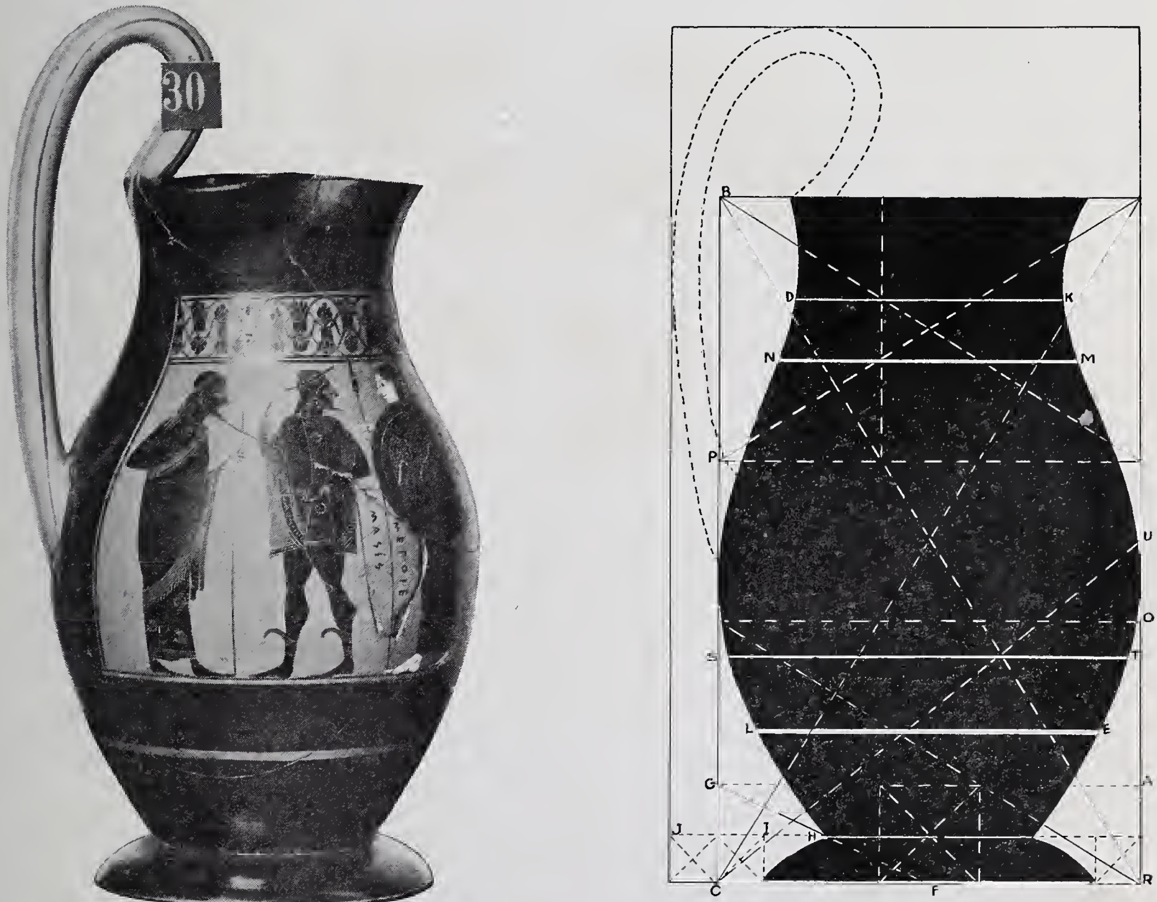
DID THE EGYPTIAN AMASIS TEACH THE GREEKS DYNAMIC PROPORTION?

ONE of the most interesting black-figured pottery designers of ancient Greece was Amasis, a man with an Egyptian name. We know nothing about him beyond the few signed examples of his work which have survived. There is one in the Louvre, one in the Bibliothèque Nationale at Paris, and two in the Museum of Fine Arts at Boston. From the style of the painting the date of this designer must be early, probably about 540 B. C.

Apart from its excellence of design this man's work is of interest to the modern artist because its symmetry is dynamic. Moreover its manipulation, in comparison with that of the Greeks themselves during this early period, shows a master mind. He antedates both Tleson and Nikosthenes, the two men whose work shows they began to design with static, and afterwards changed to dynamic, symmetry. Perhaps we have in Amasis one of the foreigners who brought knowledge of Dynamic Symmetry to Greece.

The example in the Louvre, F 30, is an exquisite jug, an olpe.

The proportion, without the handle, is 1.618, and with the handle 1.809.



(Photograph by Giraudon)

Oenochoe by Amasis in the Louvre and its proportions

The structural scheme is disclosed in Figure 1.

BR is the overall whirling square rectangle.

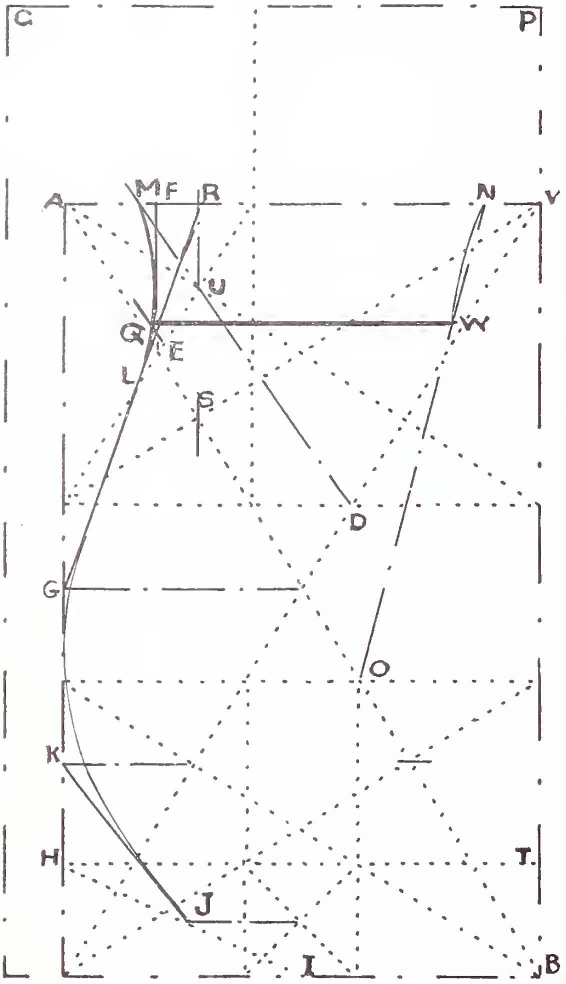
BO and RP are the major squares.

The foot details and the placing of the compositional bands DK, NM, LE and ST are clear. CU is a diagonal to half the shape BR.

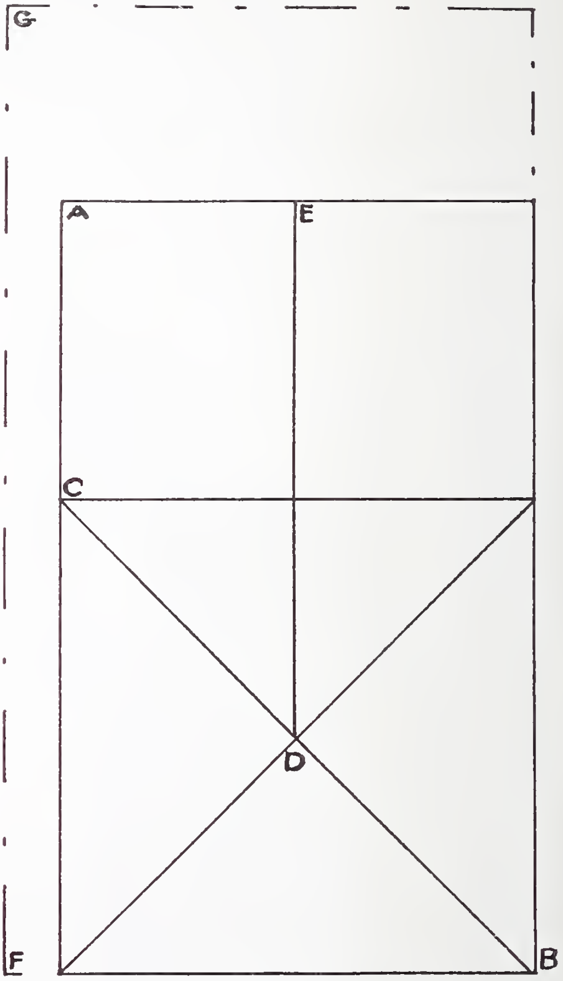
CA is a .236 strip; consequently AB is a 1.382 rectangle. (.236 from 1.618 leaves 1.382).

A line from A to B fixes the lengths of the lines DK and LE.

GF is half the .236 area.



The Curves



The Handle Proportions

In Figure 2*b* the method is shown for constructing the 1.809 rectangle GB, which includes the handle.

BC is a major square in the whirling square rectangle AB.

D is the center of this square.

Draw DE and make FB equal to it.

On FB draw the 1.809 rectangle BG.

THE CURVE TANGENTS.

Figure 2*a* shows the method of fixing the curves.

AB is the whirling square rectangle which incloses the projection of the jug, without its handle, and the dotted lines show its logical subdivisions.

The tangents are JK, KG and FE. The curve crosses the line GR.

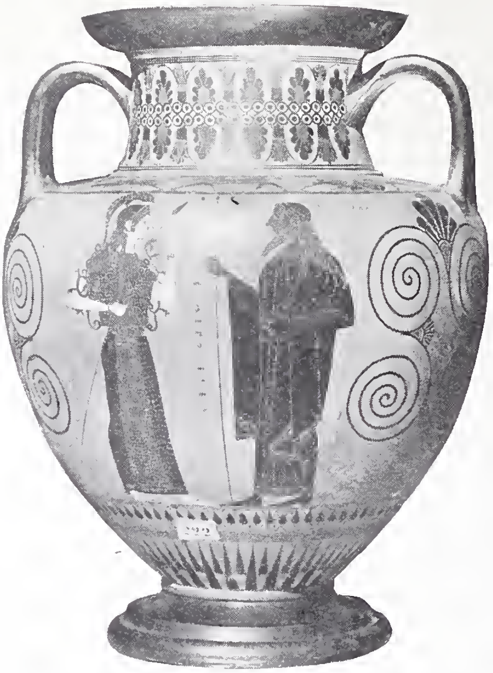
Lines from A to T and from V to H fix the length of the line QW.

From O, through W, draw ON.

From D, through U, draw DM.

MN is the lip width of the side elevation.

The error throughout is negligible.



(Photograph by Giraudon)

STRUCTURAL SCHEME FOR AN AMASIS AMPHORA

THE structural plan for the Amasis amphora in the Bibliothèque Nationale at Paris is shown in Figure 1*a*.

The overall area is 1.309 and AB is the major square of this shape.

AC is one of the excess whirling square rectangles.

The width of the foot equals the end of a root-five rectangle in the center of the 1.309 area. DE is this.

IC is one-quarter of the overall area, and its diagonal cuts a diagonal of the whole at M.

A diagonal of the whirling square rectangle AC cuts a diagonal of the whole at K and fixes the width of the lip. Draw WX. This meets a line through M at N and fixes the height of the lip.

DF is composed of two squares, or root four, in the root-five rectangle DE.

GF is a .236 area. The details of the foot as correlated by this shape are clear.

THE CURVE TANGENTS.

Figure 1*b* shows the curve tangents and other structural points.

To place a root-five rectangle in the center of a 1.309 area first draw a diagonal to the major square, as AB, Figure 1*b*. Then a diagonal to half the major area, as DE. These two diagonals intersect at F. A line through this point, parallel to a side, fixes one side of the root-five shape.

C is the intersection of a diagonal to the whole and one to the major square. It fixes the point U. Draw a line from T to U. The point T is a corner of the two squares in the root-five area YZ.

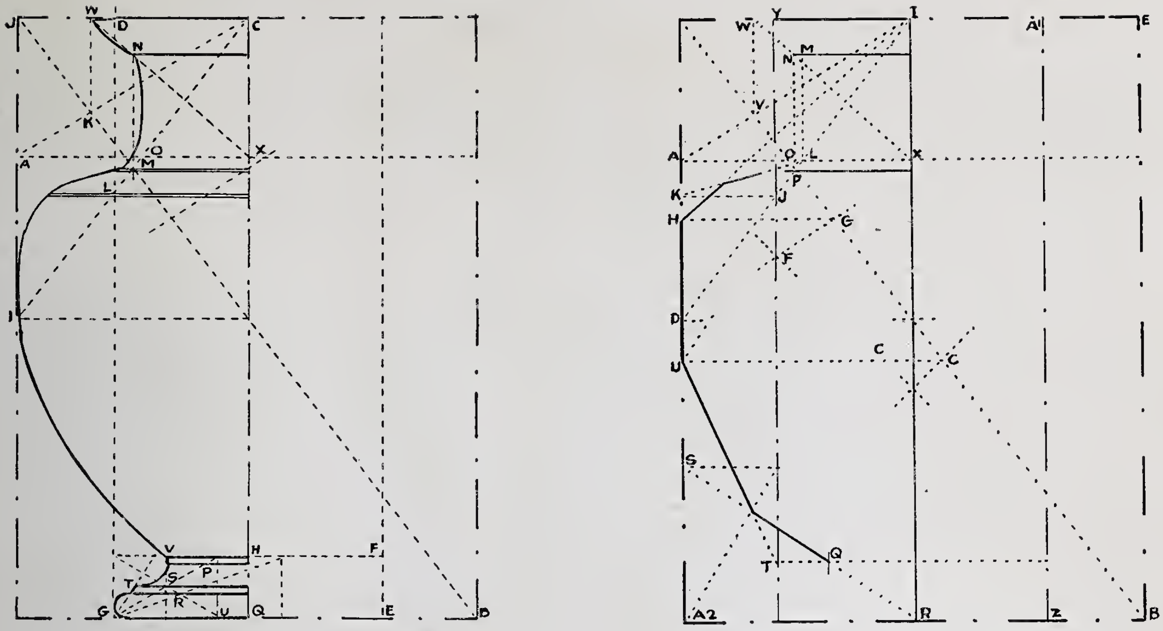


Fig. 1*a* Fig. 1*b*

Amasis Amphora in the Bibliotheque Nationale, Paris

A line from A1 to A2 fixes S. Draw RS.

A diagonal of the whole and a diagonal of the half intersect at G and fix H. Draw a line from H to I.

A line from D to I cuts YT at J and fixes K. Draw KL.

The neck curve is tangent to ML and lies between this line and the line ON.

AMASIS VASES IN AMERICA

DR. CASKEY has kindly consented to the publication here of the following notes on the two Amasis examples in the collection of the Boston Museum of Fine Arts. They are from his book on "The Geometry of Greek Vases."

"Neck Amphora. Inv. 01.8026. (*A*) Two warriors. (*B*) Athena in conversation with a draped bearded figure. Signed by Amasis as potter.

Height, 0.258 m. Diameter, 0.1865 m.

These dimensions give very accurately the ratio 1.382. This may be divided into a square and a .382 rectangle, or into two .691 rectangles, each composed of a square and a root-five rectangle. If a square be applied at the bottom of the containing area, its top coincides with a painted band below the shoulder. The intersection of the diagonal of the whole shape with the diagonal of a square applied at the end of the .382 rectangle determines the diameter of the lip. The diameter of the bottom of the body is one-third of the total diameter. Most of the other proportions are obtained from subdivisions of the root-five rectangles applied to the two .691 rectangles which make up the whole area.

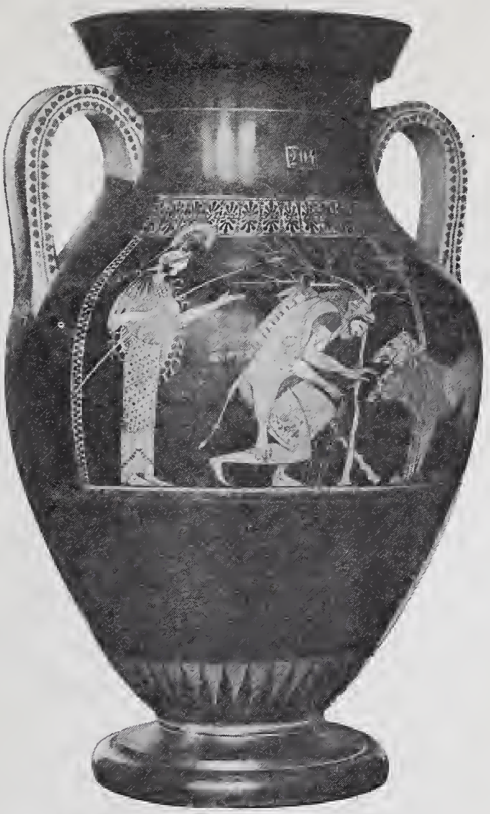
Height	1.382	Diameter of shoulder	.5528
Height to shoulder	1.0202	Diameter of bottom of body	.333
Height of neck and lip	.3618	Diameter of top of foot	.500
Diameter	1.000	Greatest diameter of foot	.5528
Diameter of lip	.679		

"Neck Amphora. Inv. 01.8027. (*A*) Hauser, Jahreshefte, 1907, pp. 1 ff., pl. I-IV. Herakles and Apollo fighting for the tripod in the presence of Hermes. (*B*) Thetis bringing the armor to Achilles in the presence of Phœnix. Signed by Amasis as potter.

Height, as restored, 0.306 m. (given by Hauser as 0.31 m.). Diameter, 0.213, 0.215 m. The exact height is uncertain, since a small portion of the vase near the bottom of the body is apparently lacking. The restoration cannot, however, have changed the height more than 1 or 2 mm. The ratio is assumed to be 1.4472. If the diameter be taken as 0.213 m., the height to give this ratio should be 0.30825 m. With these main dimensions the ratios of details work out very accurately. The height of the neck and lip is .382, the height to the shoulder, 1.0652 (.618 + .4472). The diameter of the lip equals that of the foot; both have the ratio .6584. The diameter of the shoulder is .500. The diameter of the bottom of the body is .382. Various other details are simply obtained by geometrical construction, as appears in the drawing."

From a cup by Theozotus in the Louvre

ANDOKIDES, A GREEK MASTER DESIGNER



(Photograph by Giraudon)

IN the Louvre in Paris there are two great monumental amphorae by the Greek designer Andokides. The paintings on them in black, white and red figures show them to be examples of the transition in these styles.

Probably the handsomer of the two has a height of nearly twenty-three inches. This is F 204 of the Louvre collection. The ratio of the plan for this vase is 1.559, a square plus four root-five rectangles. (See lesson on the elements in this number of THE DIAGONAL.)

Figure 1*a* shows the plan in its first stages, 1*b* the structural proportions, while Figure 2 gives the curve tangents.

In Figure 1*a* AB is a square, as is also DC.

AC and BD are each composed of four root-five rectangles.

AJ is composed of the four squares of the four root-five areas.

HC is a whirling square rectangle. (It is suggested that the student figure out why EC and FB are each composed of two whirling square rectangles.)

In Figure 1*b* the proportions of the foot are fixed by the .382 area CB. Half the width of the foot equals a side of the square AB.

The point G is fixed by a line from D to F produced to the diagonal of the square AB.

JK is an equal and similar figure to BP.

The point I, fixing the thickness of the lip, is an intersection of two diagonals, one to JK and one to half that area.

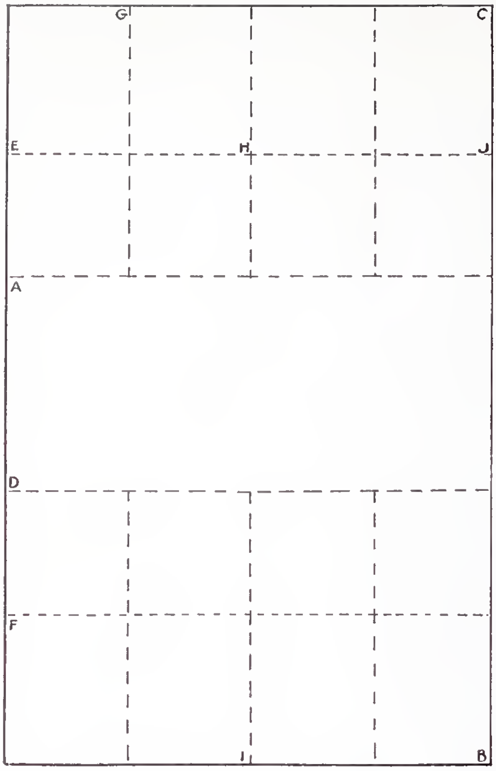


Fig. 1*a*

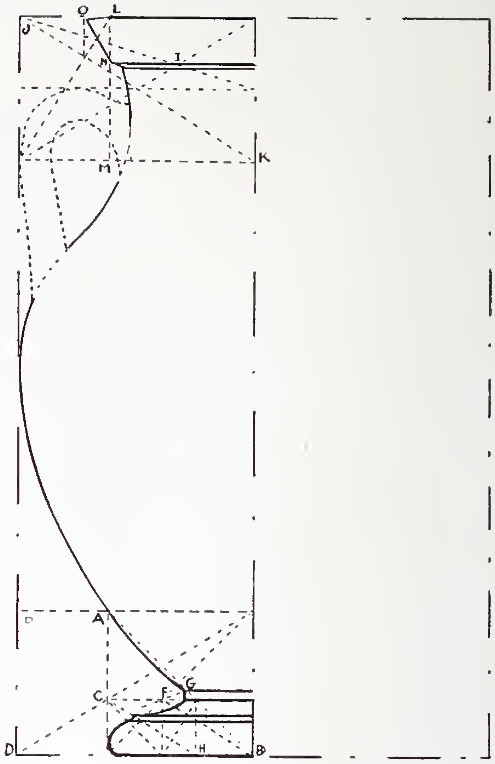


Fig. 1*b*

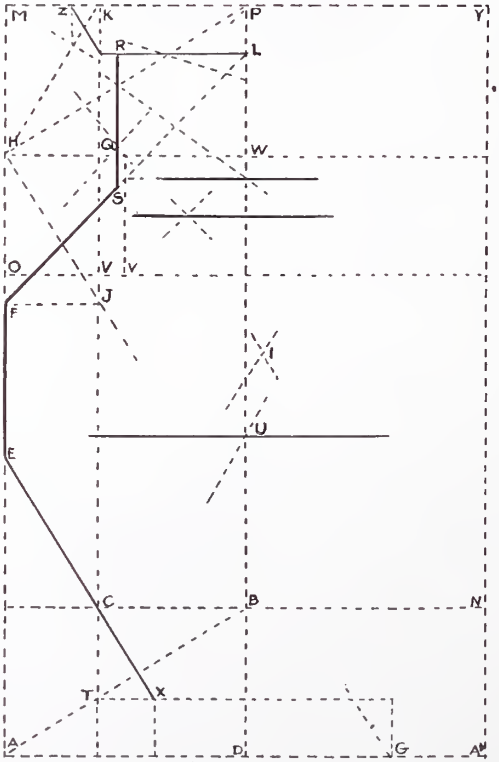


Fig. 2—How to find the curve tangents

For the curve tangents of Figure 2 we first draw a line from X through C to E.

X is a corner of a square in the .382 area of the whirling square area AB, and C is a corner of the major square in this shape.

Draw a line from H to G. The line KVC is a side of the square KW produced. It cuts the line HG at J. This fixes the point F.

L is the middle of the base of the lip. Draw FL.

Draw a line from O to P. This cuts a line from M to N at Q.

Through Q draw RS. These four lines RS, SF, FE and EX are the tangents to the curves.

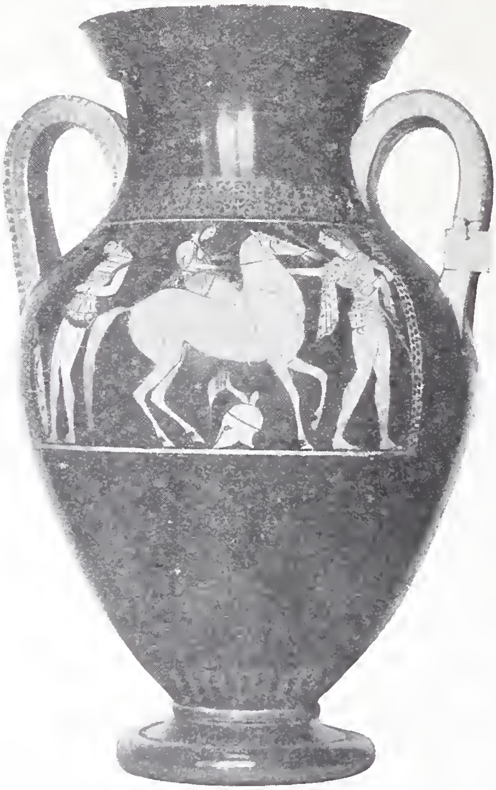
Draw a diagonal line to the whole, as AY. This cuts a line from Z to A1 at I, to fix the point where the curve touches the tangent FE. This gives the point of greatest width for the bowl.

VW is a square of one of the root-five rectangles. A line through its center fixes the top of the picture of Hercules and the double-headed dog, Cerberus.

The line FL cuts a side of this square for the top of the decorative band.

A line from T to Y fixes the point U, the base of the picture.

This example is one of the very few Greek vases where the handles are exactly the width of the bowl.



(Photograph by Giraudon)

ANDOKIDES CONTINUED

THE second Andokides amphora in the Louvre, F 203, was measured by Mlle. Everard. These measurements are:

Full height: .403, .403, .402, .401, .401, .402, .404.
Width of bowl: .248; in one place .252.
Width of lip: .175, .180, .178, .179.
Width of foot: .142, .143, .143, .142.
Width of neck at narrowest point: .130.

Width at B: .155, .153, .153, .155.
Width at D: .082.
Width at C: .082.
Width at R: .120.

Height at C: .035, .040, .040, .040, .035.
Height at D: .030, .035, .034, .037, .032, .034, .032.
Height at R: .021, .024, .021, .024, .023, .021, .021.

Assuming that the full true height is .402 and the width .2484, we obtain a 1.618 rectangle by dividing the lesser into the greater.

The full height, .402, divided by the width of the lip, .1797, results in 2.236, or the square root of five.

The full height, .402, divided by .1431, the width of the foot, produces 2.809.

The full height, .402, divided by .130, the narrowest neck width, results in 3.090, or .618 multiplied by five.

The handle width, with the full height, produces an area of 1.5 or a square and a half. The details of the plan, however, are not correlated by this area.

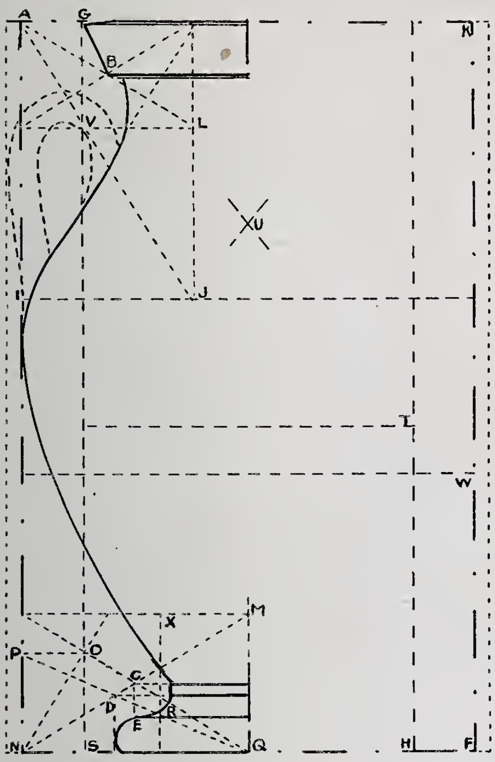


Fig. 1*a*

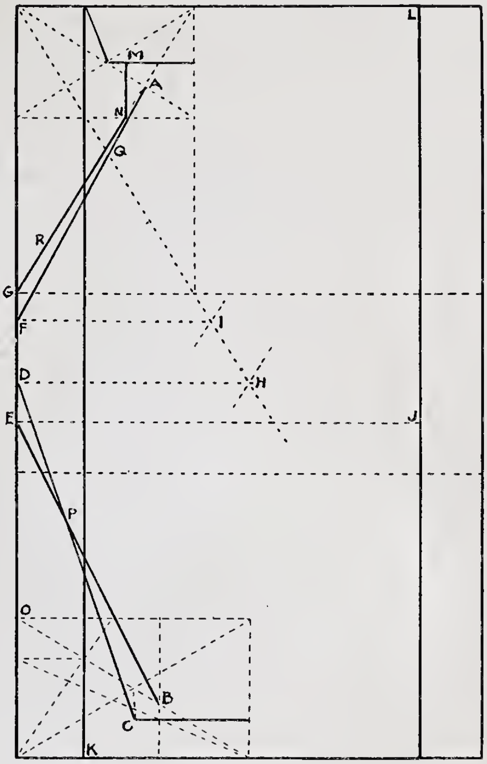


Fig. 1*b*

The construction of the vase from its plan is shown in Figure 1*a*.

IF is a square and JK, IL and LG are squares.

In the whirling square rectangle AL, B is the center.

The line GS is drawn through V. This line enables us to fix the root-five area GH.

The whirling square rectangle NM is one-fourth of the similar area NW.

O is an eye of the NM area, and the line OP enables us to fix the root-five area PQ.

C is the center of NM, and PQ cuts NM at D. Also NX is a square in NM, and a side of this cuts PQ at R. This determines the height of the foot, while D fixes the width.

In the root-five rectangle GH, the square is ST. The point U is the center of the two whirling square rectangles TG. This point marks the top of the panel containing the pictorial composition. The base of this panel is fixed by T.

THE CURVE TANGENTS.

Figure 1*b* shows the points for fixing the curve tangents.

KJ is the square of the root-five area KL, and EJ is the produced side of this square. Draw BE.

H is the center of the major whirling square area and fixes the point D. Draw CD. This is the point E of Figure 1*a*.

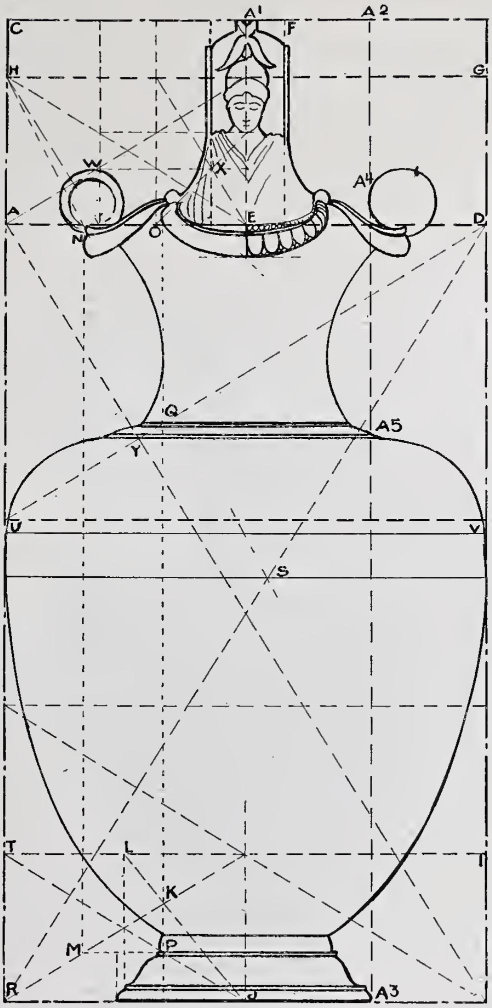
Draw a line from O to L. It cuts a diagonal to the whole at I and establishes F. Draw AF.

G is the base of a whirling square rectangle. Draw AG.

Through N draw MN.

The two lines CD and BE cross each other at P, and PD and PB are the tangent lines.

The neck curve touches NM, crosses NG and touches AF at Q. It touches AG at R and GD at F.



A REFINED GRECIAN JUG

GREEK bronze oenochoe 99.481 in the Museum of Fine Arts, Boston, was measured and drawn by Dr. L. D. Caskey, Curator of the Classic Department there. The above is a tracing of that drawing.

The overall ratio is 2.045, an area composed of 1.618 plus .427. AB is the 1.618 rectangle and CD that of .427.

RI and AG are .309 shapes. CG is .118.

The whirling square rectangle HE is subdivided to show the principal proportions of the figure on the handle.

The similar rectangle TJ fixes the proportions of the foot.

LJ equals one-half of TJ. The point K is determined by the intersection of the diagonals to LJ and TJ.

MN and PO make clear the proportional relationship between the foot and the neck.

Because AG equals RI the area HI is a 1.618 rectangle.

A diagonal to HB fixes the point S, the bottom line of the decoration on the bowl. The points Q and Y are apparent.

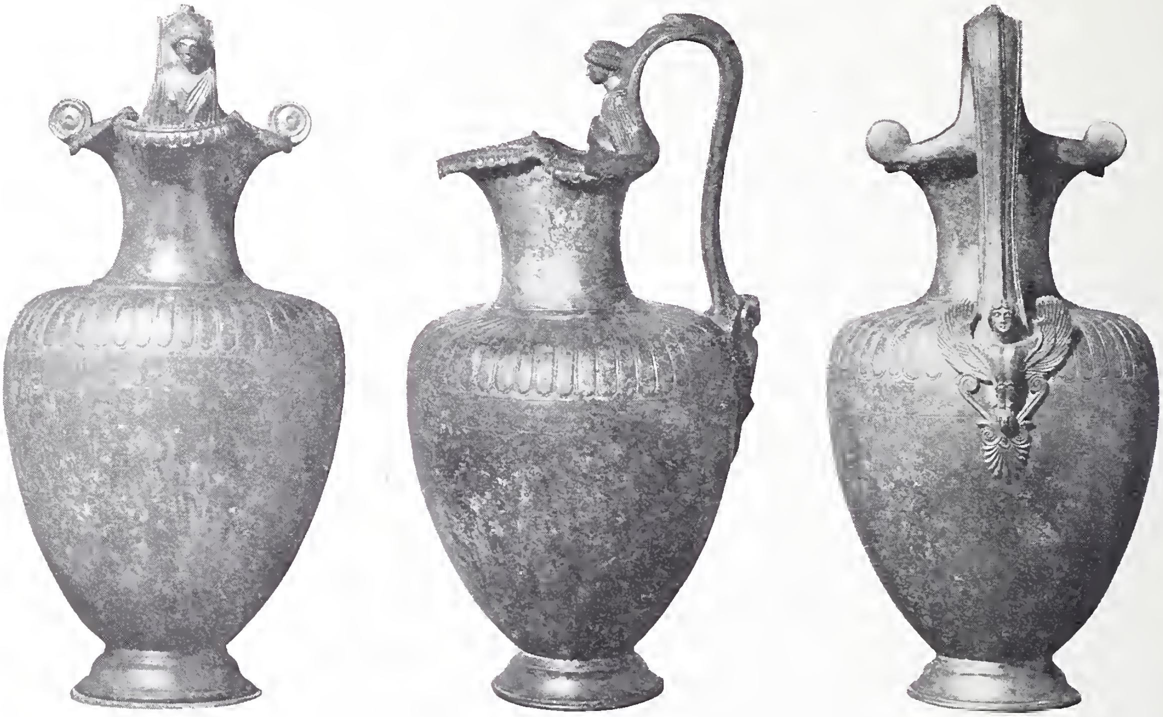
The full width of the foot fixes two whirling square rectangles in the area CD. The line A2 and A3 shows the method.

The points A4 and A5 are now clear. EA2 is a whirling square rectangle.

WX fixes the height of the circular disks on the lip.

The three photographs show the front, side and rear elevations.

These are all proportional to the main theme, but lack of space prevents publication of drawings showing this.



THE ELEMENTS OF DYNAMIC SYMMETRY

TO transfer a complement, the width of which is less than one-half the major square: Construct a square, the sides of which are equal to the width of the complement, and produce one side to the opposite side of the major square.

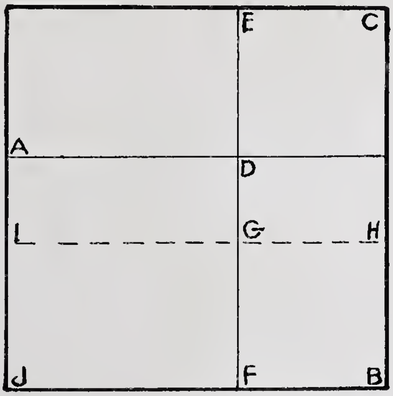


Fig. 1

AB is any rectangle; AC is its complement; CD is a square in the complement.

ED, a side of this square, is produced to F, and FG is made equal to DE. Draw HI through G, parallel to JB. The area IB is the transferred complement AC.

Suppose AB is a whirling square rectangle in the square JC. The complement AC is a .382 rectangle, *i.e.,* a 2.618 area.

AE is a whirling square rectangle, as is also DB, and CD and DJ are squares.

If IB and AC are whirling square rectangles, the area AH is .236, *i.e.,* a 4.236 shape.

Artists will frequently find it necessary in their practice to construct different rectangles in single and multiple form within the areas of their designs.

The process for doing this is very simple.

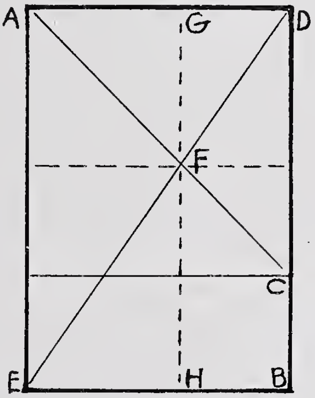


Fig. 2

Let AB, Figure 2, be a rectangle, say, 1.4472—a square plus a root-five shape.

Suppose we wish to apply to a side of this selected shape a 2.4472 area.

Apply a square on an end, and let AC be its diagonal.

ED is a diagonal to the major area, and the two diagonals cut each other at F.

Through F draw GH, parallel to AE or DB.

AH is a 2.4472 rectangle. So also is GC.

In finishing this series of lessons on the elements of Dynamic Symmetry it may not be out of place to give some attention to certain areas which the Greeks seem to have regarded as important, judging from what has been found in their buildings.

One of the most important ratios of the Parthenon exists in the cella, the room which contained the Phidian statue of the goddess Athena. This is .236, or the area which represents the difference between root five and root four.

.236 is the reciprocal of 4.236, *i.e.,* 2.236 (root five) plus 2 (root four). Hence, we see, it is an area composed of similar figures to its parent shapes.

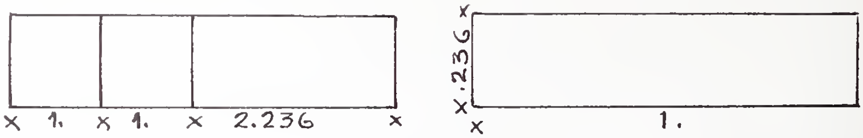
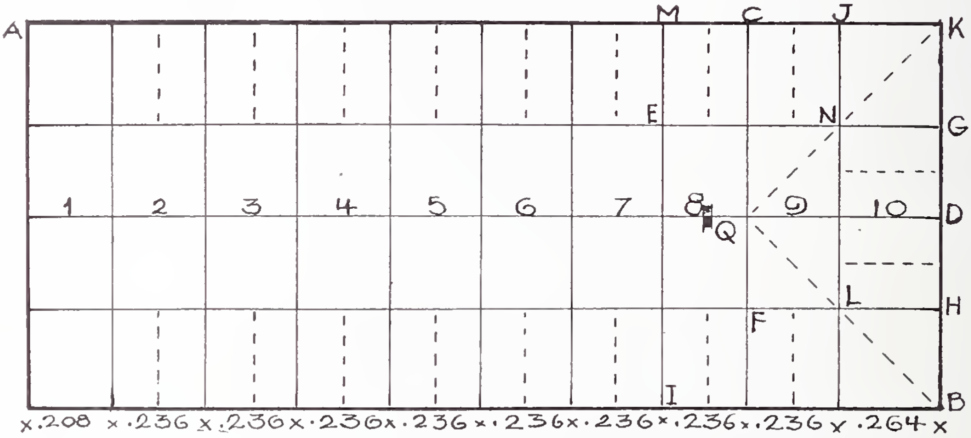


Fig. 3

The diagram, Figure 3, will make this clear.

The .236 rectangle is used in the Parthenon as a 2.36 area, that is, .236 multiplied by ten. The statue of the goddess was centered at a certain point in this area. The sacred inclosure was defined by the centers of columns and each intercolumniation fixed a .236 rectangle, except the centers of the first columns to the east and to the west. Although one of these odd areas is greater and one less than .236, the two, added, equal .472, or .236 multiplied by two. In substance the 2.36 area is divided into ten slices; eight of these are .236 rectangles and two of them, added, is a .472 area. This .472 shape is divided into two parts, one being represented by .264 and the other by .208.


A
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x.208 x.236 x.236 x.236 x.236 x.236 x.236 x.236 x.236 x .264 x

Fig. 4

In Figure 4 the .208 shape is marked 1 and the .264 marked 10. Areas 2, 3, 4, 5, 6, 7 and 8 are .236 rectangles.

The .236 rectangles are arranged with the root-four areas in the center and a half of a root-five rectangle on either end, as EF of CI. These two squares are fixed by the column centers GDH on the end of the rectangle.

It will be noticed that the area EL is a square composed of four smaller squares.

The area EJ is composed of four root-five rectangles, as is also the area LI.

And the area LG is composed of four root-five rectangles.

We now have the area EL, a square, banked on its three sides with twelve root-five rectangles.

The area ML is composed of a square plus four root-five shapes, as is also EH or NI.

It is easy to see from this arrangement that the abnormal strip JB, a .264 area, is composed of two squares, JG and LB, plus four root-five rectangles NH, but, as distinguished from the normal .236 strips, the root-five areas are in the center and the squares are on the ends of the shape. CI, for example, is a sort of reverse arrangement from this.

It is obvious in the Figure 4 plan that the variation between the two strips JB and CI must be connected with the angle column adjustment. The distance from J to K is greater than it is from J to C, *i.e.,* the angle columns are not so close to their neighbors as are the other columns.

This inclosure formed by the columns around the immediate precinct of the sacred statue is but a part of the great cella room. The areas EH, ML and NI are each composed of a square plus four root-five rectangles. Four such root rectangles in this position would have the reciprocal ratio .559, or one-fourth of 2.236, and the entire area of EH, ML or NI would be represented by the ratio 1.559. The entire area of the cella is a similar figure to the areas just mentioned. The ratio 1.559 expresses absolutely the symmetry of the most sacred part of the Parthenon. It is a theme in square and root five.

.264 is the reciprocal of 3.7888, or two squares plus four root-five rectangles (.4472 x 4), or 1.788 plus 2.

.208 is the reciprocal of 4.7888, or three squares plus four root-five rectangles.

N. B. The .264 area has a .236 strip immediately adjoining it. .264 plus .236 equals .5. Therefore the area CB must be composed of two squares, as .5 is the reciprocal of 2.

The point Q was the center of the great Athena statue and now is a rectangular hole in the cella floor.

But the hole Q is also in the center of the area CI; *i.e.,* its center is midway between C and M, and as CM equals .236, one-half of it is .118.

But we have seen that CB equals .5. If .118 be added to .5 the result is .618. That is to say, the center of the Phidian masterpiece, the most important cult statue of the Attic world, was placed on the side of a whirling square rectangle derived from the sacred precinct we have been describing.

The Parthenon is much this sort of thing from beginning to end—a carefully themed arrangement of symmetry capable of precise proof, a grand, a noble, plan, probably the most intelligent plan ever evolved by man.

Another important ratio, found in the Parthenon and elsewhere, is 1.528. This area is equal to two 1.309 rectangles standing side by side. The reciprocal of 1.309 is .764. This, multiplied by two, equals 1.528. The fraction is a little less than .528, more nearly .5278 plus. The relation of the 1.528 rectangle to the pentagonal division of the square is shown in Figure 5.

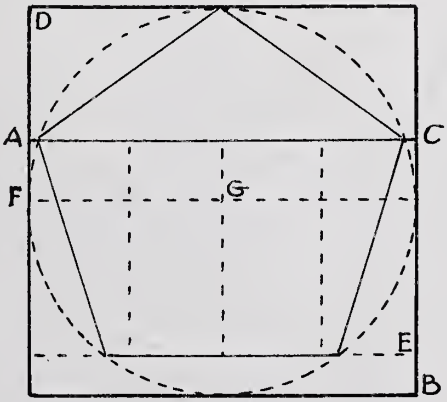

Fig. 5

Inscribe a circle in a square and a regular pentagon in the circle.

Draw the line AC through two angles of the pentagon, the length equal to a side of the square.

The area AB is a 1.528 rectangle.

The area CD is a .3455 shape. This is the reciprocal of 2.8944, two squares and two root-five rectangles.

The area AE is composed of four root-five rectangles.

If G is the center of the major square, then the area FE is composed of four whirling square rectangles and FC is four whirling square reciprocals.

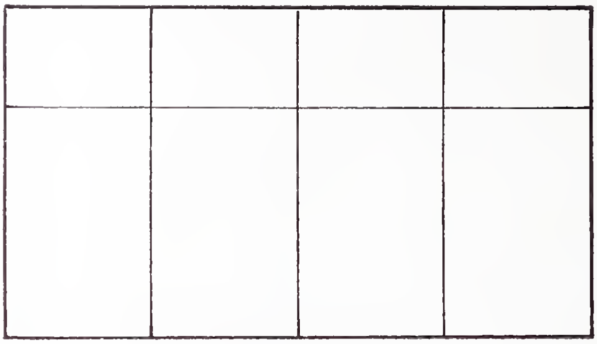

Fig. 6

The entire area AE of Figure 5 may be subdivided, as Figure 6, into eight whirling square rectangles.

In the Parthenon plan the 1.528 rectangle is divided into twelve strips by twelve intercolumniations of the peristyle columns. Twelve divided into 1.528 gives us twelve reciprocal ratios of .1273 plus. Each one of these represents the ratio 7.854 plus, *i.e.,* six squares and three whirling square rectangles. Three times .618 equals 1.854.

In the early temple at Aegina the 1.528 rectangle is used in its reciprocal form, that is, .6545. Three of these rectangles fix the general proportional scheme.

The fraction .528 is the reciprocal of 1.8944, or a square and two root-five rectangles.

[The End.]

From a garment on a statue in the Acropolis Museum, Athens

STATEMENT OF THE OWNERSHIP, MANAGEMENT, CIRCULATION, ETC., REQUIRED BY THE ACT OF CONGRESS OF AUGUST 24, 1912,

Of The Diagonal, published monthly, at New Haven, Conn., for October 1, 1920, State of Connecticut, County of New Haven, ss.

Before me, a Notary Public, in and for the State and county aforesaid, personally appeared George Parmly Day, Treasurer, who, having been duly sworn according to law, deposes and says that he is the publisher of The Diagonal, and that the following is, to the best of his knowledge and belief, a true statement of the ownership, management (and if a daily paper, the circulation), etc., of the aforesaid publication for the date shown in the above caption, required by the Act of August 24, 1912, embodied in section 443, Postal Laws and Regulations, printed on the reverse of this form, to wit:

1. That the names and addresses of the publisher, editor, managing editor, and business managers are:

Publisher, Yale University Press, 143 Elm St., New Haven, Conn.; Editor, Jay Hambidge, care of Morgan Harjes, Paris, France; Managing Editor, None; Business Managers, None.

2. That the owners are: (Give names and addresses of individual owners, or, if a corporation, give its name and the names and addresses of stockholders owning or holding 1 per cent or more of the total amount of stock.) Yale University Press, New Haven, Conn.; Yale University, New Haven, Conn.; George Parmly Day, New Haven, Conn.

3. That the known bondholders, mortgagees, and other security holders owning or holding 1 per cent or more of total amount of bonds, mortgages, or other securities are: (If there are none, so state.) Yale University Press, New Haven, Conn.; Yale University, New Haven, Conn.; George Parmly Day, New Haven, Conn.

4. That the two paragraphs next above, giving the names of the owners, stockholders, and security holders, if any, contain not only the list of stockholders and security holders as they appear upon the books of the company but also, in cases where the stockholder or security holder appears upon the books of the company as trustee or in any other fiduciary relation, the name of the person or corporation for whom such trustee is acting, is given; also that the said two paragraphs contain statements embracing affiant's full knowledge and belief as to the circumstances and conditions under which stockholders and security holders who do not appear upon the books of the company as trustees, hold stock and securities in a capacity other than that of a bona fide owner; and this affiant has no reason to believe that any other person, association, or corporation has any interest direct or indirect in the said stock, bonds, or other securities than as so stated by him.

5. That the average number of copies of each issue of this publication sold or distributed, through the mails or otherwise, to paid subscribers during the six months preceding the date shown above is . (This information is required from daily publications only.)

GEORGE PARMLY DAY, Treasurer, Yale University Press.

Sworn to and subscribed before me this 21st day of October, 1920.

L. PHELPS SOULE.
(My commission expires February 1, 1925.)

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